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Volatility of global equity indices: comparison of GARCH and Beta-*t*-EGARCH models

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Abstract. We study the volatility of nine global equity indices over the period April 2006 to July 2010. The full sample period is divided to three sub-periods before, during and after the United States (U.S.) financial crisis of October 2007 to February 2009. GARCH(1,1) and Beta-*t*-EGARCH(1,1) volatility models are estimated for all assets and all sub-periods. We compare the evolution of volatility of each asset before, during and after the U.S. financial crisis period. Furthermore, we compare the statistical performance, the in-sample and out-of-sample predictive power of both volatility models. We find that in-sample the Beta-*t*-EGARCH model is a more robust and better specification for the conditional volatility of the global equity indices than the traditional GARCH model in many cases. Nevertheless, when in-sample point and out-of-sample density forecast performance is compared, we find that the traditional GARCH model predicts with higher precision than the Beta-*t*-EGARCH model during the period of the U.S. financial crisis. These findings motivate the use of the traditional GARCH model instead of the recent Beta-*t*-EGARCH for the industry index return data considered in this work.

Keywords: Beta-*t*-EGARCH, in-sample and out-of-sample forecasts, density forecasts, U.S. financial crisis, global equity indices for different industries

1. Introduction

In the last three decades, the financial risk of investing in bonds, stocks, foreign exchange rates and their derivatives has increased significantly; see Jorion (2006). Three critical periods on financial markets were: the market crash in October 1987, the dot com boom in 2000, and the United States (U.S.) financial crisis in 2007 to 2009. Although these crises happened mostly in the U.S. financial markets, their effects were perceived around the world due to technological advances and then the beginning of the globalization process. Today countries are more interconnected and more dependent on each other than ever. In this sense, the U.S. financial crisis of 2007 to 2009 deserves special attention due to its deep and lasting effects. For this reason, it is important to have accurate estimators of financial risk (volatility) in a worldwide scale, which would allow investors to better capture the effects on volatility of an international event and its consequences in a specific industry.

Predicting and modeling volatility in the world financial markets has played a major role in the development of finance as a science in the past decades. Volatility defined as the standard deviation of the returns of an asset, can be interpreted as a statistical measurement of financial risk. Volatility is a cornerstone for approaching problems such as asset valuation, portfolio optimization and most importantly, risk management. There is no financial asset exempt from volatility although its degree may vary depending on the nature of the asset. The importance of measuring volatility in a precise way emerges from the need of investors to minimize losses and to know accurately their value at risk. This allows investors to allocate resources in a more efficient way.

Volatility is latent, so one needs to use econometric models to estimate it. There has been vast empirical and theoretical research work on this subject over the past decades. The first and simplest model for measuring volatility is the sample estimate of the standard deviation of returns. This volatility model assumes that volatility is constant over time. Nevertheless, it is well known that the dispersion of the probability distribution of returns on financial markets is time-dependent. This motivated a large number of works on dynamic volatility models. For this literature, the starting model is the Autoregressive Conditional Heteroscedasticity (ARCH) model published by the 2003 Nobel Prize in Economics winner Robert F. Engel in 1982. This model was the first to assume dynamic heteroscedasticity. This means that a) the conditional variance of errors is not constant over time and b) the conditional variance depends on past returns. The ARCH model, in general, fits better to most of financial time series data than the constant volatility model. An important property of the ARCH model is that it takes into account that the volatility is autocorrelated. It allows the conditional variance of the error term to be determined by the previous values of the squared errors. Assuming constant conditional expectation for y_t, the ARCH(q) model for volatility with q lags of past squared errors is specified as follows:

$$y_t = c + \epsilon_t \quad (1)$$

$$\epsilon_t = \sigma_t u_t \text{ where } u_t \sim N\ (0,1) \text{ i.i.d.} \quad (2)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1\, \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 \quad (3)$$

All parameters of Equation (3) are positive to ensure the positivity of the conditional variance of returns. In the specification presented, the error term is assumed to have standard normal distribution.

To extend the ARCH model, Bollerslev (1986) and Taylor (1986) developed the widely known Generalized ARCH (GARCH) model. In this work, we use the GARCH(1,1) specification of Bollerslev (1986). The GARCH(1,1) model is presented in detail in Section 3.

Following the publication of the GARCH model in 1986, a large number of extensions have been proposed. One of the most important one is the Exponential GARCH (EGARCH) by Daniel Nelson (1991). The EGARCH has additional features compared with the GARCH model. For example, in the EGARCH the conditional variance allows for asymmetric volatility. This means that falling and rising prices may have different effects on the future volatility of returns. As new methods and tools to estimate volatility have been published, such as the Beta-*t*-EGARCH by Harvey and Chakravarty (2008), it is necessary to analyze their properties and to compare them to decide which one fits better to data and which one is more useful for financial practitioners. See also the more recent book of Harvey (2013) on the Beta-*t*-EGARCH model.

The objective of this paper is to compare the classical GARCH(1,1) model with the more recent Beta-*t*-EGARCH(1,1) model. We do this by estimating these models for the Bloomberg World Index of nine different industries. We divide the data to three sub-periods: before, during and after the U.S. financial crisis. We compare the statistical and forecasting properties of both models. We find that in sample the Beta-*t*-EGARCH model is more robust than the GARCH model as it can be estimated without numerical problems for more indices than the GARCH model. Furthermore, from likelihood-based model selection metrics we also evidence that for several assets the Beta-*t*-EGARCH model dominates the traditional GARCH model in sample. Nevertheless, both the in-sample and out-of-sample forecasting performance analysis shows that the recent Beta-*t*-EGARCH model cannot beat the traditional GARCH model during the period of the U.S. financial crisis.

Remaining part of this paper is structured as follows. Section 2 describes the data set. Section 3 reviews the volatility model and presents estimation results. Section 4 compares the statistical performance of GARCH and Beta-*t*-EGARCH models. Section 5 compares the in-sample point forecasts performance of GARCH and Beta-*t*-EGARCH models during the U.S. financial crisis. Section 6 compares the out-of-sample density forecast performance of both models during the U.S. financial crisis. Finally, Section 7 concludes.

2. Data

Bloomberg calculates nine market capitalization weighted world indices since 2004. These indexes are designed to mimic the behavior of nine different industries, described in Table 1, and are translated in to a security that can be bought and sold. The main difference with more traditional indexes like the Standard and Poor's 500 (S&P 500) is that the firms considered in these indexes are from around the world, reflecting the industries in a worldwide scale. Table 1 presents the Bloomberg ticker and describes the industry for each index.

Table 1: Bloomberg world Indices

Industry	Bloomberg ticker	Description
Basic materials	BWBMAT	Companies involved in the discovery, development and processing of raw materials
Financial	BWFINL	Companies that provide financial services including credit unions, commercial banks and investment banks
Communications	BWCOMM	Companies that provide wireless and wire line services
Consumer goods cyclical	BWCCYS	Companies that rely heavily on the business cycle and economic conditions such as automotive, housing, entertainment and retail
Consumer goods non-cyclical	BWCNCY	Companies that rely on products that are consumed despite economic conditions such as beverages, food processing, crops, etc.
Energy	BWENRS	Companies involved in the production and sale of energy
Industrial	BWINDU	Companies that produce goods used in construction and manufacturing
Technology	BWTECH	Companies related to the research, development and distribution technologically based goods and services
Utilities	BWUTIL	Companies producing gas, water or power

We calculated daily log returns, $y_t = \ln(p_t/p_{t-1})$ for the nine indices, where p_t denotes the close price of the index on day t. We collected index data at the daily frequency. The full data period, April 2006 to July 2010, was divided into three sub-periods before, during and after the U.S. financial crisis. We identified these sub-periods by analyzing the evolution of the S&P 500 index. See the evolution of the S&P 500 over the period 1990 to 2012 and the definition of the three sub-periods on Figure 1.

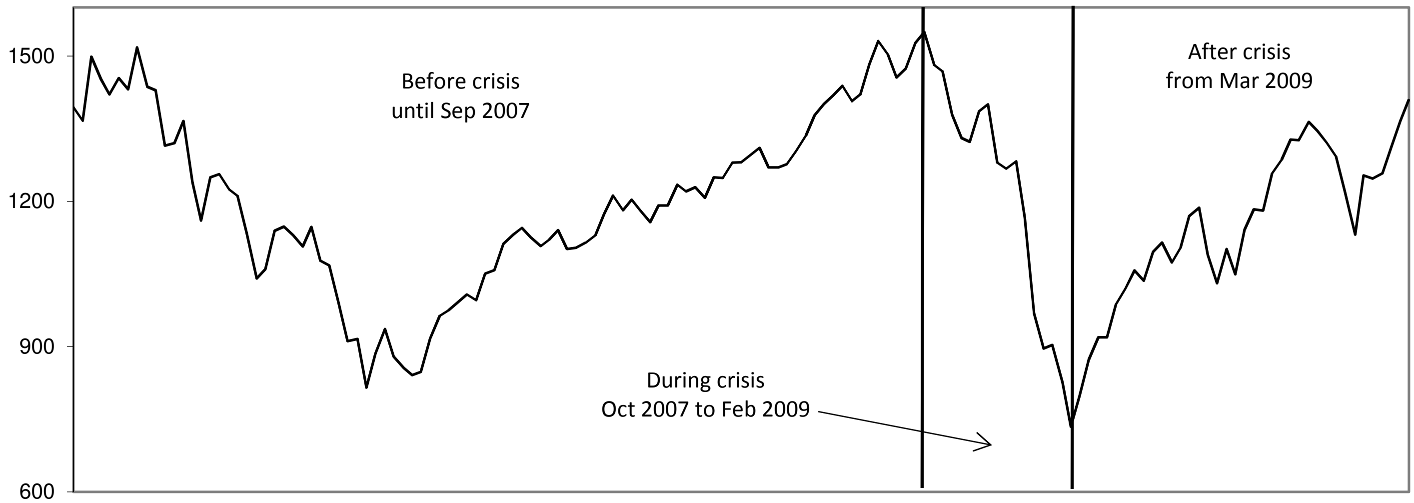

Figure 1. S&P 500 index over the period 1990 to 2012

There are $T = 369$ observations for industry index returns in each sub-period. Equal number of observations is used for each sub-period to make the statistical estimates for different sub-periods more comparable. Some details of the data set can be seen in Table 2. We present the data period, minimum, maximum, average, standard deviation, skewness and excess kurtosis for each asset. Table 2 evidences that mean return is slightly positive before and after the U.S. financial crisis, while it is negative during the crisis. We also see that during the crisis period the sample estimates of standard deviation are higher than before and after the financial crisis. Furthermore, Figures 2 to 10 show the behavior of the returns for each index over the period April 2006 to July 2010.

Table 2: Descriptive statistics

Before crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
start	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006
end	28/09/2007	28/09/2007	28/09/2007	28/09/2007	28/09/2007	28/09/2007	28/09/2007	28/09/2007	28/09/2007
sample size	369	369	369	369	369	369	369	369	369
mean	0.08%	0.03%	0.07%	0.04%	0.03%	0.07%	0.06%	0.04%	0.10%
standard dev.	1.19%	0.77%	0.79%	0.77%	0.56%	1.12%	0.78%	0.79%	0.83%
skewness	-1.213	-0.329	-0.907	-0.234	-0.409	-0.290	-0.471	-0.284	-0.198
excess kurtosis	4.394	1.472	3.995	0.525	1.598	0.982	1.208	0.327	2.633
During crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
start	01/10/2007	01/10/2007	01/10/2007	01/10/2007	01/10/2007	01/10/2007	01/10/2007	01/10/2007	01/10/2007
end	27/02/2009	27/02/2009	27/02/2009	27/02/2009	27/02/2009	27/02/2009	27/02/2009	27/02/2009	27/02/2009
sample size	369	369	369	369	369	369	369	369	369
mean	-0.25%	-0.31%	-0.16%	-0.21%	-0.11%	-0.19%	-0.25%	-0.18%	-0.16%
standard dev.	2.32%	2.11%	1.65%	1.57%	1.31%	2.50%	1.78%	1.91%	1.69%
skewness	-0.455	-0.001	0.183	1.169	-0.049	-0.398	-0.263	0.098	0.368
excess kurtosis	3.067	3.369	5.410	11.905	8.372	4.481	3.013	3.283	8.923
After crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
start	02/03/2009	02/03/2009	02/03/2009	02/03/2009	02/03/2009	02/03/2009	02/03/2009	02/03/2009	02/03/2009
end	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010
sample size	369	369	369	369	369	369	369	369	369
mean	0.16%	0.16%	0.09%	0.14%	0.09%	0.07%	0.14%	0.15%	0.04%
standard dev.	1.58%	1.54%	1.15%	1.07%	0.85%	1.45%	1.29%	1.24%	1.01%
skewness	-0.147	0.265	-0.029	0.010	-0.358	-0.048	-0.015	0.228	-0.233
excess kurtosis	0.637	3.010	4.841	1.557	2.079	1.415	1.670	2.220	1.458
Full sample	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
start	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006	28/04/2006
end	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010	29/07/2010
sample size	738	738	738	738	738	738	738	738	738
mean	0.00%	-0.04%	0.00%	-0.01%	0.00%	-0.01%	-0.01%	0.00%	0.00%
standard dev.	1.77%	1.58%	1.25%	1.19%	0.96%	1.80%	1.36%	1.40%	1.24%
skewness	-0.647	-0.104	-0.081	0.696	-0.290	-0.528	-0.409	-0.039	0.084
excess kurtosis	4.463	5.660	7.585	12.530	11.132	7.345	4.586	5.464	11.720

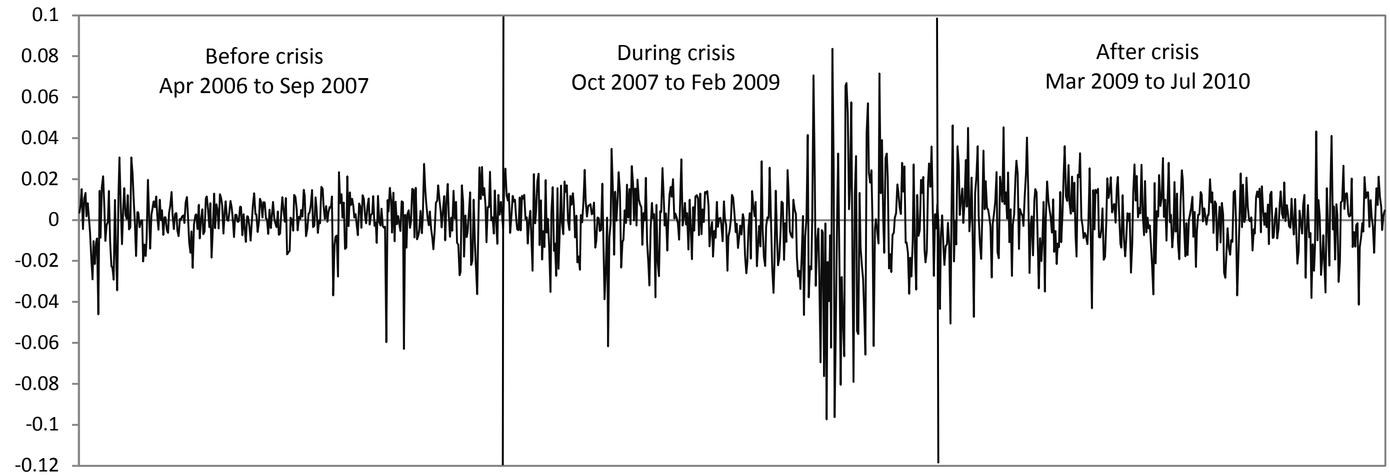

Figure 2: Basic materials, BWBMAT returns

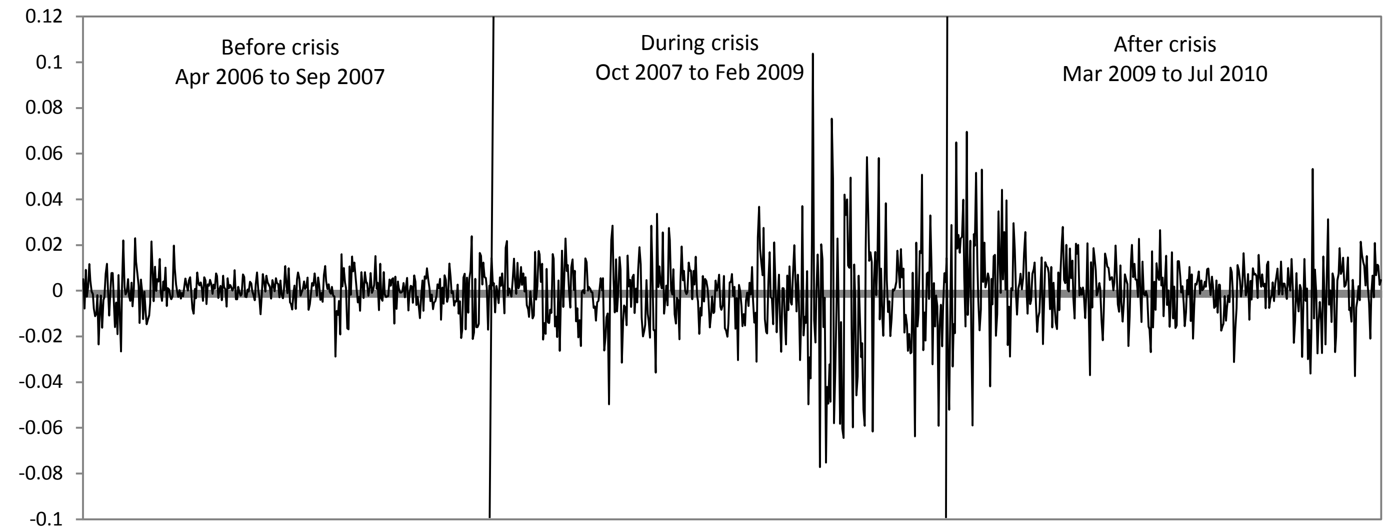

Figure 3: Financial, BWFINL

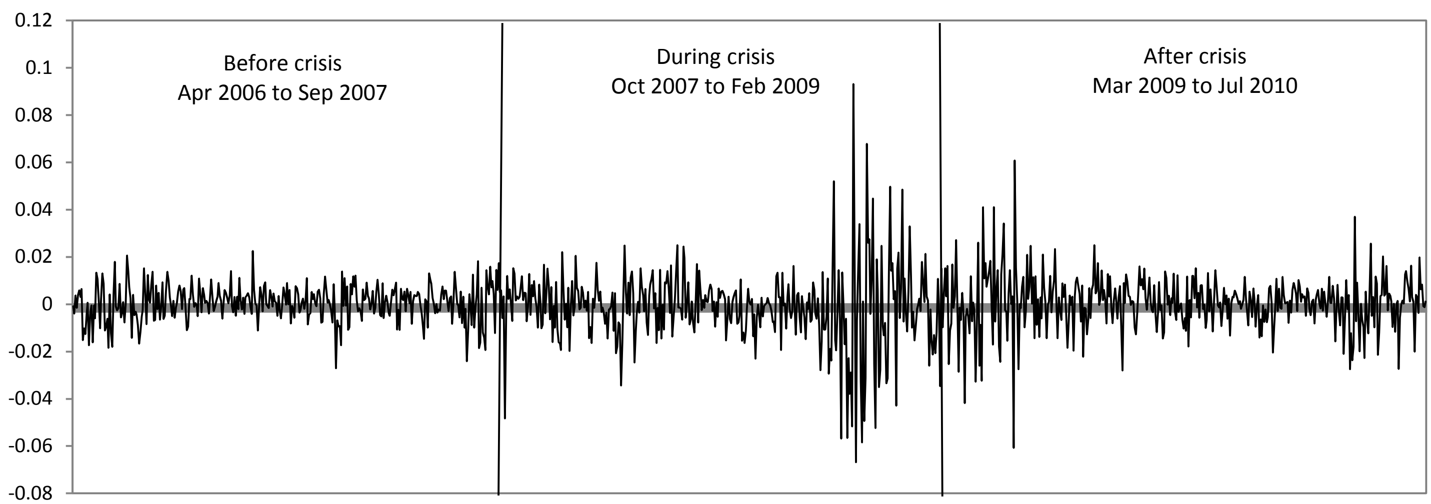

Figure 4: Communications, BWCOMM returns

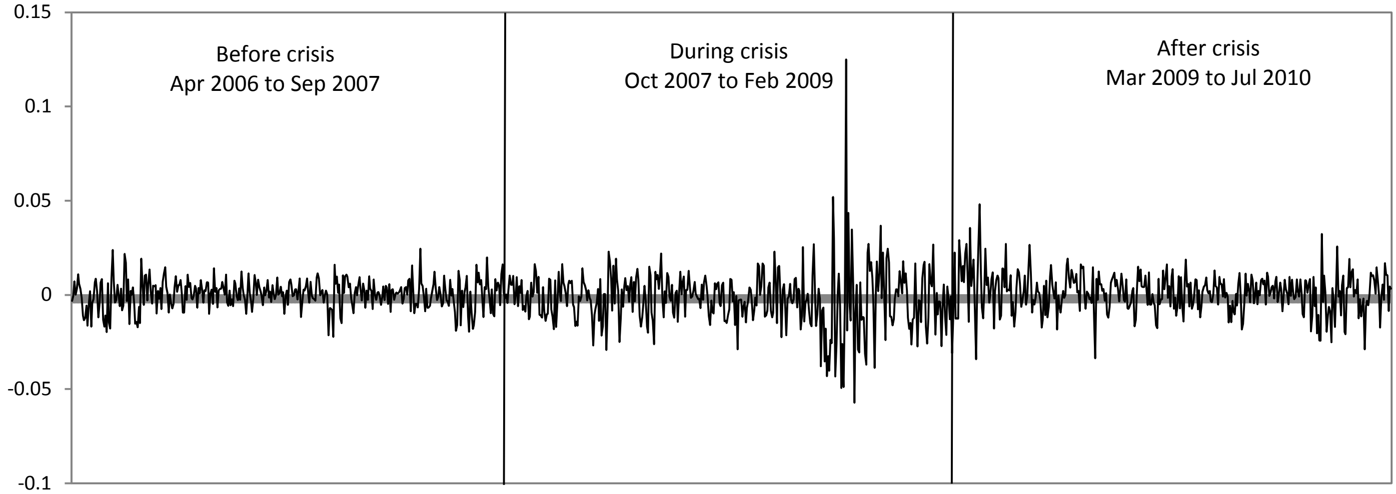

Figure 5: Consumer goods cyclical, BWCCYS returns

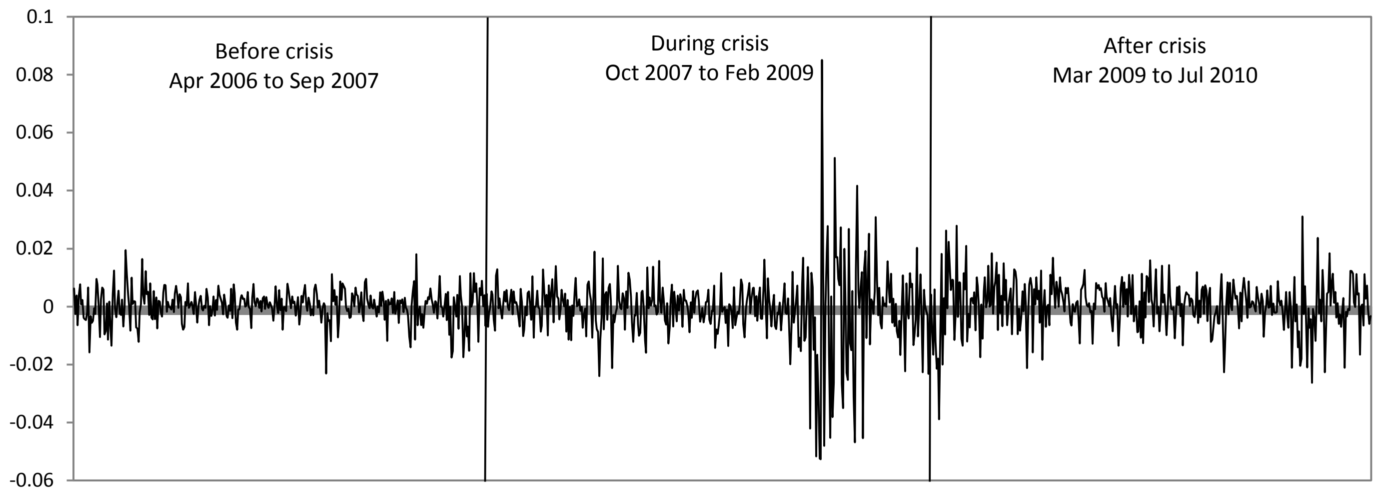

Figure 6: Consumer goods non-cyclical, BWCNCT returns

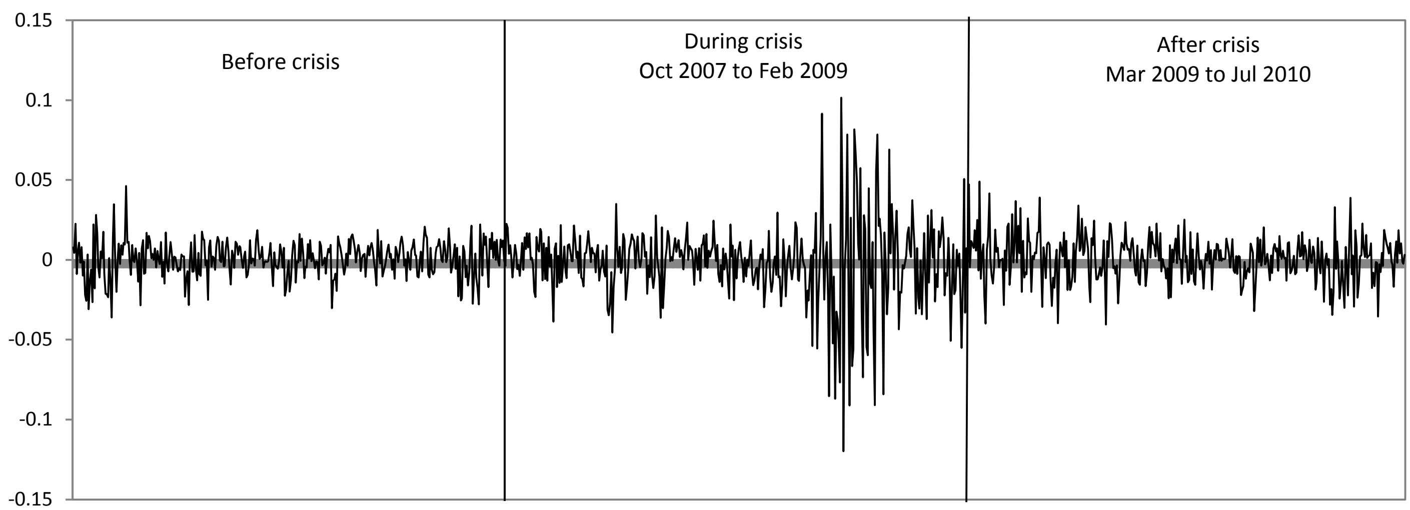

Figure 7: Energy, BWENRS returns

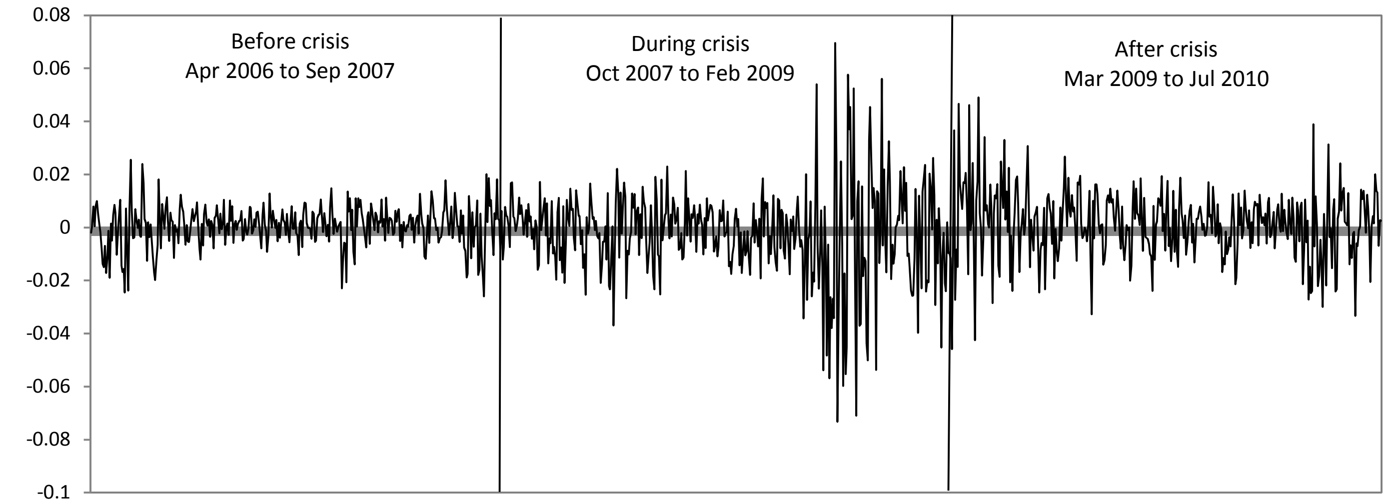

Figure 8: Industrial, BWINDU returns

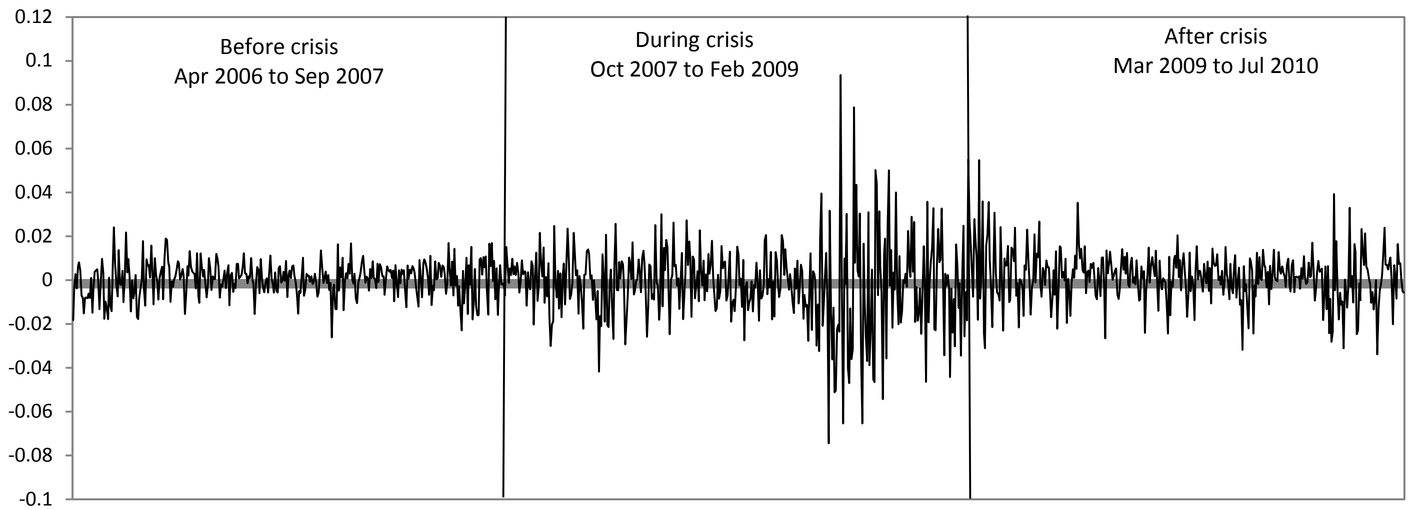

Figure 9: Technology, BWTECH returns

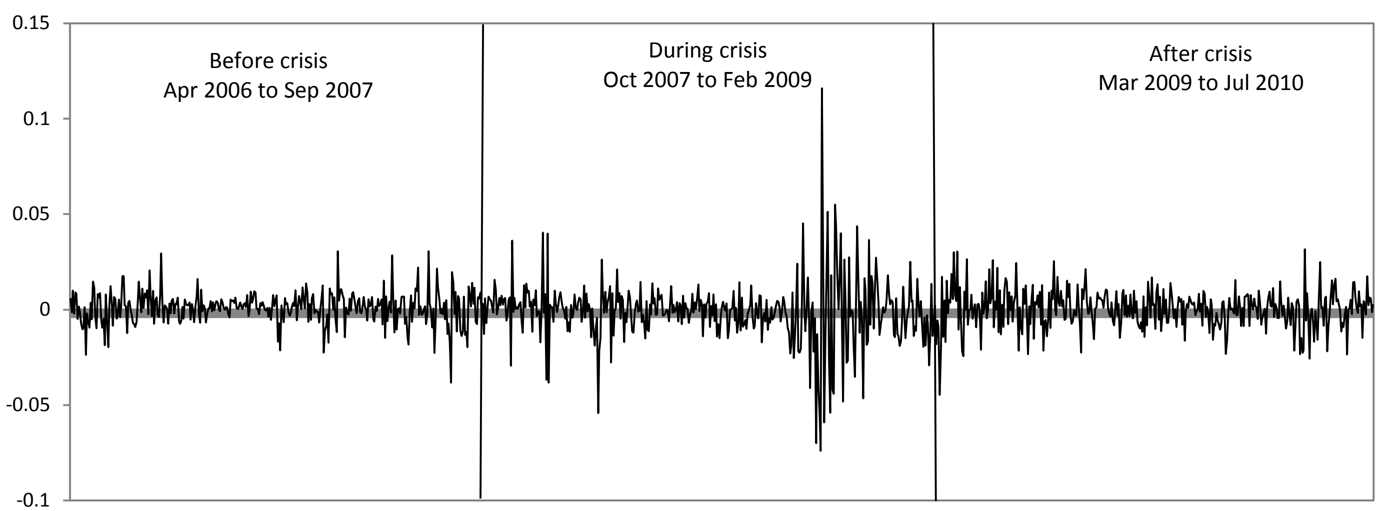

Figure 10: Utilities, BWUTIL returns

3. Volatility models

3.1. *GARCH(1,1)*

The GARCH(1,1) model specification for one conditional variance lag and one conditional error lag is as follows:

$$y_t = c + \epsilon_t \tag{4}$$

$$\epsilon_t = \sigma_t u_t \text{ where } u_t \sim N\ (0,1) \text{ i. i. d.} \tag{5}$$

$$\sigma_1^2 = \alpha_i \tag{6}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \text{ for } t > 1 \tag{7}$$

For the first period, the variance is estimated by the α_i initial condition parameter. It is assumed that all parameters in Equations (6) and (7) are positive numbers. The error term is assumed to be N(0,1) independent and identically distributed (i.i.d.). In more general specifications of the GARCH model, the error term sometimes is assumed to have *t*-distribution. In this work, we assume that errors have Gaussian distribution. This is one of the points which are extended by the Beta-*t*-EGARCH model presented in Section 3.2. The GARCH(1,1) model is covariance stationary if and only if $\alpha_1 + \beta_1 < 1$. If this condition is not met then the conditional variance of returns is not mean reverting and the model does not provide a good fit. The parameters of the GARCH(1,1) are estimated for 27 time series taking into account three sub-periods for the nine different assets. We use the Quasi Maximum Likelihood (QML, henceforth) method. This method produces robust standard errors of parameter estimates. Tables 3 to 5 present the parameter estimates for all sub-periods and all assets.

Table 3: GARCH(1,1) estimates for the before crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	0.002	0.001	NA	0.001	NA	0.001	0.001	NA	0.002
α_0	0.000	0.000		0.000		0.000	0.000		0.000
α_1	0.275	0.071		0.124		0.042	0.120		0.198
β_1	0.331	0.900		0.815		0.922	0.831		0.803
α_i	0.000	0.000		0.000		0.000	0.000		0.000
$\alpha_1 + \beta_1$	0.606	0.972		0.938		0.964	0.950		**1.001**
SE									
c	0.001	0.000		0.000		0.001	0.000		0.000
α_0	0.000	0.000		0.000		0.000	0.000		0.000
α_1	0.166	0.024		0.038		0.060	0.052		0.121
β_1	0.195	0.032		0.059		0.081	0.092		0.101
α_i	0.000	0.000		0.000		0.000	0.000		0.000
z ratio									
c	2.010**	1.732*		2.639***		1.720*	3.288***		4.730***
α_0	2.153**	1.289		1.685*		1.079	1.030		1.032
α_1	1.655*	2.983***		3.238***		0.711	2.319**		1.639
β_1	1.694*	28.532***		13.917***		11.346***	8.999***		7.926***
α_i	1.193	1.045		0.777		1.131	0.487		1.189
p-value									
c	0.040	0.083		0.008		0.085	0.001		0.000
α_0	0.031	0.197		0.092		0.281	0.303		0.302
α_1	0.098	0.003		0.001		0.477	0.020		0.101
β_1	0.090	0.000		0.000		0.000	0.000		0.000
α_i	0.233	0.296		0.437		0.258	0.626		0.235

Notes: *** Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Not Available (NA) due to numerical problems in the parameter estimation procedure. Standard Error (SE). Not covariance stationary (non-stable) estimations are indicated by bold numbers.

Table 4: GARCH(1,1) estimates for the during crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	-0.001	-0.002	-0.001	-0.001	0.000	0.000	-0.001	-0.001	0.000
α_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1	0.123	0.162	0.129	0.127	0.136	0.127	0.109	0.087	0.214
β_1	0.860	0.830	0.858	0.857	0.857	0.856	0.880	0.897	0.783
α_i	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\alpha_1 + \beta_1$	0.983	**0.992**	0.987	0.984	**0.994**	0.982	0.989	0.985	**0.997**
SE									
c	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
α_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1	0.035	0.051	0.035	0.037	0.033	0.031	0.027	0.023	0.062
β_1	0.031	0.038	0.027	0.025	0.022	0.026	0.023	0.020	0.041
α_i	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
z ratio									
c	-0.665	-2.223**	-1.351	-1.785*	-0.807	-0.350	-1.666*	-0.791	-0.581
α_0	2.232**	1.985**	1.966**	1.781*	2.009**	2.362**	2.083**	2.416**	1.929**
α_1	3.546***	3.174***	3.730***	3.431***	4.175***	4.066***	4.059***	3.725***	3.477***
β_1	27.985***	21.949***	31.530***	34.401***	39.221***	32.295***	37.606***	45.493***	19.224***
α_i	2.039**	1.317	1.471	2.381**	2.005**	1.223	2.108**	2.236**	0.781
p-value									
c	0.506	0.026	0.177	0.074	0.420	0.726	0.095	0.429	0.561
α_0	0.026	0.047	0.041	0.075	0.044	0.018	0.037	0.015	0.054
α_1	0.000	0.002	0.000	0.001	0.000	0.000	0.000	0.000	0.000
β_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_i	0.041	0.188	0.141	0.017	0.045	0.221	0.035	0.025	0.435

Notes: *** Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Standard Error (SE). Not covariance stationary (non-stable) estimations are indicated by bold numbers.

Table 5: GARCH(1,1) estimates for the after crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	NA	0.001	0.001	0.001	NA	NA	0.001	0.002	NA
α_0		0.000	0.000	0.000			0.000	0.000	
α_1		0.056	0.047	0.059			0.054	0.059	
β_1		0.903	0.924	0.903			0.914	0.893	
α_i		0.002	0.001	0.001			0.001	0.001	
$\alpha_1 + \beta_1$		0.959	0.971	0.962			0.968	0.952	
SE									
c		0.001	0.000	0.000			0.001	0.001	
α_0		0.000	0.000	0.000			0.000	0.000	
α_1		0.033	0.028	0.032			0.022	0.026	
β_1		0.033	0.028	0.034			0.024	0.027	
α_i		0.001	0.000	0.000			0.000	0.001	
z ratio									
c		1.614	1.410	2.436**			2.128**	2.796***	
α_0		2.319**	1.877*	2.009**			2.094**	2.403**	
α_1		1.681*	1.655*	1.852*			2.395**	2.225**	
β_1		27.661***	33.060***	26.224***			38.280***	33.082***	
α_i		2.134**	2.212**	2.594***			2.602***	1.984**	
p-value									
c		0.106	0.158	0.015			0.033	0.005	
α_0		0.020	0.060	0.0446			0.036	0.016	
α_1		0.093	0.098	0.064			0.017	0.026	
β_1		0.000	0.000	0.000			0.000	0.000	
α_i		0.033	0.027	0.009			0.009	0.047	

Notes: *** Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Not Available (NA) due to numerical problems in the parameter estimation procedure. Standard Error (SE).

For the period before the crisis, the GARCH model is stable for all assets besides BWUTIL. We had numerical problems with the estimation for BWCOMM, BWCNCY and BWTECH. This is probably due to the fact that the GARCH(1,1) is not a good specification for the volatility of these indices. For the period of the U.S. financial crisis, the GARCH(1,1) is not covariance stationary for BWFINL, BWCNCY and BWUTIL. However, we were able to estimate the GARCH model for all assets for the crisis period. For the after crisis period, the GARCH(1,1) model is stable for all assets. Nevertheless, we had numerical problems with the estimation of GARCH(1,1) for the BWMAT, BWCNCY and BWENRS indices.

3.2. *Beta-t-EGARCH(1,1)*

The Beta-*t*-EGARCH model is by Harvey and Chakravarty (2008). It is an attempt to overcome one of the most important problems of GARCH model: it does not capture properly the higher order moments (e.g., skewness and kurtosis) of the return distribution. The Beta-*t*-EGARCH lets the conditional variance depend on past values of the score of a *t*-distribution (see Harvey, 2013). In practice, this traduces on a conditional variance that is able to resist more extreme observations like the data sets that we use. Another benefit of using this new model is that it does not overestimate the impact of past returns on the change of volatility as may happen with the GARCH(1,1); see Harvey (2013). The Beta-*t*-EGARCH(1,1) model is specified as follows:

$$y_t = c + \epsilon_t \tag{8}$$

$$\epsilon_t = \sigma_t u_t \text{ where } u_t \sim t(v) \text{ i. i. d.} \tag{9}$$

$$\sigma_t = \exp\left(\frac{\lambda_t}{2}\right) \tag{10}$$

$$\lambda_1 = \alpha_{0i} \tag{11}$$

$$\lambda_t = \alpha_0 + \alpha_1 e_{t-1} + \beta_1 \lambda_{t-1} \text{ for } t > 1 \tag{12}$$

$$e_t = (v+1)b_t - 1 \tag{13}$$

$$b_t = \frac{\epsilon_t^2 / v \exp(\lambda_t)}{1 + \epsilon_t^2 / v \exp(\lambda_t)} \tag{14}$$

Besides the degrees of freedom parameter, $v > 0$, the parameters of the Beta-*t*-EGARCH are unrestricted. The Beta-*t*-EGARCH is covariance stationary when the absolute value of the parameter β_1 is less than one. We use the QML method to estimate parameters. This estimator produces robust standard errors for parameter estimates. Tables 6 to 8 present the estimation results.

Table 6: Beta-*t*-EGARCH(1,1) estimates for the before crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
α_{0i}	-9.589	-9.703	-10.270	-10.390	-10.554	-8.306	-10.424	-9.144	-10.229
α_0	-0.716	-0.338	-0.440	-0.606	-0.635	-0.383	-0.572	-0.277	-0.547
α_1	0.174	0.118	0.112	0.120	0.130	0.070	0.132	0.057	0.186
β_1	0.924	0.967	0.957	0.939	0.941	0.959	0.943	0.972	0.947
ν	5.265	7.588	7.062	21.483	6.503	13.582	17.855	16.997	4.665
SE									
c	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
α_{0i}	1.514	0.649	1.987	1.108	0.478	1.399	1.208	0.505	0.801
α_0	0.984	0.229	0.488	0.351	0.551	0.507	0.531	0.150	0.587
α_1	0.112	0.039	0.058	0.032	0.067	0.075	0.051	0.020	0.108
β_1	0.104	0.022	0.049	0.036	0.051	0.055	0.053	0.015	0.058
ν	1.464	3.777	2.354	22.010	2.623	7.435	18.303	15.027	0.904
z ratio									
c	0.000***	2.558**	3.537***	2.771***	2.909***	2.026*	3.867***	2.046**	5.175***
α_{0i}	-6.334***	-14.956***	-5.169***	-9.380***	-22.062***	-5.938***	-8.627***	-18.123***	-12.765***
α_0	-0.728	-1.474	-0.903	-1.725*	-1.152	-0.756	-1.076	-1.854*	-0.932
α_1	1.554	3.041***	1.923*	3.718***	1.946*	0.938	2.564**	2.825***	1.721*
β_1	8.886***	43.335***	19.719***	26.308***	18.335***	17.442***	17.753***	63.851***	16.472***
ν	3.597***	2.009**	3.000***	0.976	2.479**	1.827*	0.976	1.131	5.162***
p-value									
c	0.000	0.011	0.000	0.006	0.004	0.043	0.000	0.041	0.000
α_{0i}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_0	0.467	0.141	0.367	0.085	0.249	0.450	0.282	0.064	0.351
α_1	0.120	0.002	0.055	0.000	0.052	0.348	0.010	0.005	0.085
β_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ν	0.000	0.045	0.003	0.329	0.013	0.068	0.329	0.258	0.000

Notes: ***Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Standard Error (SE).

Table 7: Beta-*t*-EGARCH(1,1) estimates for the during crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	-0.001	-0.002	-0.001	-0.001	0.000	0.000	NA	-0.001	-0.001
α_{0i}	-9.498	-9.736	-9.910	-9.930	-11.084	-9.457		-10.014	-9.364
α_0	-0.158	-0.218	-0.175	-0.208	-0.188	-0.145		-0.167	-0.303
α_1	0.108	0.145	0.128	0.106	0.127	0.119		0.088	0.202
β_1	0.980	0.973	0.980	0.976	0.980	0.982		0.979	0.967
ν	27.740	9.735	9.457	18.083	14.501	18.492		23.579	5.748
SE									
c	0.001	0.001	0.001	0.001	0.000	0.001		0.001	0.001
α_{0i}	0.548	0.737	0.606	0.414	0.558	0.643		0.558	1.885
α_0	0.086	0.129	0.104	0.108	0.091	0.081		0.088	0.169
α_1	0.028	0.037	0.033	0.029	0.031	0.030		0.025	0.049
β_1	0.011	0.015	0.011	0.012	0.009	0.010		0.010	0.019
ν	29.760	4.520	5.116	11.019	16.147	16.522		31.502	2.488
z ratio									
c	-0.784	-2.415**	-1.379	-1.999**	-0.681	-0.144		-0.827	-1.147
α_{0i}	-17.343***	-13.220***	-16.346***	-23.961***	-19.857***	-14.713***		-17.962***	-4.969***
α_0	-1.852*	-1.690*	-1.682*	-1.928*	-2.063**	-1.793*		-1.912*	-1.789*
α_1	3.832***	3.912***	3.882***	3.637***	4.127***	3.942***		3.567***	4.156***
β_1	90.777***	63.920***	85.683***	80.678***	104.189***	97.663***		93.759***	52.141***
ν	0.932	2.154**	1.848*	1.641	0.898	1.119		0.748	2.311**
p-value									
c	0.433	0.016	0.168	0.046	0.496	0.886		0.408	0.251
α_{0i}	0.000	0.000	0.000	0.000	0.000	0.000		0.000	0.000
α_0	0.064	0.091	0.093	0.054	0.039	0.073		0.056	0.074
α_1	0.000	0.000	0.000	0.000	0.000	0.000		0.000	0.000
β_1	0.000	0.000	0.000	0.000	0.000	0.000		0.000	0.000
ν	0.351	0.031	0.065	0.101	0.369	0.263		0.454	0.021

Notes: ***Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Not Available (NA) due to numerical problems in the parameter estimation procedure. Standard Error (SE).

Table 8: Beta-*t*-EGARCH(1,1) estimates for the after crisis period

Parameters	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
c	NA	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001
α_{0i}		-6.365	-7.200	-7.435	-7.510	-6.808	-6.888	-6.893	-7.508
α_0		-0.264	-0.264	-0.313	-0.514	-0.282	-0.235	-0.321	-0.389
α_1		0.072	0.062	0.065	0.075	0.048	0.058	0.083	0.048
β_1		0.971	0.973	0.967	0.949	0.968	0.975	0.966	0.960
ν		8.728	5.634	14.692	7.735	13.378	23.116	6.308	14.656
SE									
c		0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
α_{0i}		0.511	0.413	0.390	0.538	0.517	0.423	0.567	0.568
α_0		0.135	0.118	0.145	0.199	0.165	0.116	0.120	0.224
α_1		0.046	0.038	0.033	0.031	0.025	0.026	0.034	0.028
β_1		0.015	0.012	0.015	0.020	0.019	0.013	0.013	0.023
ν		3.938	1.900	7.723	3.018	8.802	23.186	1.761	14.123
z ratio									
c		2.021**	2.224**	2.756***	3.187***	1.039	2.236**	3.753***	1.212
α_{0i}		-12.451***	-17.418***	-19.086***	-13.954***	-13.174***	-16.301***	-12.162***	-13.211***
α_0		-1.956*	-2.233**	-2.154**	-2.582**	-1.716*	-2.030**	-2.673***	-1.741*
α_1		1.577	1.660*	1.955*	2.408**	1.894*	2.285**	2.471**	1.736*
β_1		64.474***	80.177***	62.830***	48.056***	52.125***	76.573***	75.317***	41.200***
ν		2.216**	2.965***	1.902*	2.562**	1.520	0.997	3.583***	1.038
p-value									
c		0.043	0.026	0.006	0.001	0.299	0.025	0.000	0.226
α_{0i}		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_0		0.050	0.026	0.031	0.010	0.086	0.042	0.008	0.082
α_1		0.115	0.097	0.051	0.016	0.058	0.022	0.014	0.083
β_1		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ν		0.027	0.003	0.057	0.010	0.129	0.319	0.000	0.299

Notes: ***Significant at the 1% level. **Significant at the 5% level. *Significant at the 10%. Not Available (NA) due to numerical problems in the parameter estimation procedure. Standard Error (SE).

We find that for all assets and all sub-periods $|\beta_1|$ is lower than one. This shows that the Beta-*t*-EGARCH model was stable for all cases. Furthermore, we only find estimation problems for two cases: BWINDU (during crisis) and BWBMAT (after crisis). These findings suggest that the Beta-*t*-EGARCH model is a more robust model than the GARCH(1,1) one.

Figures 11 to 19, we compare the evolution of volatility estimates for a) constant volatility model (Table 2); b) GARCH(1,1) model; c) Beta-*t*-EGARCH(1,1) model.

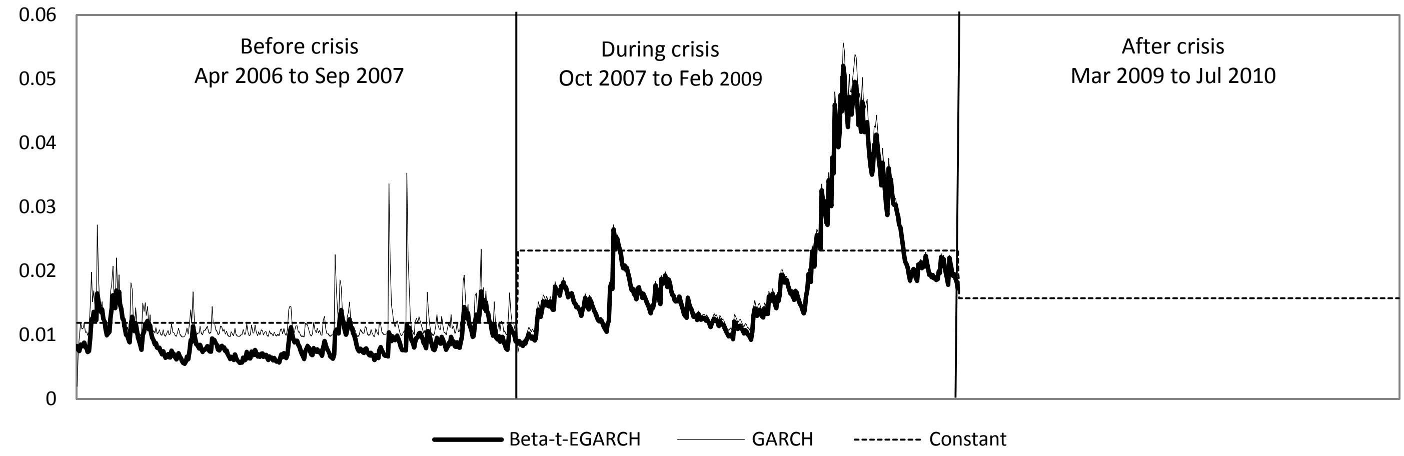

Figure 11: BWBMAT volatility. *Note*: Beta-*t*-EGARCH and GARCH are not available for the after crisis period due to numerical errors in parameter estimation.

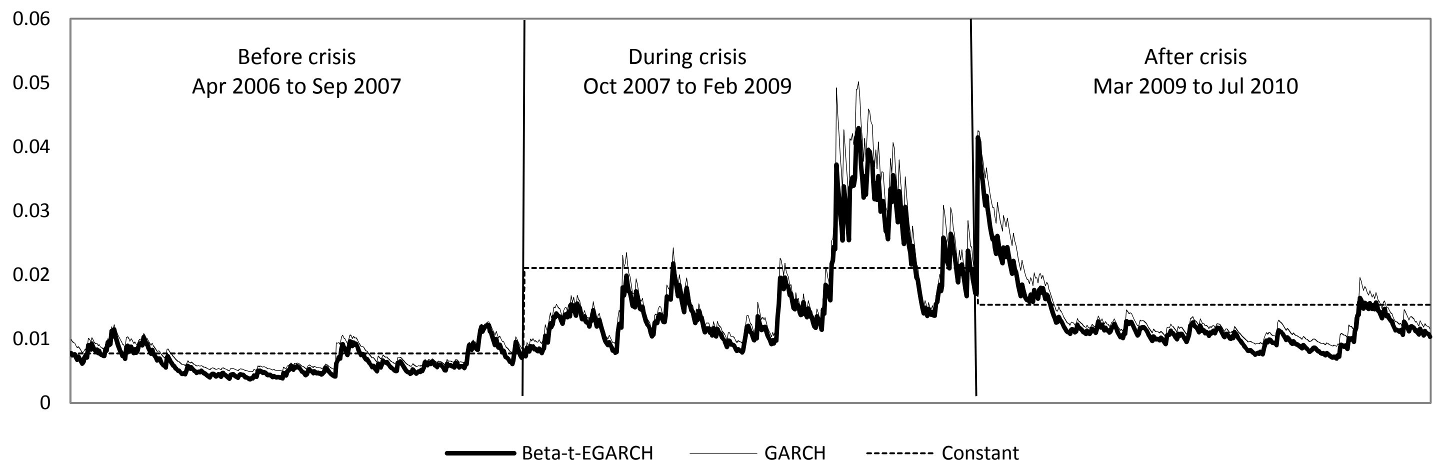

Figure 12: BWFINL volatility

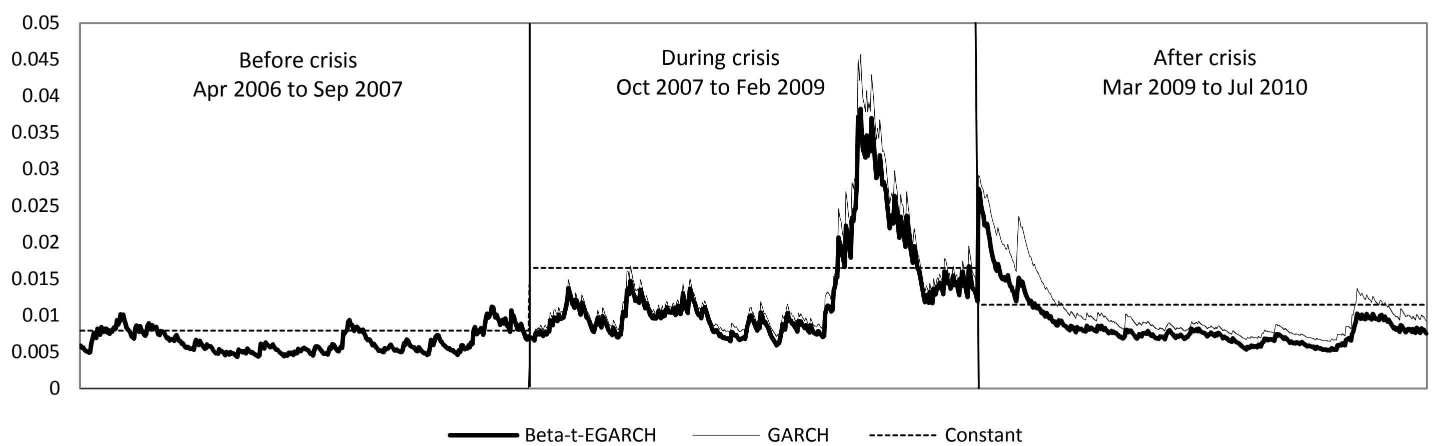

Figure 13: BWCOMM volatility. *Note*: GARCH for the before crisis period is not available due to numerical errors in parameter estimation.

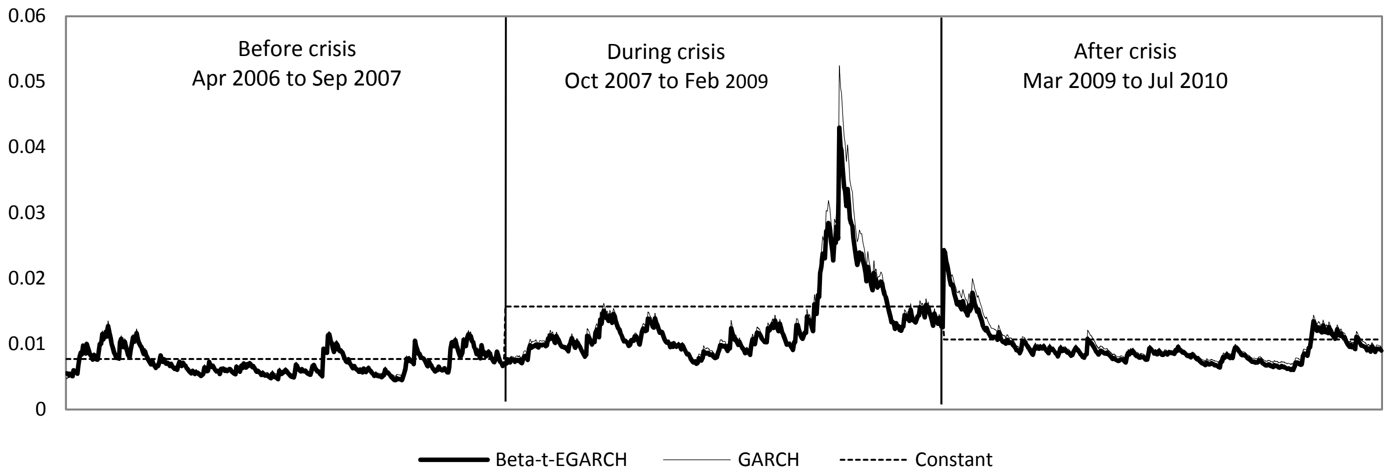

Figure 14: BWCCYS volatility

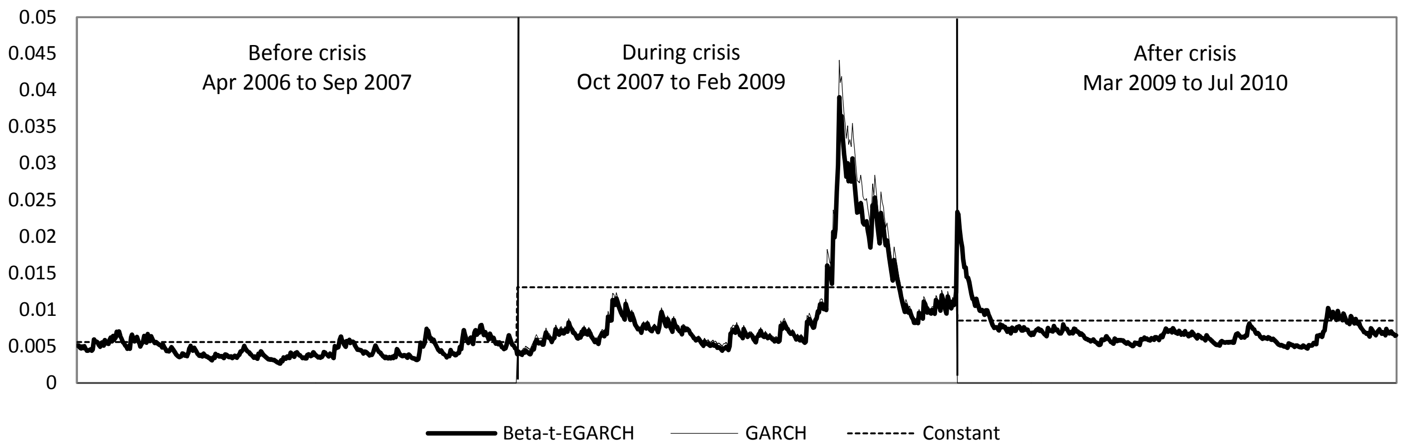

Figure 15: BWCNCY volatility. *Note:* GARCH is not available for the before and after crisis periods due to numerical errors in parameter estimation.

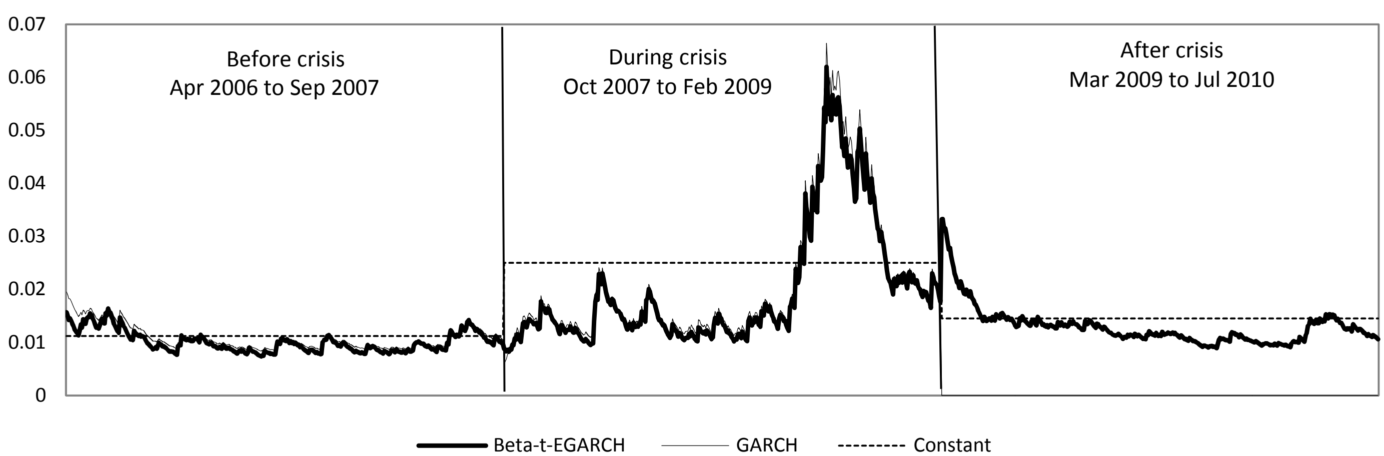

Figure 16: BWENRS volatility. *Note:* GARCH is not available for the after crisis period due to numerical errors in parameter estimation.

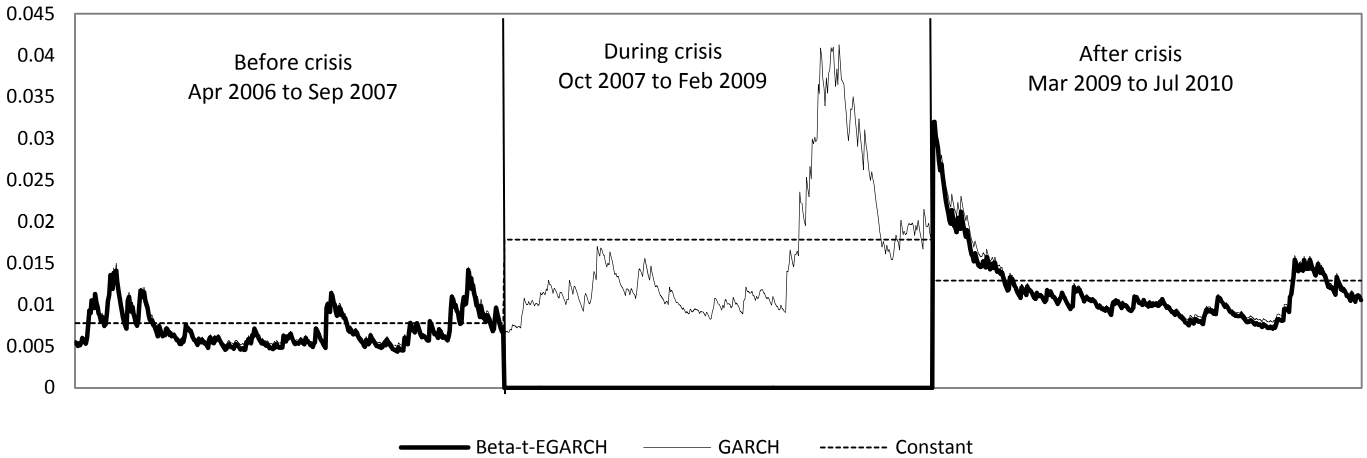

Figure 17: BWINDU volatility. Note: Beta-*t*-EGARCH is not available for the during crisis period due to numerical errors in parameter estimation.

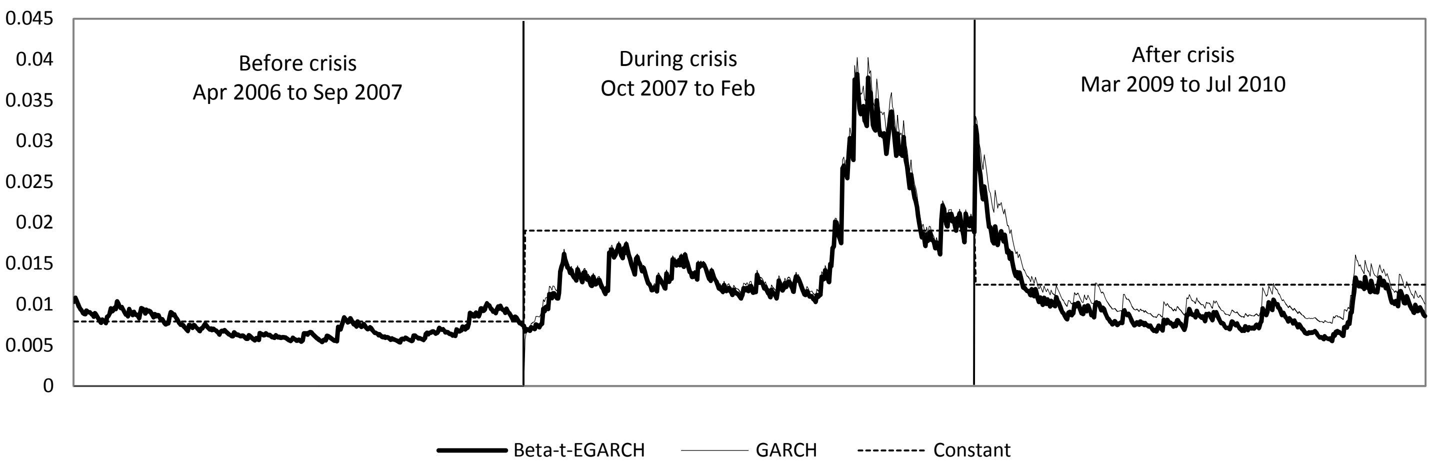

Figure 18: BWTECH volatility. *Note*: GARCH is not available for the before crisis period due to numerical errors in parameter estimation.

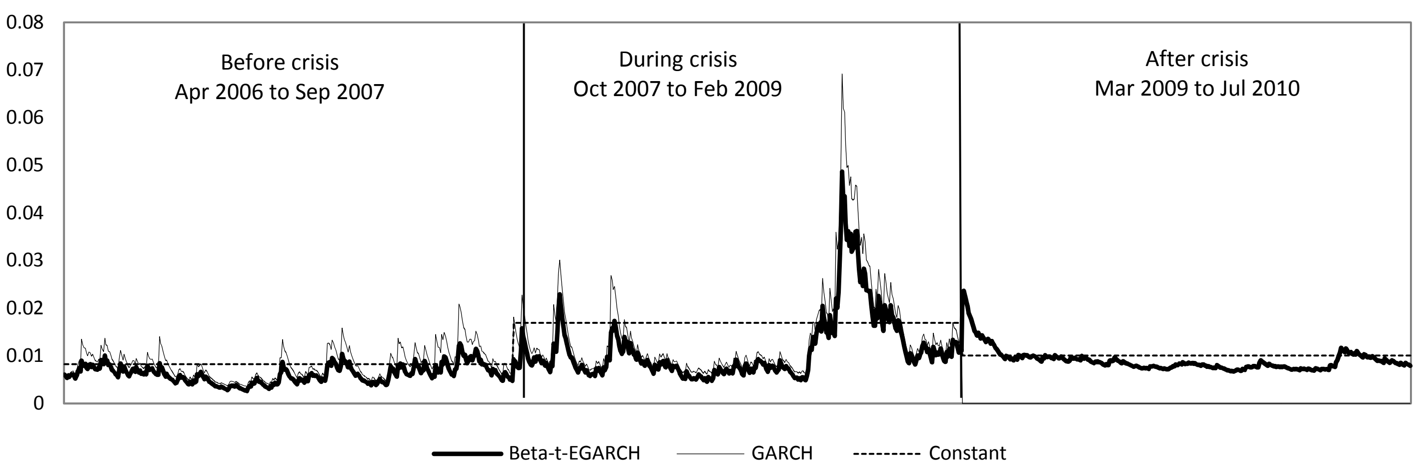

Figure 19: BWUTIL volatility. *Note*: GARCH is not available for the after crisis period due to numerical errors in parameter estimation.

4. In-sample statistical performance

We use two likelihood-based model selection metrics to compare the statistical properties of both models. The first one is the Log Likelihood (LL) of data which is based on the probability that the model will be accurate on explaining the dependent variable; i.e. index return. Higher values of LL indicate better model performance. The second metric is the Bayesian Information Criterion (BIC), $BIC = K\ln(T) - 2LL$, where K is the number of parameters included in the specification and T is the number of time periods observed. Lower BIC value indicates better model performance. The likelihood of data increases when a new parameter is added to the model. The BIC introduces a penalty term for the number of parameters included in a model. Table 9 shows the likelihood-based models performance metrics for all assets and all sub-periods.

Table 9: LL and BIC for GARCH(1,1) and Beta-*t*-EGARCH(1,1)

Before crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
Beta-*t*-EGARCH (1,1)									
LL	**1152.8**	**1304.9**	**1291.6**	**1292.1**	**1412.2**	**1149.8**	**1300.5**	**1275.2**	**1285.6**
BIC	**-2270.2**	**-2574.3**	**-2547.8**	-2548.7	-2789.0	-2264.1	-2565.6	-2514.8	**-2535.7**
GARCH (1,1)									
LL	1121.2	1296.0	NA	1291.1	NA	1148.7	1298.1	NA	1267.7
BIC	-2218.9	-2568.3	NA	**-2558.6**	NA	**-2273.8**	**-2572.5**	NA	-2511.7
During crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
Beta-*t*-EGARCH (1,1)									
LL	946.2	**973.2**	**1081.4**	**1082.3**	**1204.9**	**947.4**	NA	**1000.3**	**1106.9**
BIC	-1857.0	-1911.0	-2127.4	-2129.1	-2374.3	-1859.2	NA	-1965.1	**-2178.4**
Garch(1,1)									
LL	**946.9**	970.6	1080.0	1081.3	1202.6	946.8	**1048.9**	999.5	1095.1
BIC	**-1870.1**	**-1917.5**	**-2136.3**	**-2139.0**	**-2381.5**	**-1870.0**	**-2074.1**	**-1975.4**	-2166.6
After crisis	BWBMAT	BWFINL	BWCOMM	BWCCYS	BWCNCY	BWENRS	BWINDU	BWTECH	BWUTIL
Beta-*t*-EGARCH (1,1)									
LL	NA	**1067.1**	**1181.7**	**1178.5**	**1265.4**	**1062.0**	**1116.5**	**1139.7**	**1195.0**
BIC	NA	-2098.8	**-2327.9**	-2321.4	**-2495.3**	**-2088.5**	-2197.6	**-2243.9**	**-2354.6**
Garch(1,1)									
LL	NA	1064.8	1174.6	1177.3	NA	NA	1116.3	1133.4	NA
BIC	NA	**-2105.9**	-2325.5	**-2331.0**	NA	NA	**-2209.0**	-2243.1	NA

Notes: Not Available (NA) due to numerical problems in the parameter estimation procedure. Better model performance is indicated by bold numbers in the table.

We were able to estimate the Beta-*t*-EGARCH model for more cases than the GARCH model due to the fact that we have less model specification problems. This shows that the Beta-*t*-EGARCH is a more robust volatility model than the GARCH model. Based on the LL metric we find that for 24 estimations out of 27, the Beta-*t*-EGARCH model is superior. Moreover, we also compare model performance by the BIC metric since it penalizes for the number of parameters. Under the BIC model selection criterion, we find that for 17 estimations out of 27, the GARCH model is superior. These results show that the LL and BIC based model comparison does not evidence the clear dominance of Beta-*t*-EGARCH versus the GARCH. For further comparison, in the following sections we compare the in-sample point forecasts and the out-of-sample density forecasts of GARCH and Beta-*t*-EGARCH models.

5. In-sample point forecast performance

In this section, we compare the one-step ahead and in-sample predictive performance of GARCH and Beta-*t*-EGARCH models. We focus on the crisis period of October 2007 to February 2009. To evaluate the predictive performance of competing models, we need a benchmark true volatility to which we can compare the estimated ones. In order to approximate the true volatility of index returns, we follow the approach suggested by Pagan and Schwert (1990) and Day and Lewis (1992). A proxy for the true volatility is given by $\dot{\sigma}_t = |y_t - \bar{y}|$, where $\bar{y}$ is the average return over the sample period. For all assets, we evaluate the distance of the estimated volatility from the absolute return by the Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma_t - \dot{\sigma}_t)^2} \tag{15}$$

where σ_t is the estimate of volatility either by the GARCH(1,1) model or by the Beta-*t*-EGARCH model. Lower RMSE value evidences more precise one-step-ahead in-sample forecasting performance. The RMSE loss function is used because Hansen and Lunde (2006) and Patton (2011) show that the MSE is a robust loss function (Patton 2011) of volatility forecasts evaluation. The RMSE estimates for both models are presented in Table 10.

Table 10. In-sample forecast evaluation by RMSE

Index	GARCH(1,1)	Beta-*t*-EGARCH(1,1)
BWBMAT	**1.47%**	1.49%
BWFINL	**1.34%**	1.43%
BWCOMM	**1.07%**	1.12%
BWCCYS	**1.04%**	1.10%
BWCNCY	**0.85%**	0.88%
BWENRS	**1.57%**	1.61%
BWINDU	NA	1.11%
BWTECH	**1.24%**	1.27%
BWUTIL	**1.11%**	1.19%

Notes: Bold numbers indicate the lower RMSE, i.e. the more precise in-sample forecasting performance.

The results of Table 10 show that for all assets, the in-sample predictive performance of the GARCH model is superior to that of the Beta-*t*-EGARCH model during the period of financial crisis.

6. Out-of-sample density forecast performance

For practitioners, out-of-sample forecasts are more useful since they estimate volatility models for a historical data window, take their portfolio decisions in the present and invest over a future investment time horizon. When we evaluate out-of-sample forecasting performance, we focus on the density forecasts instead of the point forecasts. We do this since, given that a model provides better density forecasts, we can perform Monte Carlo simulation experiments using the parameter estimates obtained for past data. This would allow the investor to study better the possible future evolution of asset returns than simple point forecasts of future returns. In this section, we build on the results of Amisano and Giacomini (2007) who suggest comparing out-of-sample density forecast performance of competing models by loss functions based on the log score function.

We define the loss function used for model performance evaluation as follows. Let $\ln f(y_t|\theta_{t-})$ denote the log density of future returns evaluated according to the parameter estimates of GARCH model obtained for a historical time window.

Let $\ln g(y_t|\theta_{t-})$ denote the log density of future returns evaluated according to the parameter estimates of the Beta-*t*-EGARCH model obtained for a historical time window.

In this study, we use the after crisis sub-period as future returns and the during crisis period as the historical data window. In other words, we estimate each volatility model using data from the during crisis period. Then, we substitute into the log density function (log score) estimated for the crisis period the data set collected from the after crisis period. If

$$T^{-1}\sum_{t=1}^{T}\ln f(y_t|\theta_{t-}) > T^{-1}\sum_{t=1}^{T}\ln g(y_t|\theta_{t-}) \tag{16}$$

then the out-of-sample density forecasting performance of the GARCH model, on average, is superior to that of the Beta-*t*-EGARCH model. Otherwise, the out-of-sample density predictive performance of the Beta-*t*-EGARCH model dominates on average. Table 11 show the average log scores for the GARCH and Beta-*t*-EGARCH models.

Table 11. Out-of-sample forecast performance evaluation by mean log scores

Index	GARCH(1,1)	Beta-*t*-EGARCH(1,1)
BWBMAT	**2.680**	2.565
BWFINL	**2.789**	2.637
BWCOMM	**3.124**	2.931
BWCCYS	**3.127**	2.933
BWCNCY	3.265	**3.265**
BWENRS	**2.742**	2.567
BWINDU	**2.921**	NA
BWTECH	**2.975**	2.711
BWUTIL	**3.200**	3.000

Notes: Bold numbers indicate the higher mean log score, i.e. the more precise out-of-sample density forecasting performance.

The table evidences that, during the crisis period, the out-of-sample density forecast performance of the GARCH model is superior to that of the Beta-*t*-EGARCH model for all assets, besides the BWCNCY index where the two mean log scores are practically identical. This motivates the use of the traditional GARCH model to conduct Monte Carlo simulation experiments of future returns of global industry indices.

7. Conclusions

The objective of this paper is to compare the recent Beta-*t*-EGARCH(1,1) model with the classical GARCH(1,1) model. We give the reader an overview of the dynamic volatility models applied and define their specifications. We calculate the returns for the World Bloomberg Index for nine different industries and divide them in three sub-periods: before, during and after the U.S. financial crisis. We calculate the parameters for Beta-*t*-EGARCH(1,1) and GARCH(1,1) using the QML method. To compare both models, we use the LL and the BIC likelihood-based model performance metrics. We find that by the LL model performance metric, the Beta-*t*-EGARCH(1,1) is the better model. However, after penalizing by the number of parameters (BIC measure), the GARCH(1,1) becomes the better model for several assets. We also evaluate the in-sample forecast performance of the two volatility models by the RMSE metric. We find that in the sample predictive performance of the GARCH(1,1) is superior to that of the Beta-*t*-EGARCH model during the period of U.S. financial crisis. We also compare the out-of-sample density forecasting performance of the two volatility models for the after crisis sub-period. We find that the GARCH(1,1) model dominates the out-of-sample density forecasting performance of the Beta-*t*-EGARCH model during the period of the U.S. financial crisis. It is important to note that in this paper we only studied two competing volatility models: the traditional GARCH(1,1) model and the more recent Beta-*t*-

EGARCH(1,1) model. Future studies should perform a comprehensive analysis of statistical and predictive performance for the Beta-*t*-EGARCH model involving a large number of competing volatility models.

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