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An application for Guatemalan Quetzal to US Dollar

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Abstract. In this paper, we contribute to the body of literature of Dynamic Conditional Score (DCS) models as follows. We introduce the dynamic Skew-Gen- t (skewed generalized t distribution) and the dynamic NIG (Normal-Inverse Gaussian distribution) location models, which are alternatives to the dynamic Student's- t and the dynamic EGB2 (Exponential Generalized Beta distribution of the second kind) location models, respectively. The DCS models of this paper include the stochastic local level component, the stochastic seasonality component, and the irregular component with DCS-EGARCH (exponential generalized autoregressive conditional heteroscedasticity) scale dynamics. We show that the Skew-Gen- t location model performs a smooth form of trimming of extreme observations, similar to the Student's- t location model. We also show that the NIG location model performs a smooth form of Winsorizing of extreme observations, similar to the EGB2 location model. As an illustration, we use data from the Guatemalan Quetzal (GTQ) to United States Dollar (USD) exchange rate for period 4 January 1994 to 30 June 2017. The use of these data is interesting for the new DCS models, since: (i) GTQ/USD has significant jumps and falls for the data window; (ii) GTQ/USD has a stochastic annual seasonality component; (iii) GTQ/USD has significant volatility dynamics. Our main results are the following: (i) The statistical performance of Skew-Gen- t -DCS is superior to that of Student's- t -DCS. (ii) The statistical performance of NIG-DCS is superior to that of EGB2-DCS. (iii) For the new DCS models, we find that Skew-Gen- t -DCS discounts extreme observations more effectively than NIG-DCS.

Keywords: Dynamic Conditional Score (DCS) models, trimming, Winsorizing, stochastic seasonality component, Guatemalan Quetzal (GTQ) to United States Dollar (USD) exchange rate

JEL Classification: C22, C52, C58, F31

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1. Introduction

Harvey (2013, Chapter 3.6) and Harvey and Luati (2014) introduce the dynamic Student's- t location model that includes both stochastic local level and stochastic seasonality components. The Student's- t location model is in the family of Dynamic Conditional Score (DCS) models (Harvey 2013). For DCS models, each dynamic equation is updated by the conditional score of the Log-Likelihood (LL) (hereinafter, score function) with respect to a time-varying parameter. The score function discounts extreme observations, by reducing the effect of the irregular time-series component v_t when dynamic equations are updated. Therefore, DCS models are robust to extreme observations (Harvey 2013).

More recently, Caivano et al. (2016) introduce the dynamic EGB2 (Exponential Generalized Beta distribution of the second kind) location model, which also includes stochastic local level and stochastic seasonality components. Caivano et al. (2016) compare the dynamic Student's- t location and the dynamic EGB2 location models, and demonstrate that extreme observations are discounted in different ways in those models. For the Student's- t location model, the score function converges to zero as $|v_t| \rightarrow \infty$; this type of discounting of extreme observations is described as a soft form of trimming. For the EGB2 location model, the score function converges to a positive or negative non-zero value as $|v_t| \rightarrow \infty$; this type of discounting of extreme observations is described as a soft form of Winsorizing. As to which type of discounting is more effective, it is an open question in the body of the relevant DCS literature (Harvey 2013, Chapter 3.6; Harvey and Luati 2014; Caivano et al. 2016).

In this paper, we contribute to the body of literature on the following points: (i) We introduce the dynamic Skew-Gen- t (skewed generalized t distribution) location model with stochastic local level and stochastic seasonality components, and show that this model performs a soft form of trimming, similar to the dynamic Student's- t location model. (ii) We introduce the dynamic NIG (Normal-Inverse Gaussian distribution) (Barndorff-Nielsen and Halgreen 1977) location model with stochastic local level and stochastic seasonality components, and show that this model performs a soft form of Winsorizing, similar to the dynamic EGB2 location model. (iii) We

use DCS-EGARCH (Exponential Generalized Autoregressive Conditional Heteroscedasticity) (Harvey 2013) scale dynamics for the irregular component in all models that include stochastic local level and stochastic seasonality components.

As an illustration, we use daily data from the Guatemalan Quetzal (GTQ) to United States Dollar (USD) exchange rate (hereinafter, GTQ/USD or p_t) for period 4 January 1994 to 30 June 2017. The use of these data is interesting for the new DCS models with stochastic local level, stochastic seasonality and EGARCH, since: (i) GTQ/USD has significant jumps and falls for the data window; (ii) GTQ/USD has a stochastic annual seasonality component; (iii) GTQ/USD has significant volatility dynamics. We compare the statistical performance of alternative DCS specifications, analyze the determinants of the stochastic seasonality of GTQ/USD, and compare the trimming and Winsorizing properties of different score functions.

Our main results are the following: (i) We find that the statistical performance of Skew-Gen- t -DCS is superior to that of t -DCS. (ii) We also find that the statistical performance of NIG-DCS is superior to that of EGB2-DCS. (iii) With respect to the new DCS models, we find that Skew-Gen- t -DCS discounts extreme observations more effectively than NIG-DCS.

The remainder of this paper is organized as follows. Section 2 presents the econometric framework. Section 3 presents the model specifications. Section 4 presents the corresponding methods of statistical inference. Section 5 presents the empirical results. Section 6 concludes.

2. Econometric framework

The DCS model used in this paper is formulated as: $p_t = \mu_t + s_t + v_t = \mu_t + s_t + \exp(\lambda_t)\epsilon_t$ for days $t = 1, \dots, T$, where T denotes the number of time-series observations. This model includes three score-driven components: the stochastic local level component μ_t , the stochastic annual seasonality component s_t , and the irregular component v_t . The irregular component is factorized to the product of the dynamic scale parameter $\exp(\lambda_t)$ and the standardized error term ϵ_t . For ϵ_t , we consider the Student's- t , Skew-Gen- t , EGB2 and NIG distributions (see Section 3).

Firstly, the local level component $\mu_t = \mu_{t-1} + \delta u_{\mu,t}$ is updated by the score function $u_{\mu,t}$ with respect to μ_t ($u_{\mu,t}$ is specified in Section 3), and $u_{\mu,t}$ is scaled by parameter δ . We initialize μ_t

by using the first observation p_1 . It is noteworthy that, as an alternative, we also consider the use of parameter μ_0 to initialize μ_t . We obtain very similar results for both cases, thus, in this paper we only report results for $\mu_1 = p_1$.

Secondly, the annual seasonality component is $s_t = D_t' \rho_t = (D_{\text{Jan},t}, D_{\text{Feb},t}, \dots, D_{\text{Dec},t})' \rho_t$, where the monthly dummies $D_{j,t}$ with $j \in \{\text{Jan}, \dots, \text{Dec}\}$ select an element from the 12×1 vector of dynamic variables ρ_t . The vector ρ_t is formulated as $\rho_t = \rho_{t-1} + \gamma_t u_{\mu,t}$.

As suggested in the works of Harvey (2013, Chapter 3.6) and Harvey and Luati (2014), we initialize ρ_t by estimating the equation $p_t = a + bt + c_{\text{Jan}} D_{\text{Jan},t} + \dots + c_{\text{Dec}} D_{\text{Dec},t} + \epsilon_t$, under the restriction $c_{\text{Jan}} + \dots + c_{\text{Dec}} = 0$. For this equation, we use data from the first year of the full data window (i.e. the first 259 observations of the sample from year 1994), and estimate the parameters by using the Non-linear Least Squares (NLS) method. The initial values of ρ_t are given by the NLS estimates of $(c_{\text{Jan}}, \dots, c_{\text{Dec}})'$. It is noteworthy that, as an alternative, Harvey (2013, Chapter 3.6) and Harvey and Luati (2014) also suggest estimating ρ_1 as parameters of the DCS model, jointly with the rest of the parameters. We aimed at estimating the parameters of initial conditions of seasonality in this way, but the parameters of the joint model were not identified for our dataset. Thus, we undertake the NLS estimation of ρ_1 in a first step.

The vector ρ_t is updated by the score function $u_{\mu,t}$ with respect to μ_t (Section 3), and $u_{\mu,t}$ is scaled by using the 12×1 vector of dynamic parameters γ_t . Each element of the γ_t vector is given by $\gamma_{jt} = \gamma_j$ for $D_{jt} = 1$ and $\gamma_{jt} = -\gamma_j / (12 - 1)$ for $D_{jt} = 0$, where γ_j with $j \in \{\text{Jan}, \dots, \text{Dec}\}$ are seasonality parameters to be estimated. This specification ensures that the sum of the seasonality parameters is zero, hence, s_t has mean zero and it is effectively separated from μ_t .

Thirdly, we model the time-varying scale of the irregular component v_t by using the DCS-EGARCH model $\lambda_t = \omega + \beta \lambda_{t-1} + \alpha u_{\lambda,t-1}$, which is updated by the score function $u_{\lambda,t}$ with respect to λ_t ($u_{\lambda,t}$ is specified in Section 3). We initialize λ_t by using parameter λ_0 . It is noteworthy that, as an alternative, we also consider DCS-EGARCH with leverage effects (Harvey 2013, Chapter 4.3). However, we find that the parameter measuring leverage effects is not significantly different from zero for the GTQ/USD dataset used in this paper. In the body of literature, DCS-

EGARCH for the Student's- t , Skew-Gen- t , EGB2 and NIG distributions is named as Beta- t -EGARCH (Harvey and Chakravarty 2008), Skew-Gen- t -EGARCH (Harvey and Lange 2017), EGB2-EGARCH (Caivano and Harvey 2014) and NIG-EGARCH. To the best of our knowledge, NIG-EGARCH has not yet been considered in the body of DCS literature.

3. Model specifications

We use four alternative probability distributions for ϵ_t . For each alternative, we present the log of the conditional density of p_t , and the score functions $u_{\mu,t}$ and $u_{\lambda,t}$. Firstly, $\epsilon_t \sim t[0, 1, \exp(\nu) + 2]$ is the Student's t -distribution, where $\nu \in \mathbb{R}$ influences tail-thickness. This specification of degrees of freedom ensures a finite conditional variance of p_t . The log-density of p_t is

$$\begin{aligned} \ln f(p_t|p_1, \dots, p_{t-1}) &= \ln \Gamma \left[\frac{\exp(\nu) + 3}{2} \right] - \ln \Gamma \left[\frac{\exp(\nu) + 2}{2} \right] \\ &\quad - \frac{\ln(\pi) + \ln[\exp(\nu) + 2]}{2} - \lambda_t - \frac{\exp(\nu) + 3}{2} \ln \left\{ 1 + \frac{\epsilon_t^2}{\exp(\nu) + 2} \right\} \end{aligned} \quad (1)$$

where $\Gamma(x)$ is the gamma function. The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t|p_1, \dots, p_{t-1})}{\partial \mu_t} = \frac{\exp(\lambda_t)\epsilon_t}{\epsilon_t^2 + \exp(\nu) + 2} \times \frac{\exp(\nu) + 3}{\exp(2\lambda_t)} = u_{\mu,t} \times \frac{\exp(\nu) + 3}{\exp(2\lambda_t)} \quad (2)$$

where $u_{\mu,t}$ is defined by the second equality. The score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1, \dots, p_{t-1})}{\partial \lambda_t} = \frac{[\exp(\nu) + 3]\epsilon_t^2}{\exp(\nu) + 2 + \epsilon_t^2} - 1 \quad (3)$$

Secondly, $\epsilon_t \sim \text{Skew-Gen-}t[0, 1, \tanh(\tau), \exp(\nu) + 2, \exp(\eta)]$ (McDonald and Michelfelder 2017), where $\tanh(x)$ is the hyperbolic tangent function, and $\tau \in \mathbb{R}$, $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ influence the asymmetry, tail-thickness and peakedness, respectively. Our degrees of freedom specification ensures a finite conditional variance of p_t . The log-density of p_t is

$$\ln f(p_t|p_1, \dots, p_{t-1}) = \eta - \lambda_t - \ln(2) - \frac{\ln[\exp(\nu) + 2]}{\exp(\eta)} - \ln \Gamma \left[\frac{\exp(\nu) + 2}{\exp(\eta)} \right] \quad (4)$$

$$\begin{aligned}
& -\ln \Gamma[\exp(-\eta)] + \ln \Gamma \left[\frac{\exp(\nu) + 3}{\exp(\eta)} \right] \\
& - \frac{\exp(\nu) + 3}{\exp(\eta)} \ln \left\{ 1 + \frac{|\epsilon_t|^{\exp(\eta)}}{[1 + \tanh(\tau) \operatorname{sgn}(\epsilon_t)]^{\exp(\eta)} \times [\exp(\nu) + 2]} \right\}
\end{aligned}$$

where $\operatorname{sgn}(x)$ is the signum function. The score function $u_{\mu,t}$ is given by

$$\begin{aligned}
\frac{\partial \ln f(p_t | p_1, \dots, p_{t-1})}{\partial \mu_t} &= \frac{\exp(\lambda_t) \epsilon_t |\epsilon_t|^{\exp(\eta)-2}}{|\epsilon_t|^{\exp(\eta)} + [1 + \tanh(\tau) \operatorname{sgn}(\epsilon_t)]^{\exp(\eta)} [\exp(\nu) + 2]} \times \frac{\exp(\nu) + 3}{\exp(2\lambda_t)} \quad (5) \\
&= u_{\mu,t} \times \frac{\exp(\nu) + 3}{\exp(2\lambda_t)}
\end{aligned}$$

where $u_{\mu,t}$ is defined by the second equality. The score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t | p_1, \dots, p_{t-1})}{\partial \lambda_t} = \frac{|\epsilon_t|^{\exp(\eta)} [\exp(\nu) + 3]}{|\epsilon_t|^{\exp(\eta)} + [1 + \tanh(\tau) \operatorname{sgn}(\epsilon_t)]^{\exp(\eta)} [\exp(\nu) + 2]} - 1 \quad (6)$$

Thirdly, $\epsilon_t \sim \text{EGB2}[0, 1, \exp(\xi), \exp(\zeta)]$, where $\xi \in \mathbb{R}$ and $\zeta \in \mathbb{R}$ influence both asymmetry and tail-thickness. The log-density of p_t is

$$\begin{aligned}
\ln f(p_t | p_1, \dots, p_{t-1}) &= \exp(\xi) \epsilon_t - \lambda_t - \ln \Gamma[\exp(\xi)] - \ln \Gamma[\exp(\zeta)] \quad (7) \\
&+ \ln \Gamma[\exp(\xi) + \exp(\zeta)] - [\exp(\xi) + \exp(\zeta)] \ln[1 + \exp(\epsilon_t)]
\end{aligned}$$

The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t | p_1, \dots, p_{t-1})}{\partial \mu_t} = u_{\mu,t} \times \{ \Psi^{(1)}[\exp(\xi)] + \Psi^{(1)}[\exp(\zeta)] \} \exp(2\lambda_t) \quad (8)$$

$$u_{\mu,t} = \{ \Psi^{(1)}[\exp(\xi)] + \Psi^{(1)}[\exp(\zeta)] \} \exp(\lambda_t) \left\{ [\exp(\xi) + \exp(\zeta)] \frac{\exp(\epsilon_t)}{\exp(\epsilon_t) + 1} - \exp(\xi) \right\} \quad (9)$$

where $\Psi^{(1)}(x)$ is the trigamma function. Furthermore, the score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t | p_1, \dots, p_{t-1})}{\partial \lambda_t} = [\exp(\xi) + \exp(\zeta)] \frac{\epsilon_t \exp(\epsilon_t)}{\exp(\epsilon_t) + 1} - \exp(\xi) \epsilon_t - 1 \quad (10)$$

Fourthly, $\epsilon_t \sim \text{NIG}[0, 1, \exp(\nu), \exp(\nu)\tanh(\eta)]$, where $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ influence tail-thickness and asymmetry, respectively. The log-density of p_t is

$$\begin{aligned} \ln f(p_t|p_1, \dots, p_{t-1}) &= \nu - \lambda_t - \ln(\pi) + \exp(\nu)[1 - \tanh^2(\eta)]^{1/2} \\ &+ \exp(\nu)\tanh(\eta)\epsilon_t + \ln K^{(1)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right] - \frac{1}{2} \ln(1 + \epsilon_t^2) \end{aligned} \quad (11)$$

where $K^{(1)}(x)$ is the modified Bessel function of the second kind of order 1. The score function $u_{\mu,t}$ is given by

$$\begin{aligned} \frac{\partial \ln f(p_t|p_1, \dots, p_{t-1})}{\partial \mu_t} &= -\exp(\nu - \lambda_t)\tanh(\eta) + \frac{\epsilon_t}{\exp(\lambda_t)(1 + \epsilon_t^2)} \\ &+ \frac{\exp(\nu - \lambda_t)\epsilon_t}{\sqrt{1 + \epsilon_t^2}} \times \frac{K^{(0)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right] + K^{(2)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right]}{2K^{(1)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right]} \\ u_{\mu,t} &= \frac{\partial \ln f(p_t|p_1, \dots, p_{t-1})}{\partial \mu_t} \times \exp(2\lambda_t) \end{aligned} \quad (12)$$

where $K^{(0)}(x)$ and $K^{(2)}(x)$ are the modified Bessel functions of the second kind of orders 0 and 2, respectively. The score function $u_{\lambda,t}$ is

$$\begin{aligned} u_{\lambda,t} &= \frac{\partial \ln f(p_t|p_1, \dots, p_{t-1})}{\partial \lambda_t} = -1 - \exp(\nu)\tanh(\eta)\epsilon_t + \frac{\epsilon_t^2}{1 + \epsilon_t^2} \\ &+ \frac{\exp(\nu)\epsilon_t^2}{\sqrt{1 + \epsilon_t^2}} \times \frac{K^{(0)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right] + K^{(2)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right]}{2K^{(1)} \left[\exp(\nu)\sqrt{1 + \epsilon_t^2} \right]} \end{aligned} \quad (14)$$

4. Statistical inference

The DCS specifications of this paper are estimated by using the Maximum Likelihood (ML) method (Davidson and MacKinnon 2003). The ML estimator is given by

$$\hat{\Theta}_{\text{ML}} = \arg \max_{\Theta} \text{LL}(p_1, \dots, p_T; \Theta) = \arg \max_{\Theta} \sum_{t=1}^T \ln f(p_t|p_1, \dots, p_{t-1}) \quad (15)$$

where Θ denotes the vector of time-constant parameters. The standard errors of parameters are estimated by using the inverse information matrix (Harvey 2013). For some parameters, we estimate their transformed values, and use the delta method to estimate the standard errors of those parameters (Davidson and MacKinnon 2003). For DCS-EGARCH(1,1), sufficient conditions for the consistency and asymptotic normality of the ML estimates are demonstrated in the work of Harvey (2013). For the first-order DCS model, Harvey (2013, Chapter 2.3.2) defines $C_\lambda = \beta^2 + 2\beta\alpha E(\partial u_{\lambda,t}/\partial \lambda_t) + \alpha^2 E[(\partial u_{\lambda,t}/\partial \lambda_t)^2]$. We estimate C_λ for each DCS specification of the present paper. Two conditions for DCS-EGARCH(1,1) are $|\beta| < 1$ and $C_\lambda < 1$.

5. Empirical results

5.1. GTQ/USD exchange rate

As an illustration, we use GTQ/USD exchange rate data in this paper. The source of those data is the Bank of Guatemala (we present some specific details of the data source in the notes of Table 1). The GTQ/USD exchange rate p_t is available from 6 November 1989, when GTQ/USD started to float in the foreign currency market. Until 1994, the Bank of Guatemala used a pegged float exchange rate regime, for which the rate was allowed to fluctuate within a specific band. For period 6 November 1989 to 31 December 1993, the GTQ/USD time series shows constant level periods with zero volatility, step-function like evolution in other periods, and significant jumps or falls on some days (Figs. 1(a-b)). The new DCS models suggested in this paper are not adequate for the GTQ/USD time series of this period.

From 1994, a managed float regime was introduced, and GTQ/USD became more volatile. In 1997 and 1998, GTQ depreciated in relation to the effects of the Asian Financial Crisis and the Russian Financial Crisis, respectively (Figs. 1(c-d)). In 1999, both demand and price of the goods exported from Guatemala decreased significantly, and GTQ significantly depreciated again (Figs. 1(c-d)) due to a negative current account and a negative capital account in the same year. As a consequence, the Bank of Guatemala intervened in the GTQ/USD exchange rate market in August 1999.

In May 2001, the Congress of the Republic of Guatemala approved the Law of Free For-

Foreign Currency Transactions (Act N°: 94-2000), and created the Institutional Foreign Currency Market (hereinafter, MID). Those institutions that participate in MID are obliged to report all foreign currency transactions, on a daily basis, to the Bank of Guatemala. The current exchange rate regime in Guatemala allows the participation of the Bank of Guatemala in MID. Since 2006, the rules of intervention of the Bank of Guatemala are officially published, and are known by the participants of MID.

In this study, we use data for period 4 January 1994 to 30 June 2017 (Figs. 1(c-d)). The Bank of Guatemala reports bid and ask prices for GTQ/USD for seven days of the week (it is noteworthy that Guatemalan banks are open seven days in every week). We use the average of bid and ask prices for each day. We use data for every Monday to Friday from the data window. We do not include bank holidays and weekends in the dataset, since MID undertakes foreign currency transactions only from Monday to Friday (thus, GTQ/USD does not change during the weekend). It is noteworthy that, as an extension of our model, we have also considered a weekly stochastic seasonality component of GTQ/USD, for the data used in this paper. However, we have found that the weekly seasonality is not significantly different from zero.

We present some descriptive statistics for the GTQ/USD level p_t and the GTQ/USD log-return $\ln(p_t/p_{t-1})$ time series in Table 1. We also present results for the Augmented Dickey–Fuller (1979) (hereinafter, ADF) test in Table 1, which suggest that p_t is a I(1) process, and thus motivate the use of the local level component with unit root in the DCS model. In Table 1, we also present the mean p_t for each month of the year. Those mean p_t estimates indicate the following annual seasonality effects: (i) strengthening GTQ from December to May; (ii) relatively stable GTQ from June to August; (iii) weakening GTQ from September to November. These results motivate the use of the annual seasonality component s_t in the DCS model. Significant jumps and falls in GTQ/USD are also observed in Fig. 1(c), which motivate the use of different DCS specifications that differently discount extreme observations. Finally, significant volatility clustering is observed in Fig. 1(d), which motivates the use of DCS-EGARCH.

[APPROXIMATE LOCATION OF TABLE 1 AND FIGURE 1]

5.2. Estimation results

Parameter estimates and model diagnostics are presented in Table 2. We find that all parameters of the alternative DCS specifications are significantly different from zero. For all specifications, the EGARCH estimates support the consistency and asymptotic normality of the ML estimator (i.e. $|\beta| < 1$ and $C_\lambda < 1$) (Table 2).

We use the following metrics to compare statistical performance: mean LL, mean Akaike Information Criterion (AIC), mean Bayesian Information Criterion (BIC) and mean Hannan-Quinn Criterion (HQC) (Davidson and MacKinnon 2003). These metrics suggest that (i) Skew-Gen- t -DCS is superior to t -DCS; (ii) NIG-DCS is superior to EGB2-DCS; (iii) all LL-based statistical performance metrics support the use of Skew-Gen- t -DCS (Table 2).

We also perform a Likelihood-Ratio (LR) test for non-nested models (Vuong 1989). In the LR test, we estimate the linear regression $d_t = c + \epsilon_t$, where d_t is the difference between the log-densities of two models for day t . We estimate this equation by using the OLS-HAC estimator (Ordinary Least Squares Heteroscedasticity and Autocorrelation Consistent) (Newey and West 1987). We find that Skew-Gen- t -DCS is superior to the alternative DCS models (Table 2).

[APPROXIMATE LOCATION OF TABLE 2]

5.3. Stochastic seasonality

Significant stochastic annual seasonality s_t estimates are shown in Figs. 2(a-d) for t -DCS, Skew-Gen- t -DCS, EGB2-DCS and NIG-DCS, respectively. The annual seasonality component can be explained by the evolution of agricultural product exports within each year. During period December to May, the amount of USD entering Guatemala increases due to coffee, sugar, banana and cardamom exports (it is noteworthy that Guatemala is the biggest cardamom producer and exporter in the world). Therefore, during period December to May, GTQ becomes stronger with respect to USD (Figs. 2(a-d)). For period June to August, the GTQ/USD exchange rate is relatively stable. For period September to November, the amount of USD entering Guatemala reduces due to the finish of agricultural product harvests. As a consequence, during period September to November, GTQ becomes weaker with respect to USD (Figs. 2(a-d)).

It is noteworthy that, in principle, the annual seasonality of GTQ/USD should not exist, due to arbitrage trading in GTQ/USD. The reason why it still exists for the data window of the present study is that Guatemalan banks offer to their clients GTQ/USD prices with a large bid-ask spread. Thus, clients do not make use of arbitrage trading due to high transaction costs.

The amplitude of the annual seasonality component of GTQ/USD is dynamic (Figs. 2(a-d)). The following three regimes with different amplitudes can be identified: (R1) 1994 to 2000; (R2) 2001 to 2008; (R3) 2009 to 2017 (Figs. 2(a-d)). These regimes are mainly due to the changing importance of agricultural goods in Guatemalan exports. In what follows, we study these regimes in three points, where each point is related to different foreign currency movements.

Firstly, in Table 3, we present the evolution of total exports from Guatemala and total imports to Guatemala, for period 1993 to 2016. The growth rate of total exports from Guatemala decreases over time: the mean growth rates of total exports for (R1), (R2) and (R3) are 15.1%, 7.8% and 3.4%, respectively (Table 3). This reduction in the growth rate of total exports suggests a decreasing amplitude of the annual seasonality for periods (R1) to (R3).

Secondly, the relative importance of total exports, with respect to total currency inflows and outflows, decreases for the data window. In Fig. 3, we present the relative importance of the following foreign currency movements for (R1), (R2) and (R3): (i) total exports from Guatemala (Fig. 3(a)); (ii) total imports to Guatemala (Fig. 3(b)); (iii) receipt of loans to Guatemala (Fig. 3(c)); (iv) payment of loans from Guatemala (Fig. 3(d)); (v) remittances to Guatemala of Guatemalans working abroad (Fig. 3(e)). For all cases, we compute relative importance with respect to the sum of total inflows and total outflows of foreign currency. For each period (R1), (R2) and (R3), we estimate average relative importance separately for each month. We find that the relative importance of total exports, on average, significantly decreases from (R1) to (R2) and (R3) (Fig. 3(a)). Furthermore, we also find that the relative importance of loans and remittances that do not have a significant seasonality component, on average, significantly increases from (R1) to (R3) (Figs. 3(c-e)). These results also support that the amplitude of the seasonality component of GTQ/USD reduces for the data window.

Thirdly, a further explanation for the reduction of the amplitude of the annual seasonality component is the reduction of the relative importance of agricultural product exports within total exports. In Table 4, we present the export income from coffee, sugar, banana and cardamom, which, as aforementioned, are the main agricultural export products of Guatemala. The relative importance of these products has significantly reduced during period 1994 to 2016 (Table 4).

[APPROXIMATE LOCATION OF TABLES 3, 4 AND FIGURES 2, 3]

5.4. *Trimming and Winsorizing*

The score functions $u_{\mu,t}$ and $u_{\lambda,t}$ of t -DCS, Skew-Gen- t -DCS, EGB2-DCS and NIG-DCS are presented in Figs. 4(a-d), respectively. We evaluate all score functions by using the ML estimates of the shape parameters, and for λ_t we use its unconditional mean estimate $\hat{\omega}/(1 - \hat{\beta})$.

For t -DCS and Skew-Gen- t -DCS, $u_{\mu,t}$ undertakes a smooth form of trimming (Fig. 4(a)). In the central part of the distribution, observations are discounted more for t -DCS than for Skew-Gen- t -DCS (Fig. 4(a)). In the extreme parts of the distribution, observations are discounted in similar ways for t -DCS and Skew-Gen- t -DCS (Fig. 4(a)). Furthermore, for t -DCS and Skew-Gen- t -DCS, $u_{\lambda,t}$ undertakes a smooth form of Winsorizing (Fig. 4(c)). In the central part of the distribution, observations are discounted in similar ways for t -DCS and Skew-Gen- t -DCS (Fig. 4(c)). In the extreme parts of the distribution, observations are discounted more for Skew-Gen- t -DCS than for t -DCS (Fig. 4(c)). The LL-based metrics of Table 2 suggest that discounting of extreme values of Skew-Gen- t -DCS is more effective than that of t -DCS.

For EGB2-DCS and NIG-DCS, $u_{\mu,t}$ undertakes a smooth form of Winsorizing (Fig. 4(b)), and $u_{\lambda,t}$ increases linearly as $|\epsilon_t| \rightarrow \infty$ (Figs. 4(b, d)). We find that, for both $u_{\mu,t}$ and $u_{\lambda,t}$, observations are discounted more for EGB2-DCS than for NIG-DCS (Figs. 4(b, d)). Moreover, for both EGB2-DCS and NIG-DCS, observations are discounted in an asymmetric way with respect to the left and right tails of the distribution (i.e. observations in the right tail are discounted more than observations in the left tail by both $u_{\mu,t}$ and $u_{\lambda,t}$) (Fig. 4(d)). The LL-based metrics presented in Table 2 suggest that discounting of extreme values of NIG-DCS is more effective than that of EGB2-DCS.

It is noteworthy that the functional form of $u_{\lambda,t}$ for EGB2-EGARCH and NIG-EGARCH can be compared with the functional form of the updating term of the GARCH model (Bollerslev 1986; Taylor 1986), that is a quadratic function of ϵ_t . EGB2-EGARCH and NIG-EGARCH are more robust for extreme observations than GARCH, due to more significant discounting of extreme values (Fig. 4(d)). Moreover, Beta- t -EGARCH and Skew-Gen- t -EGARCH are more robust to extreme observations than EGB2-EGARCH and NIG-EGARCH, due to more significant discounting of extreme values (Fig. 4(c-d)).

[APPROXIMATE LOCATION OF FIGURE 4]

6. Conclusions

In this paper, we have introduced the dynamic Skew-Gen- t and the dynamic NIG location models, which are alternatives to the dynamic Student's- t and the dynamic EGB2 location models, respectively. We have shown that the Skew-Gen- t location model performs a smooth form of trimming of extreme observations. We have also shown that the NIG location model performs a smooth form of Winsorizing of extreme observations. We have considered DCS-EGARCH scale dynamics in these DCS location models.

As an illustration, we have used daily data from GTQ/USD for period 4 January 1994 to 30 June 2017. We have found a significant stochastic seasonality component with a decreasing amplitude over time. This decreasing amplitude may be explained by the following points: (i) reducing growth rate of total exports; (ii) reducing relative importance of total exports, and increasing relative importance of non-seasonal foreign currency movements (i.e. loans and remittances); (iii) reducing relative importance of agricultural product exports to total exports.

For GTQ/USD, we have shown that the Skew-Gen- t -DCS model is superior to the Student's- t -DCS model, and we have also shown that the NIG-DCS model is superior to the EGB2-DCS model. Furthermore, with respect to the new DCS models, we have shown that Skew-Gen- t -DCS discounts extreme observations more effectively than NIG-DCS. The results presented in this paper may motivate the use of the new Skew-Gen- t -DCS model for the GTQ/USD exchange rate, by central bankers, policy makers, financial investors or private firms.

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Table 1. Descriptive statistics

Statistics	GTQ/USD p_t	GTQ/USD $\ln(p_t/p_{t-1})$	Month	Mean p_t
Start date	4 January 1994	4 January 1994	January	7.4087
End date	30 June 2017	30 June 2017	February	7.3819
Sample size	6,128	6,128	March	7.3590
Minimum	5.4939	-0.0199	April	7.3545
Maximum	8.3948	0.0192	May	7.3411
Average	7.3904	0.0000	June	7.3712
Standard deviation	0.7476	0.0017	July	7.3656
Skewness	-1.2354	0.6494	August	7.3840
Excess kurtosis	0.0049	14.8366	September	7.4312
ADF statistic, constant	-2.1142(0.2391)	-13.6809***(0.0000)	October	7.4501
ADF statistic, constant plus trend	-0.8073(0.9637)	NA	November	7.4298
ADF statistic, constant plus quadratic trend	-2.2003(0.7293)	NA	December	7.4126

Guatemalan Quetzal (GTQ); United States Dollar (USD); Augmented Dickey–Fuller (ADF); Not Available (NA). p -values of the ADF test are reported in parentheses. *** indicates significance at the 1% level. Source of data: Bank of Guatemala, <http://www.banguat.gob.gt/cambio/default.asp>. Accessed 29 July 2017.

Table 2. Parameter estimates and model diagnostics

<i>t</i> -DCS		Skew-Gen- <i>t</i> -DCS		EGB2-DCS		NIG-DCS	
δ	11.6767*** (0.2957)	δ	9.6840*** (0.2568)	δ	0.9985*** (0.0116)	δ	0.3326*** (0.0105)
γ_{Jan}	-1.0548*** (0.0734)	γ_{Jan}	-0.8595*** (0.0578)	γ_{Jan}	-0.0835*** (0.0051)	γ_{Jan}	-0.0279*** (0.0020)
γ_{Feb}	0.4225*** (0.0341)	γ_{Feb}	0.3582*** (0.0284)	γ_{Feb}	0.0361*** (0.0031)	γ_{Feb}	0.0119*** (0.0011)
γ_{Mar}	-0.6163*** (0.0295)	γ_{Mar}	-0.5109*** (0.0242)	γ_{Mar}	-0.0525*** (0.0028)	γ_{Mar}	-0.0176*** (0.0011)
γ_{Apr}	-0.3205*** (0.0903)	γ_{Apr}	-0.3008*** (0.0741)	γ_{Apr}	-0.0156* (0.0092)	γ_{Apr}	-0.0051* (0.0030)
γ_{May}	0.8895*** (0.0639)	γ_{May}	0.7650*** (0.0549)	γ_{May}	0.0736*** (0.0052)	γ_{May}	0.0242*** (0.0019)
γ_{Jun}	1.1123*** (0.0656)	γ_{Jun}	0.8921*** (0.0494)	γ_{Jun}	0.0982*** (0.0057)	γ_{Jun}	0.0326*** (0.0022)
γ_{Jul}	3.2336*** (0.1538)	γ_{Jul}	2.8908*** (0.1377)	γ_{Jul}	0.2633*** (0.0114)	γ_{Jul}	0.0868*** (0.0045)
γ_{Aug}	1.3050*** (0.0493)	γ_{Aug}	1.0514*** (0.0383)	γ_{Aug}	0.1137*** (0.0035)	γ_{Aug}	0.0378*** (0.0016)
γ_{Sep}	1.2205*** (0.1380)	γ_{Sep}	0.9919*** (0.1059)	γ_{Sep}	0.1131*** (0.0113)	γ_{Sep}	0.0372*** (0.0038)
γ_{Oct}	-0.2428*** (0.0483)	γ_{Oct}	-0.2224*** (0.0382)	γ_{Oct}	-0.0191*** (0.0044)	γ_{Oct}	-0.0067*** (0.0015)
γ_{Nov}	-0.6171*** (0.0494)	γ_{Nov}	-0.5120*** (0.0387)	γ_{Nov}	-0.0562*** (0.0041)	γ_{Nov}	-0.0182*** (0.0015)
γ_{Dec}	4.0804*** (0.1674)	γ_{Dec}	3.4212*** (0.1369)	γ_{Dec}	0.3637*** (0.0108)	γ_{Dec}	0.1215*** (0.0044)
ω	-0.9301*** (0.0484)	ω	-1.0078*** (0.0495)	ω	-0.8338*** (0.0438)	ω	-0.6769*** (0.0368)
β	0.7976*** (0.0102)	β	0.7800*** (0.0105)	β	0.8281*** (0.0089)	β	0.8273*** (0.0092)
α	0.2004*** (0.0064)	α	0.2229*** (0.0068)	α	0.1298*** (0.0043)	α	0.1385*** (0.0046)
λ_0	-2.9936*** (0.8533)	λ_0	-2.9098*** (0.8859)	λ_0	-3.4120*** (0.5187)	λ_0	-2.4672*** (0.5576)
ν	1.8338*** (0.0368)	τ	0.0387*** (0.0084)	ξ	0.3544*** (0.0370)	ν	1.0697*** (0.0349)
		ν	1.4936*** (0.0451)	ζ	0.2505*** (0.0404)	η	0.0559*** (0.0125)
		η	0.8207*** (0.0117)				
C_λ	0.3642	C_λ	0.3318	C_λ	0.4791	C_λ	0.4674
LL	3.1134	LL	3.1211	LL	3.0922	LL	3.0925
AIC	-6.2209	AIC	-6.2356	AIC	-6.1783	AIC	-6.1788
BIC	-6.2011	BIC	-6.2137	BIC	-6.1574	BIC	-6.1580
HQC	-6.2140	HQC	-6.2280	HQC	-6.1710	HQC	-6.1716
LR	0.0077*** (0.0021)	LR	NA	LR	0.0288*** (0.0067)	LR	0.0285*** (0.0060)

Dynamic Conditional Score (DCS); Exponential Generalized Beta distribution of the second kind (EGB2); Normal-Inverse Gaussian distribution (NIG); mean Log-Likelihood (LL); mean Akaike Information Criterion (AIC); mean Bayesian Information Criterion (BIC); mean Hannan-Quinn Criterion (HQC); Likelihood-Ratio (LR); Not Available (NA). For EGARCH, $|\beta| < 1$ and $C_\lambda < 1$ are required for the consistency and asymptotic normality of the ML estimates. Bold numbers indicate a superior statistical performance. The LR test is always with respect to the best performing Skew-Gen-*t*-DCS model. For LR, we estimate the linear regression $d_t = c + \epsilon_t$ by using OLS-HAC, where d_t is the difference between the two log-density functions for day t . Standard errors are reported in parentheses. * and *** indicate significance at the 10% and 1% levels, respectively.

Table 3. Total exports from Guatemala and total imports to Guatemala

Year	Regime	Exports	Exports %	Exports mean %	Imports	Imports %	Imports mean %	Mean p_t
1993		1,249,135.00			1,825,336.80			5.6354
1994	R1	1,719,461.80	37.7%		2,416,061.50	32.4%		5.7595
1995	R1	2,314,620.36	34.6%		2,855,612.18	18.2%		5.8106
1996	R1	2,356,943.45	1.8%		2,581,500.20	-9.6%		6.0912
1997	R1	3,147,110.12	33.5%		3,551,071.23	37.6%		6.0637
1998	R1	3,502,412.56	11.3%		4,288,290.71	20.8%		6.3944
1999	R1	2,663,945.10	-23.9%		3,361,987.53	-21.6%		7.3853
2000	R1	2,954,127.01	10.9%	R1: 15.1%	3,406,576.45	1.3%	R1: 11.3%	7.7632
2001	R2	2,496,071.04	-15.5%		3,595,573.30	5.5%		7.8586
2002	R2	2,218,061.44	-11.1%		3,845,055.97	6.9%		7.8216
2003	R2	2,661,740.70	20.0%		4,305,029.40	12.0%		7.9408
2004	R2	3,074,419.20	15.5%		4,588,573.70	6.6%		7.9481
2005	R2	3,644,831.80	18.6%		6,010,208.40	31.0%		7.6338
2006	R2	3,813,656.50	4.6%		7,279,563.30	21.1%		7.6024
2007	R2	4,219,396.20	10.6%		9,363,544.70	28.6%		7.6736
2008	R2	5,034,553.30	19.3%	R2: 7.8%	11,695,311.00	24.9%	R2: 17.1%	7.5584
2009	R3	4,795,305.10	-4.8%		9,362,202.80	-19.9%		8.1640
2010	R3	5,490,744.44	14.5%		11,169,889.52	19.3%		8.0556
2011	R3	6,576,115.10	19.8%		13,451,267.10	20.4%		7.7832
2012	R3	6,561,021.10	-0.2%		13,767,708.70	2.4%		7.8311
2013	R3	6,464,898.00	-1.5%		13,791,808.10	0.2%		7.8565
2014	R3	6,640,461.10	2.7%		14,239,560.90	3.2%		7.7309
2015	R3	6,409,639.40	-3.5%		14,058,324.00	-1.3%		7.6542
2016	R3	6,421,918.00	0.2%	R3: 3.4%	13,924,693.80	-1.0%	R3: 2.9%	7.5985

Total exports and total imports are measured in thousands of USD. Source of data: Bank of Guatemala, <http://www.banguat.gob.gt/inc/ver.asp?id=/estaeco/bc/hist/bc11.htm&e=132516>. Accessed 29 July 2017.

Table 4. Exports of the main agricultural products of Guatemala

Year	Total exports	Coffee	Sugar	Banana	Cardamom	Other products	CSBC (%)	Other (%)
1994	1,502.60	318.3	161.5	113.9	42.3	866.6	42.33%	57.67%
1995	1,935.50	539.3	238.2	138.6	40.7	978.7	49.43%	50.57%
1996	2,030.70	472.4	202.1	155.2	39.4	1,161.60	42.80%	57.20%
1997	2,344.10	589.5	255.4	151.1	38	1,310.10	44.11%	55.89%
1998	2,581.70	586.6	316.7	191.4	36.7	1,450.30	43.82%	56.18%
1999	2,460.40	562.6	195.2	135.4	56.5	1,510.70	38.60%	61.40%
2000	2,699.00	575	190.8	167.5	79.4	1,686.30	37.52%	62.48%
2001	2,411.70	306.5	212.6	185	96.1	1,611.50	33.18%	66.82%
2002	4,162.10	261.8	227	216.3	93.3	3,363.70	19.18%	80.82%
2003	4,459.40	299.4	212.3	210	78.9	3,658.80	17.95%	82.05%
2004	5,033.60	328	188	229.7	73.8	4,214.10	16.28%	83.72%
2005	5,380.90	464.1	236.6	238.1	70.4	4,371.70	18.76%	81.24%
2006	6,012.80	464	298.6	216.8	83.4	4,950.00	17.68%	82.32%
2007	6,897.70	577.3	358.1	300.2	137.1	5,525.00	19.90%	80.10%
2008	7,737.40	646.2	378.1	317.1	208	6,188.00	20.02%	79.98%
2009	7,213.70	582.3	507.7	414.8	304.1	5,404.80	25.08%	74.92%
2010	8,462.50	713.9	726.7	353.3	308.1	6,360.50	24.84%	75.16%
2011	10,400.90	1,174.20	648.8	475.3	296.9	7,805.70	24.95%	75.05%
2012	9,978.70	958.1	803	499.8	250.3	7,467.50	25.17%	74.83%
2013	10,024.80	714.5	941.9	594.7	215.6	7,558.10	24.61%	75.39%
2014	10,803.50	668.2	951.7	651.8	239.8	8,292.00	23.25%	76.75%
2015	10,674.80	663	850.8	715.1	243	8,202.90	23.16%	76.84%
2016	10,449.40	649.1	816.7	702.6	229	8,052.00	22.94%	77.06%

Coffee, Sugar, Banana and Cardamom (CSBC); exports are measured in millions of USD. Source of data: Bank of Guatemala, <http://www.banguat.gob.gt/inc/ver.asp?id=/pim/expfob&e=133863>. Accessed 11 August 2017.

Fig. 1 GTQ/USD level p_t and GTQ/USD log-return $\ln(p_t/p_{t-1})$

Fig. 1(a) p_t from 6 Nov 1989 to 31 Dec 1993

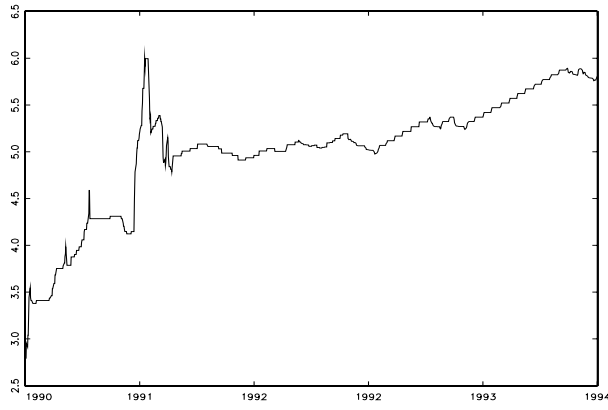


Fig. 1(b) $\ln(p_t/p_{t-1})$ from 6 Nov 1989 to 31 Dec 1993

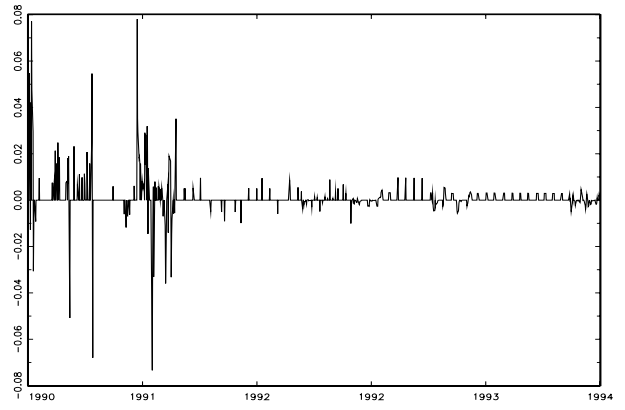


Fig. 1(c) p_t from 4 Jan 1994 to 30 Jun 2017



Fig. 1(d) $\ln(p_t/p_{t-1})$ from 4 Jan 1994 to 30 Jun 2017

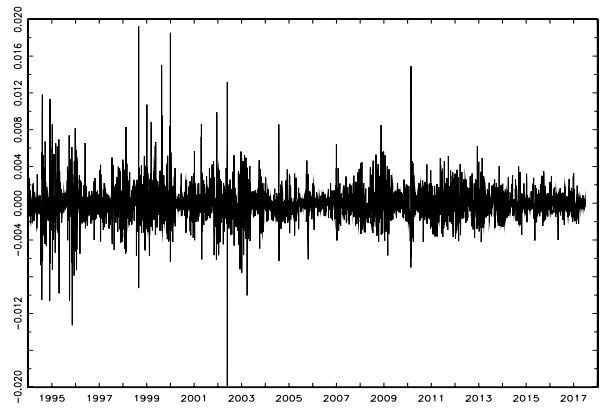


Fig. 2 GTQ/USD stochastic annual seasonality component s_t for period 4 January 1994 to 30 June 2017

Fig. 2(a) Annual seasonality s_t for t -DCS

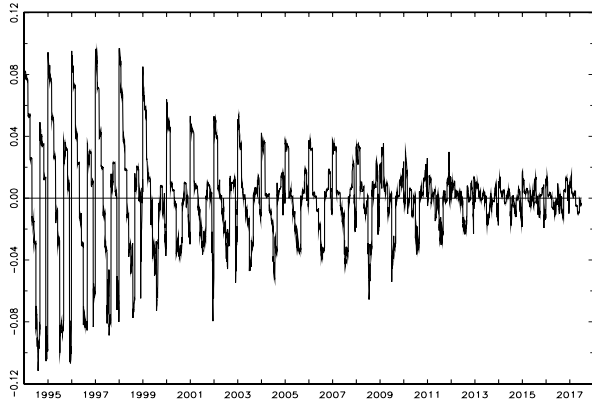


Fig. 2(b) Annual seasonality s_t for Skew-Gen- t -DCS

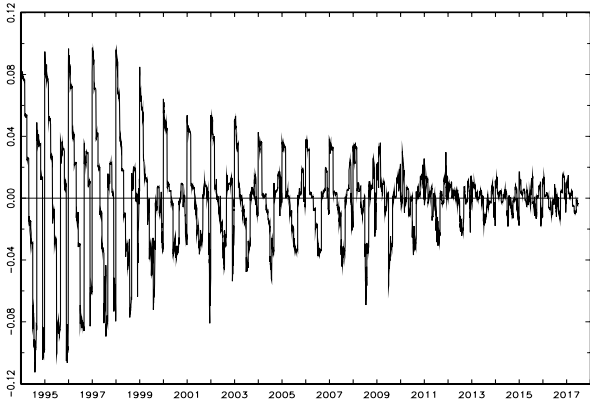


Fig. 2(c) Annual seasonality s_t for EGB2-DCS

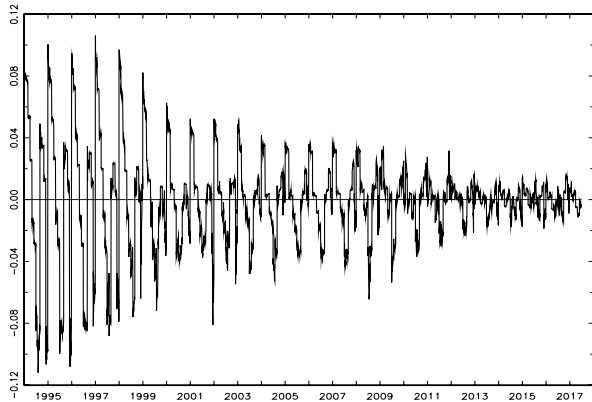


Fig. 2(d) Annual seasonality s_t for NIG-DCS

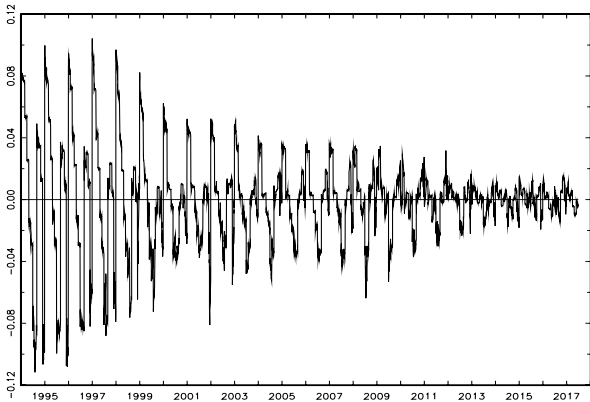


Fig. 3 Relative importance of specific foreign currency movements

Notes: R1 is 1994 to 2000 (solid thin); R2 is 2001 to 2008 (dashed thin); R3 is 2009 to 2016 (solid thick).

Fig. 3(a) Total exports from Guatemala

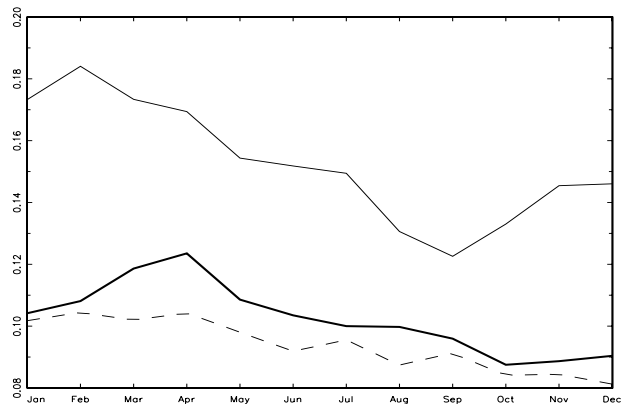


Fig. 3(b) Total imports to Guatemala

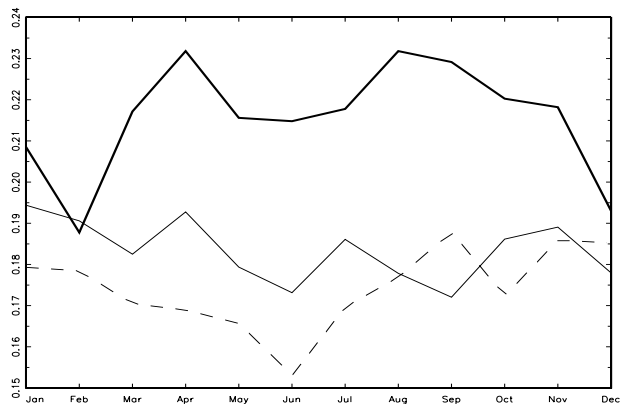


Fig. 3(c) Receipt of loans to Guatemala

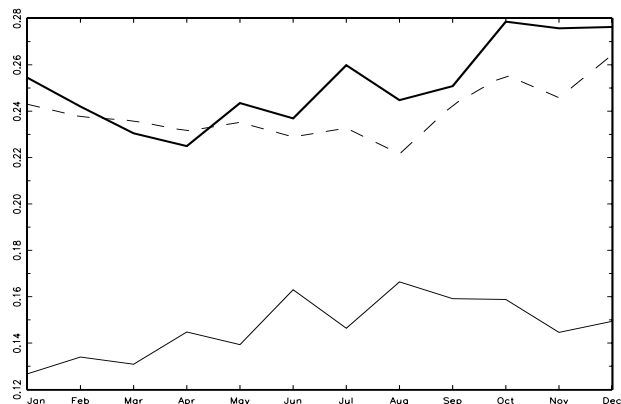


Fig. 3(d) Payment of loans from Guatemala

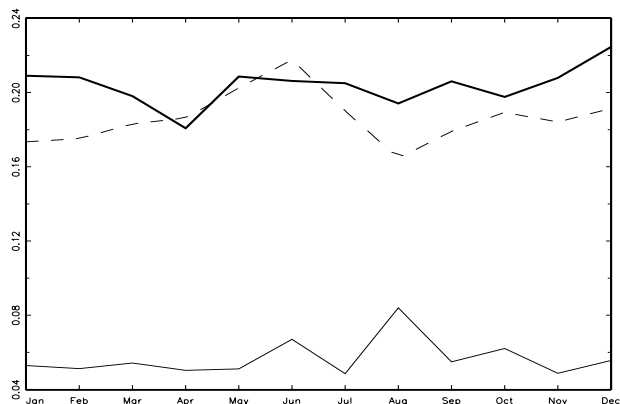


Fig. 3(e) Remittances to Guatemala

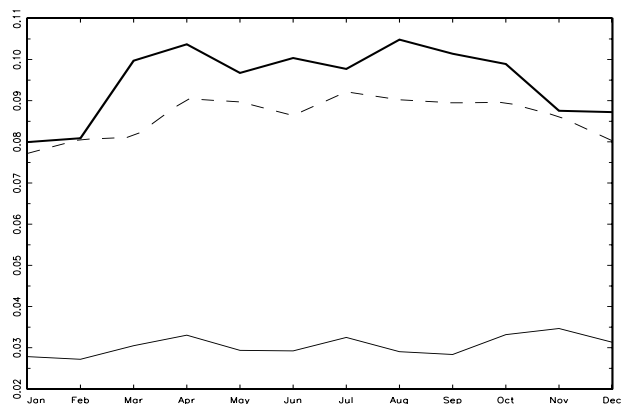


Fig. 4 Score functions of t -DCS, Skew-Gen- t -DCS, EGB2-DCS and NIG-DCS; estimated for the GTQ/USD time series
 Notes: Score functions are presented as a function of the standardized error term ϵ_t .

Fig. 4(a) $u_{\mu,t}$ for t -DCS (thin) and Skew-Gen- t -DCS (thick)

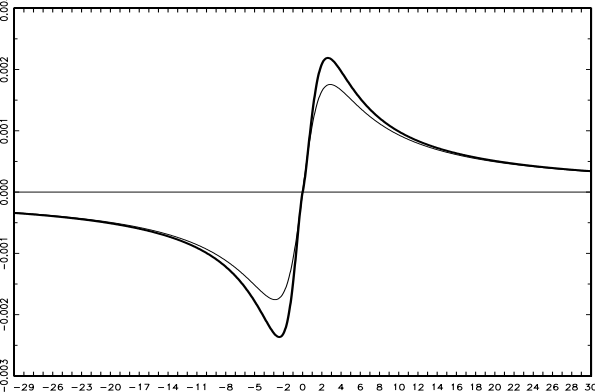


Fig. 4(b) $u_{\mu,t}$ for EGB2-DCS (thin) and NIG-DCS (thick)

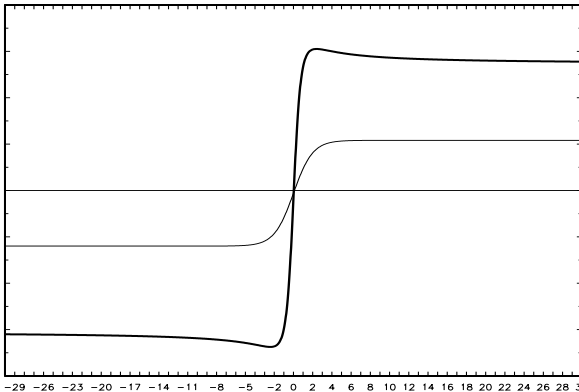


Fig. 4(c) $u_{\lambda,t}$ for t -DCS (thin) and Skew-Gen- t -DCS (thick)

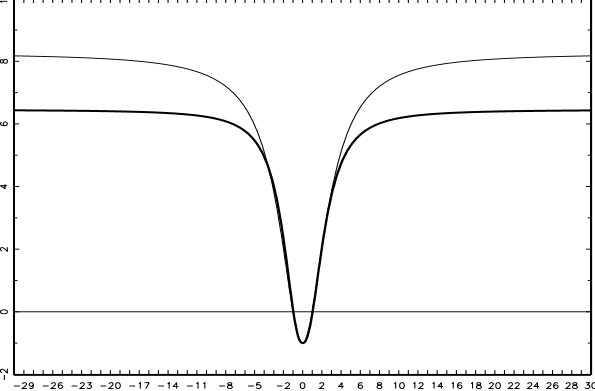


Fig. 4(d) $u_{\lambda,t}$ for EGB2-DCS (thin) and NIG-DCS (thick)

