Discussion Paper 1/2015

Guatemalan Econometric Study Group

Universidad Francisco Marroquín

March 2015

Analysis of systematic risk factors for cost of equity estimation on German enterprises

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March 2015

Abstract. The question of how much the minimum expected return in a company or an investment has to be, is academically well discussed and elaborated. But it still stays a question which has its challenge to be answered precisely. The financial market, specifically the stock market gives us plenty of information and historical data which let the investors be capable to make reasonable comparisons between firms and let them consider the best option for them. Besides the expected return of an investment, one always needs to focus on the risk. People are considered risk-averse. That is why risk it is a key figure in any investment decision. Several econometric models, among them the Capital Asset Pricing Model (CAPM), Arbitrage Pricing Theory (APT) and another multifactor models like the Fama-French model or the Carhart model have one important factor in common. They compare an investment with the market, observe the performance, characteristics, peculiarities, and with that they calculate so called risk-factors which should lead to a forecast about how the investment might perform in the future. The present document has its focus on the risk factors, which are calculated as part of the CAPM. The main objective of the analysis is to demonstrate the importance and application of the volatility of these risk-factors. There will be introduced and presented an alternative to the standard and traditional estimation considering the time variance of the different factors used in the models. This alternative will be applied with real data in order to test and evaluate its utility.

1 Overview and objectives

This document is split into two parts, a theoretical part, where the basic methods and ideas of the topic will be argued and presented. The existing tools and approaches are discussed and analyzed. In the second part of the paper, the focus is based on presenting a new approach of estimating the cost of equity or the estimation of the rate of return of public traded stocks. This approach will be tested with real data and by analyzing the results, conclusions will be made. Firstly, it will be defined what the cost of capital actually is. Public traded companies and the expected rate of return of their stock is central part of the paper. The composition and origin of the weighted average cost of capital can serve as an answer to this question. The understanding of the Capital Asset Pricing Model (CAPM) is essential because it is the tool which is going to be used in the Applications Study. CAPM is introduced in Section 3. The main objective of this work is to apply time-series of risk factors on the CAPM. To achieve this, a methodology needs to be introduced which makes it possible to get daily information of a specific time-window of risk factors. This will be done with the Kaman filter and its subjacent algorithm. Section 4 gives a breve introduction in this matter. The tools presented until Sections 5 and 6 are going to be applied on the data of 30 German enterprises which are listed in the main German stock index (DAX30). These data is going to be verified and time-series of risk factors will be analyzed. The last part is the comparison between the different methods of estimating the rate of return of a public traded share. Finally, it will be discussed if estimating the time-series is useful or can be discarded.

2 Cost of capital and its compounds

2.1 Weighted Average Cost of Capital (WACC)

The difficulty of knowing and calculating, or at least estimating, a company's cost of capital is pervasive but due to steadily increasing competition in the global market it is getting more and more important to have a precise and efficient method for this undertaking. The cost of capital rate is a classical value-driver in the Value Based Management (VBM) where the companies evaluate their different business activities with the objective to create a higher Return on Invested Capital (ROIC) as the costs of the capital, which is usually the weighted one, calculated by the cost of debt and the cost of equity. It can be used to get to the Net Present Value (NPV) of the new project, business opportunities or to value a firm in context of a Merger & Acquisitions (M&A) with the discounting cash flow model. Especially M&A scenarios, where companies get legally consolidated, by purchasing of one company by another or by a so called merger of equals, the information how much one company values, in monetary terms, plays a significant and often underestimated role. In these scenarios and company evaluations the principal idea is to know as precisely as possible how much values the investment.

In other words, the cost of capital is defined as the opportunity cost of all capital invested in an enterprise[1]; see Figure 1.

$$WACC = \frac{E}{V} * Re + \frac{D}{V} * Rd * (1 - Tc)$$

Figure 1: Weighted Average Cost of Capital

Taking a closer look on this formula we can divide its composition into the equity and debt components. The equity part is defined by the cost of equity (Re) which is weighted by the amount of equity in relation to the enterprise value. This relation is expressed by the division of equity and enterprise value (E/V). On the other side, the interest rate paid for a debt (Rd) is multiplied with the debt-enterprise-value relation (D/V). An important additional point to consider while getting to know the WACC formula is the fact that in most countries of the world the debt a company has is suspended by tax payments. In the WACC formula in Figure 1, this is included by multiplying the cost of debt weighting with the tax reduced due to the debt (1-Tc).

The weighted average cost of capital and its parameters listed in Figure 1 reflect the cost the firm is charged when raising additional capital, considering the proportion of debt and equity. This proportion is one factor where uncertainty plays a role, depending on who is trying to realize the calculations. For this you use the firms target capital structure and this information is usually just known inside the firm which makes it diffi-

[1] Ian Giddy, Stern School of Business, New York University (accessed on: 1 April 2014)

cult for outside standing persons and therefore assumptions are necessary to make. These assumptions need to be based on the available information about the company's capital structure; trends of historical data or an average capital structure of comparable firms can be used.

2.2 Cost of debt and cost of equity

The cost of debt reflects the interest rate which the firm pays annually for its debt with consideration of the tax-discount. Cost of debt is usually tax deductible; the cost of equity is not. This is reflected in Figure 1, in the formula of the WACC, by (1-Tc).

To calculate the cost of equity there are several methods. Amongst others, there are the dividend discount model (also termed as the Gordon growth model) and the CAPM.

With the Gordon growth model one can calculate the cost of equity by valuing a firms stock by discounting dividends which are distributed on a stable basis to the stockholder. There already lies its restriction, it is just applicable for firms which have been paying dividends steadily and also could have reached a stable economic growth and have no or just little changes in its financial policies. There needs to be some kind of trust and certainty that dividends are paid to perpetuity.[2] If all this is the case one can apply the Gordon growth model with the formula showed in Figure 2.

$$\text{StockValue (P)} = \frac{D}{k - G}$$

Where:
D = Expected dividend per share one year from now
k = Required rate of return for equity investor
G = Growth rate in dividends (in perpetuity)

Figure 2: Gordon Growth Model for Valuing Stocks[3]

The factor *k*, defined in Figure 2 as the required rate of return for equity investor is now the cost of equity we want to calculate. By simply forming the equation, we come to the following formula of Figure 3.

[2] http://people.umass.edu/nkapadia/FINOPMGT304/Week2.pdf (accessed on 1 April 2014)

[3] http://www.investopedia.com/terms/g/gordongrowthmodel.asp (accessed on 1 April 2014)

$$\text{Cost of Equity (k)} = \frac{D - (P * g)}{P}$$

Where:
D = Expected dividend per share one year from now
P = Stock Value
G = Growth rate in dividends (in perpetuity)

Figure 3: Cost of Equity with Gordon Growth Model

When counting with the conditions and having the already mentioned requirements fulfilled, this method has the advantage that no assumptions need to be made by using this model.

On the other hand, the CAPM uses another approach. It works with a rate of a so called "risk-free" investment as the initial point and adds a premium risk factor. As the risk-free rate usually a 10-year government bond rate is used. At this point, the uncertainty starts cause in the real world nobody really knows how trustable and risk carried the government bonds are or will be in the future.

The risk premium factor is calculated and estimated by observations, comparisons and analysis of the stock prices of other companies, industries, branches and other factors like company sizes and performances. Interesting to mention is also that there are models which includes a so called country risk-factor, for example if you consider an investment in an development country you need to apply this factor reflecting the additional risk due to the different and more uncertain conditions in the particular country. There exist several multifactor-risk models like the in the beginning of this work mentioned Arbitrage Pricing Theory (APT) and other multifactor models like the Fama-French or Carhart models. These models are not further explained in continuity of this work, but we will focus in specific on the particularity of the CAPM, which considers only one risk-factor, the so called market risk premium or β-factor. Section 3.1 is dedicated to the CAPM because it will be the tool which will be deployed in the application study further on in this document.

Professor Aswath Damodaran from the Stern School of Business at the New York University continuously publishes and updates new statistics and analysis in this area. He did this for the cost of capital of U.S firms by sector. He used the financial data of 7766 firms from different sources like S&P Capital IQ, Bloomberg and the Federal Bank of the United States. With this data Damodaran calculated the Cost of Capital by sector and with the result of an average cost of equity of 8.07% as well as an average cost of debt of 6.04%. Weighted and with consideration of the capital structures of the firms he comes to the result of and average cost of capital of 6.94%.[4]

In conclusion, the cost of capital is the cost for a company of receiving monetary funds via debts or equity with the objective to use it in investment projects or business realizations. In the same moment it is the opportunity cost for an investor for not investing money into another company or project with same risk.

[4] http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/wacc.htm (accessed in January 2014)

3 Estimation of cost of equity with expected rate of return

3.1 Capital Asset Pricing Model (CAPM)

This section has its focus on the brief explanation and the characteristics of this method used to calculate or at least estimate the cost of equity. The CAPM is chosen because it is doubtless the most famous way to estimate the cost of equity. In a survey, conducted in the U.S. for members of the Association for Financial Professionals (AFP), 90% of the participants uses the CAPM to estimate their cost of equity[5]. Secondly the CAPM is used in the application study of this work where the costs of equity of German enterprises will be estimated and analyzed.

The CAPM is based on the portfolio-selection-theory of Markowitz as well as the separation-theorem of Tobin. It was developed by Sharpe, Lintner and Mossin in the 1970s. In this model, the return of an investment depends on its risk; the value of the return consists out of the forecasted future expected return adjusted with the risk, measured by the standard deviation or volatility of the returns. This will be done with a risk-premium and therefore it is necessary to divide the non-systematic risk and the systematic risk from each other.

Based on this financial theory we can use the CAPM for the expected return estimation by application of the following formula:

$$E(r_i) - r_f = \beta_i [E(r_m) - r_f]$$

Figure 4: Expected return of a stock[6]

The main notation of this figure is $E(r_i)$, which is the return expected from a specific investment like a stock of a public traded company. R_f stands for the risk free rate, the rate one can achieve by investing in something which does not require risk. This might be a government bond or a long term bond emitted by a stable and trustable company. $E(r_m)$ is the expected market returns for example the return of a stock-index or an industry index. In order to use this market-index for the own investment it is necessary to know how the behavior and sensibility of the investment in comparison to the market. There for β_i, the risk factor is used and is the key figure of this work. It reflects the systematic risk and is the measure for the volatility of the stock compared to the market volatility.

Risk in general can be defined as the uncertainty of achieving an expected outcome. According to that definition, the more uncertainty exist, the more risk exists. Furthermore, from an investor's point of view, the more risk an investment has, the investor is willed to pay less or expect more in return. Both the non-systematic and the systematic risk in addition sum the total risk of an asset. These two types of risks will be

[5] Jacobs et al. (2012)

[6] Blazsek (2014)

discussed and illuminated; nevertheless, the further course and the econometric model used in this study will focus on the market-specific risk and its methods of calculations.

3.2 Firm specific risk versus systematic risk

The firm specific risk or also called non-systematic risk is determined by so called internal factors which are not related to the total market and just affect the firm itself. The firm specific risk is related to a specific investment and is determined by the quality of the management, internal decisions and strategic planning. For an investor of this specific investment, this risk can be reduced by having the right risk-management strategy applied to the portfolio. For the investor, this risk actually can be reduced to zero by diversification. This is not the case, if e.g. a company owner invests in his own company without any other investments in order to establish diversification.

On the other hand, the systematic risk has impact on all companies of the market and is influenced by global, macroeconomic factors like world economy, inflation, interest rates, as well as, currency exchange rates, amongst others. Due to the listed factors, it cannot be reduced or diversified. Because of the same reason, we will use in continuity the beta, the market risk factor, as the overall risk measure. Figure 5 illustrates the two different types of risks which are responsible for the volatility, σ of a security.

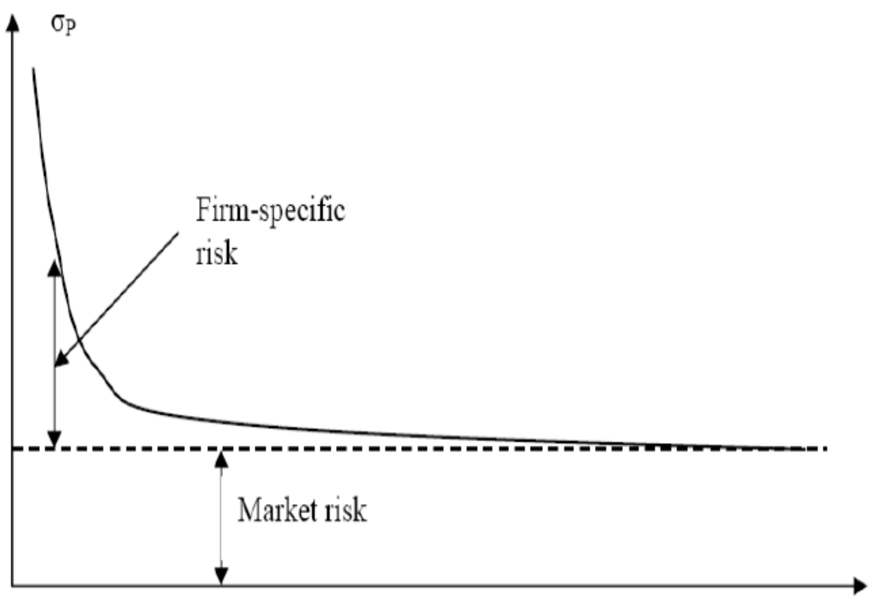

Figure 5: Decomposition of Firm-specific and Market risk[7]

[7] Blazsek, S., UFM Presentation, Econometrics Class, 2014

3.3 Calculation of the expected return of a security with the CAPM

Since the actual objective of this work is not the creation of an investment portfolio with several different securities, the firm specific risk will not be considered any further and it will be focused on the market risk represented by the β-coefficient.

Observing the regression equation in order to calculate the expected return of a security we have to find out what the different parameters stand for. Bodie et al. (2009) explain the equation in Figure 6.

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

Figure 6: Regression equation of single factor model[8]

They explain α as the expected excess return in the case when the market excess return is zero. This is due to the valuation of the security or share. If the result is that the security or share is underpriced, α is larger than as it would be overpriced. In the literature of Bodie et al. α is also called "nonmarket premium" and it is assumed that market analysts come up with different and own estimates of α.

In the time series analysis, we will use in the following section of this study, we will see that the α-factor is nearby zero and therefore negligible for further interpretations. In the Application Study of Section 6 in this document, α is going to be calculated but only β is considered in the analysis and interpretation. When it is about calculating the equity cost of capital of the firms later on, it also will be done with only β. e is a disturbance value which reflects unobservable effects like human behavior measurement imprecision. In this model, e has zero mean and it is called the firm specific surprise of the security. This should not be considered in the following section.

β is the sensitivity of the security to the market index used in the model. The importance of the β-coefficient, which determines the systematic or market risk, is to observe it constantly. When this observation is guaranteed, it is possible to estimate the volatility and the sensitivity of the returns of shares or securities to the variation of its value.[9]

In general terms, the β-coefficient reflects the sensibility of the stock-price in relation to the market. Therefore if result a company's β-coefficient as one it means that the share-price moves with no significant difference to the market. When the beta is significantly under one, the volatility of the firms share price is less than the market and the investment of the company's shares can be declared less risky. On the contrary, if the β-coefficient is significantly higher than one it also means that the investment in this kind of shares brings a higher risk.

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[8] Bodie, Kane and Marcus, 2009, *Investments*, Chapter 8, page 247.

[9] Bodie, Kane and Marcus, 2009, *Investments*, Chapter 8, page 247.

3.4 Multiple factor models for calculation of expected return

"The success of factor models in predicting returns in financial asset markets and analyzing risk depends on both the choice of risk factors and the method for estimating factor sensitivities"[10]

In the same way as the CAPM, the objective of multiple factor models like the mentioned Fama-French model, APT or the Carhart model is the estimation of the expected return of an investment, a security or a stock portfolio. The motivation of doing this with factor models is the application of not just one but more factors which have an impact on the return on the firm's share price. No matter if you use one factor or more, the outcome depends on the quality of the chosen factors and of the econometric specification which is used to estimate the parameters of the factor model. Besides using the factor model for expected return estimation, it also is useful to estimate the covariance matrix for a portfolio of shares or to decompose the total risk into the systematic and the non-systematic risk.[11] In the following paragraph, I specify different factors used in the three multi-factor models mentioned above.

The authors of the Fama-French-Model (1993), Fama and French, claimed, that the explanation of realized return with just one factor like in the CAPM was not enough and lead to a poor performance. Against that, they hold the statement that it needs three factors[12]:

1. The excess market portfolio return, which is the same one of the CAPM, calculated with the covariance of the portfolio and the market.
2. The reflection of the difference between the excess return on a portfolio with small stocks and the excess return on a portfolio with larger stocks (measured by the market capitalization). The name of the factor is called Small-Minus-Big (SMB).
3. The difference between the excess return on a portfolio with high-book-to-market stocks and the excess return on a portfolio with low-book-to-market stocks. This factor is called High-Minus-Low (HML).

Figure 7 illustrates the Fama-French formula to estimate the expected return of a portfolio.

$$E(R_i) - R_f = b_i[E(R_M - R_f)] + s_i E(SMB) + h_i E(HML)$$

where

$E(R_i)$: Expected rate of portfolio return.

R_f: Risk-free rate of return.

$E(R_M - R_f)$: Expected rate of excess market portfolio return.

E (SMB): Expected value of the difference between the excess return on a portfolio of small stocks and the excess return on a portfolio of big stocks.

E (HML): Expected value of the difference between the excess return on a portfolio of high-book-to-market stocks and the excess return on a portfolio of low-book-to-market stocks

Figure 7: Fama French Formula with Explanation[13]

[10] Alexander, Market Models, Chapter 8, page, 229, 2001

[11] Blazsek, S., UFM Presentation Econometrics Class, 2014

[12] Eraslan, V. (2013)

As explained above, the Fama-French model adds two more factors to explain the expected return of a security or a portfolio in order to try to absorb the anomalies and deficiencies of the one factor CAPM.[14]

APT, introduced by the economist Stephen Ross in 1976, uses as well more than one factor to estimate the expected return of a security or a portfolio. The APT relates the price of an asset, with several “systematic” factors. This different common factors influence the returns which is reflected in the formula of the APT. Another significant difference is that the APT is derived from a statistical model, while the CAPM is an equilibrium model, it assumes market equilibrium. Factors which can be used to determine could be for example: consumer behavior index; inflation rate; interest rate of the central bank.

Other factors could be the gold- or the oil price. With consideration of all of these mentioned factors there will be created a formula of a linear regression and the expected return can be estimated. See Figure 8.

$$E(r_p) = r_f + \beta_{p1}[E(r_1) - r_f] + \beta_{p2}[E(r_2) - r_f]$$

We need to find the risk premium, RP, for each of the two factors; $RP_1 = [E(r_1) - r_f]$ and $RP_2 = [E(r_2) - r_f]$.

Figure 8: Arbitrage-Pricing-Theory Formula [15]

The last multifactor model worth to mention is the Carhart model which is also called the Carhart-four-factor-model. It is very similar to the Fama-French model but includes and considers besides the size and book-value factors also another factor called the momentum factor, also known as the MOM-factor.

Normally, people hold or buy assets or securities that have had a consistent appreciation, in other words, a consistent growth in return or price. This behavior incorporates the momentum factor so that the positive performance has a positive influence in the Factor-Model and a negative performance a negative one. The momentum factor intents to quantify human behavior tendencies in order to estimate the expecting future return of an investment[16].

[13] Eraslan (2013)

[14] Eraslan (2013)

[15] http://eml.berkeley.edu/users/webfac/shomali/e136_s04/136.3.pdf (accessed in April 2014)

[16] http://www.atlasca.com/education/portfolio-strategy/factor-models/ (accessed in April 2014)

4 Kalman filter for time series analysis of market risk factors

The Harvard Business Review article of Jacobs et al. and especially the realized survey with more than 300 respondents of its global membership firms makes it clear[17]. 90% of the respondents use in their companies the CAPM for the cost of equity estimation but there seems to be no consent at all when it is about the estimations to be made using this popular model. When using this model there must be realized a future expected market return forecast, the answers of which time horizon are used in the companies goes from 5 years until 15 years. What risk free rate to use? U.S. Treasury Bonds with 90 days or with 30 years period? When calculating the company's β-coefficient, what period do the companies use? See Figure 9.

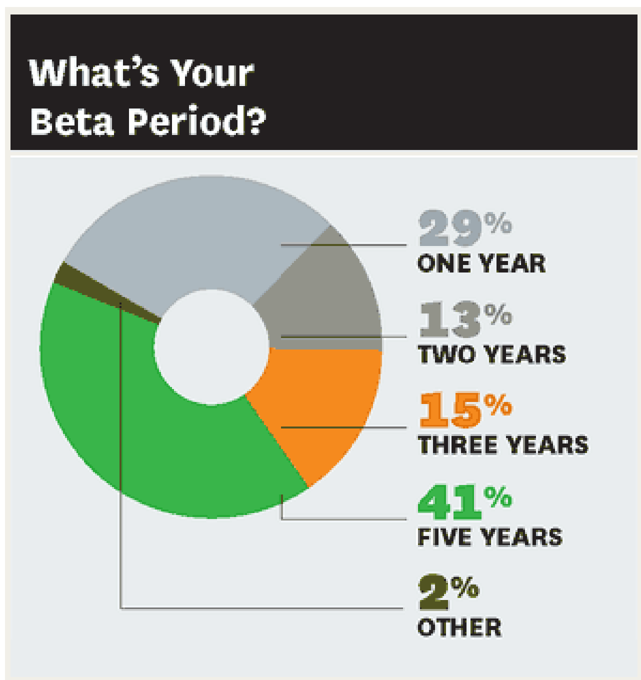


Figure 9: AFP Survey: β-period[18]

It is obvious that all this uncertainties have a significant impact on strategic decisions which are going to be made regarding enterprise valuation or accepting or rejecting an investment project. Jacobs et al. exemplified, that a 1% underestimated cost of capital can turn the sign of the Net Present Value of a $20 million investment from $1 million into $-1 million. Therefore it is important to contemplate about the different assumptions and estimations used for calculating the cost of equity, the cost of debt and the resultant Weighted Cost of Capital. In the further course of this paper I will just focus on the risk β-coefficient.

Central point is the introduction and application of a method which makes it possible to analyze and calculate the cost of equity of a company with the financial information of the historical stock prices as well as market information; e.g. of the S&P 500. The main idea is to visualize the market specific risk factor of a

[17] Jacobs et al., HBR. July 2012

[18] Jacobs et al., HBR. July 2012

company in function of the cycles wished to be analyzed; this might be monthly, weekly or daily. This happens with an algorithm, designed and introduced by R. E. Kalman. In this section, I will present and describe this method in order to continue with the application and event study in the following section.

4.1 Variation of the β-coefficient over time; example of Google

The traditional approach of calculating possible future returns of an investment, in this case company shares can be done with the already described CAPM. Actually, the beta-factor has very different values in the progress of time. This approach is based on the assumption that it is possible to calculate always one true-value for every single point or moment in time.[19] Exactly this is what is going to be observed in the actual part of this work.

If you, for example, observe on yahoo finance[20] the share-price of Google (GOOGL), you will find a suggested beta of 0.89. At this moment you don´t know against which market index this were calculated, and especially not what historical time period was used. Assuming that the market index is the Nasdaq Composite Index we now want to realize some calculations focusing on the different periods for beta calculation. We use the historical prices on a daily base of the last five years (from 7 April 2009 to 7 April 2014) from Google but as well as from the Nasdaq Composite, because Google is listed in this index. The beta for the different time periods will now be calculated with the following formula on Figure 10.

$$\beta = \frac{\mathrm{Cov}(r_a, r_b)}{\mathrm{Var}(r_b)}$$

where Cov and Var are the covariance and variance operators.

Figure 10: Formula for beta calculation

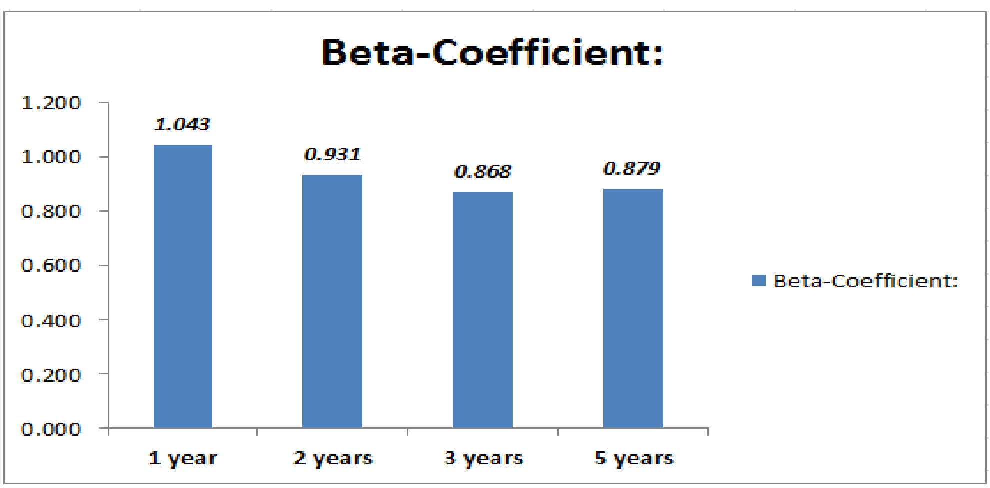


Figure 11: Betas calculated for different historical time periods

[19] Alexander (2001), *Market Models*, Chapter 8, page 238.

[20] See Yahoo Finance, 8 April 2014.

Figure 11 shows clearly the variation of different betas for the different time periods. It looks that the beta of 0.89 published in yahoo finance is calculated with the historical data of the last 3 to 5 years while observing the beta factor of the last one to two years it is in the range of from 0.93 to 1.04. This leads to a 14% gap between the beta of 0.89 and the 1-year beta of 1.04.

Processing this data in the mentioned Kalman filter algorithm and visualizing the beta-factor over time, it is quite obvious that there does not exist any consistent continuity.

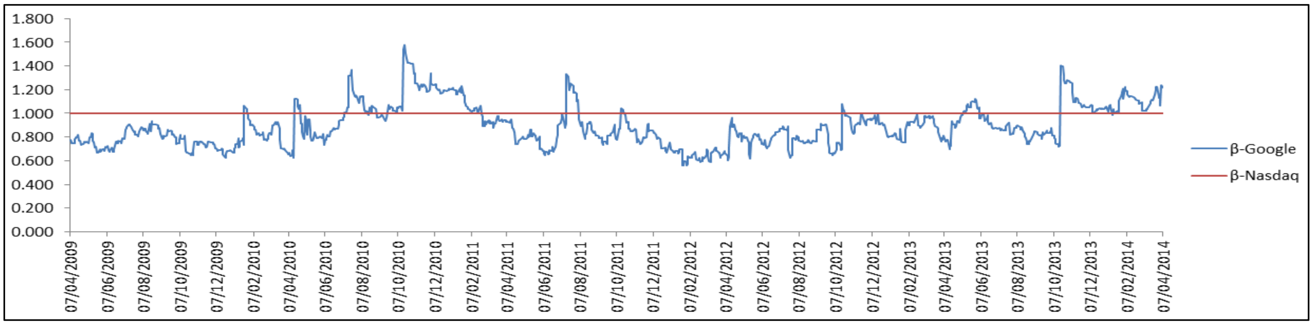


Figure 12: 5-year time series of beta (Google)

The methodology of calculating the time series of these beta as well as its applications will be the focus of the next sections of this work.

4.2 Methodology and *β*-calculation

From this point on, we will just focus on the β-calculation with the already mentioned Kalman filter algorithm. The Kalman filter algorithm, developed and introduced in 1960 of R. E. Kalman is a set of mathematical equations which provides a mean to estimate a state of a process, doing so by minimizing the mean of the squared error of the historical data. This so called "discrete" Kalman filter algorithm realizes an estimation of a process by applying a mechanism of feedback control. The algorithm calculates the process state at a specific moment and obtains feedback in form of (noise) measurements. Welch and Bishop explain the approach with two groups of equations: a) time update equations and b) measurement update equations. The first mentioned time update equations are necessary to project the current state of the process and the measurement equations are responsible for the feedback.[21]

The following two application-studies intent to approve or to disprove the functionality and to show the benefits of the time-series-analysis of the risk-factor beta and therefor the CAPM with utilizing the Kalman-filter method.

[21] Welch, Bishop (2006)

5 Case studies: application and event studies with Kalman filter on German DAX enterprises

5.1 Objective and approach

The objective of this section is to apply the already described Kalman filter on historical data of German DAX-30 enterprises in order to visualize both mentioned important factors α and β. These graphs will be analyzed and interpreted as well as the different companies compared. The main objective of this study will be to evaluate if conclusions can be made about the time-series of the factors and if yes, what consequences this has on the estimation of the company respectively the traded shares. In transition to the next section this time series analysis will be used in order to calculate the cost of equity of each firm and compare it with the one calculated by the traditional linear-regression-analysis and the CAPM.

5.2 DAX-30 enterprises

The DAX-30 is the leading index of the German stock market; it includes titles which have the highest market capitalization and the highest stock turnover of the existing German public traded companies. To get listed into the DAX-30, there needs to be fulfilled further criteria. The companies need to be listed in the prime standard of the German Stock Exchange, need to have at least 10% widely held stock and the legal place of business has to be in Germany. The rate of the DAX-30 is composed by the underlying firms; however the firms are weighted differently, so a company with a higher market capitalization determines the DAX-30 stronger than a lower one.[22]

I chose the mentioned index because due to its diversity and its compactness regarding the quantity of the underlying firms it reflects well and is a good indicator of the German economy[23].

[22] See also: http://www.finanzen.net/index/DAX/30-Werte

[23] Please note that the share of Volkswagen (VOW3.DE) is not considered, since no complete historical data have been available on yahoo finance. Therefore, 29 DAX enterprises are used in this study.

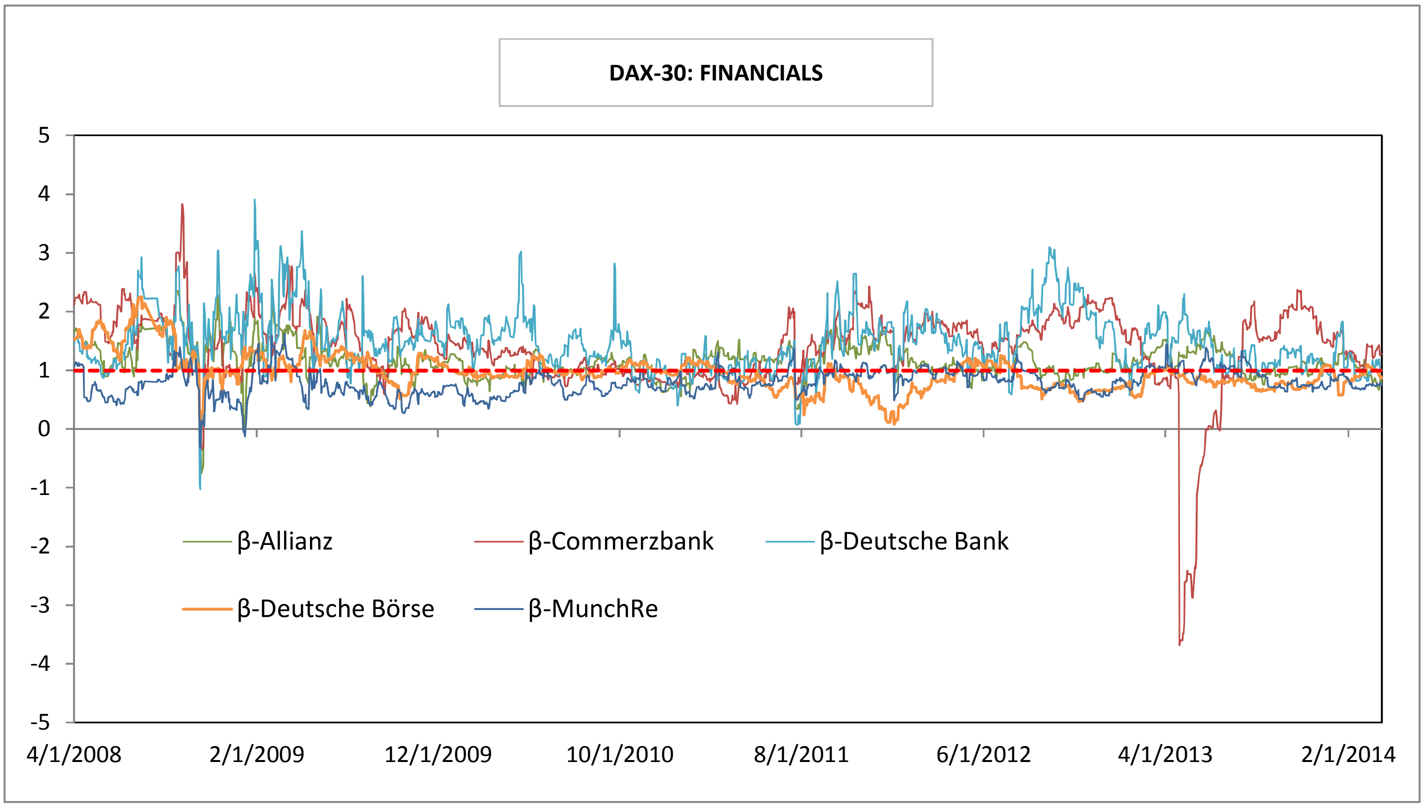
DAX-30-firm	Symbol	Industry	Revenue 2012
Adidas	ADS.DE	Clothing	14.9
Allianz	ALV.DE	Insurance	72.1
BASF	BAS.DE	Chemicals	78.7
Bayer	BAYN.DE	Pharmaceutical & Chemicals	39.8
Beiersdorf	BEI.DE	Consumer Goods	6
BMW	BMW.DE	Manufacturing	76.8
Commerzbank	CBK.DE	Banking	10
Continental	CON.DE	Manufacturing	32.7
Daimler	DAI.De	Manufacturing	114.3
Deutsche Bank	DBK.DE	Banking	33.7
Deutsche Börse	DB1.DE	Securities	2.2
Deutsche Post	DPW.DE	Communications	55.5
Deutsche Telekom	DTE.DE	Communications	58.2
E.ON	EOAN.DE	Energy	132.1
Fresenius Medical Care	FME.DE	Medical	10.7
Fresenius	FRE.DE	Medical	19.3
HeidelbergCement	HEI.DE	Building	14
Henkel	HEN3.DE	Consumer goods	16.5
Infineon Technologies	IFX.DE	Manufacturing	3.9
K+S	SDF.DE	Chemicals	3.9
Lanxess	LXS.DE	Chemicals	9.1
Linde	LIN.DE	Industrial gases	15.3
Lufthansa	LHA.DE	Transport Aviation	30.1
Merck	MRK.DE	Pharmaceuticals	11.2
Munich Re	MUV2.DE	Insurance	52
RWE	RWE.DE	Energy	53.2
SAP	SAP.DE	IT	16.2
Siemens	SIE.DE	Industrial, electronics	78.3
ThyssenKrupp	TKA.DE	Industrial, manufacturing	40.1
Volkswagen	VOW3.DE	Manufacturing	192.7

Table 1: DAX-30 Enterprises with Industry and Revenue in 1000 millions of Euro[24]

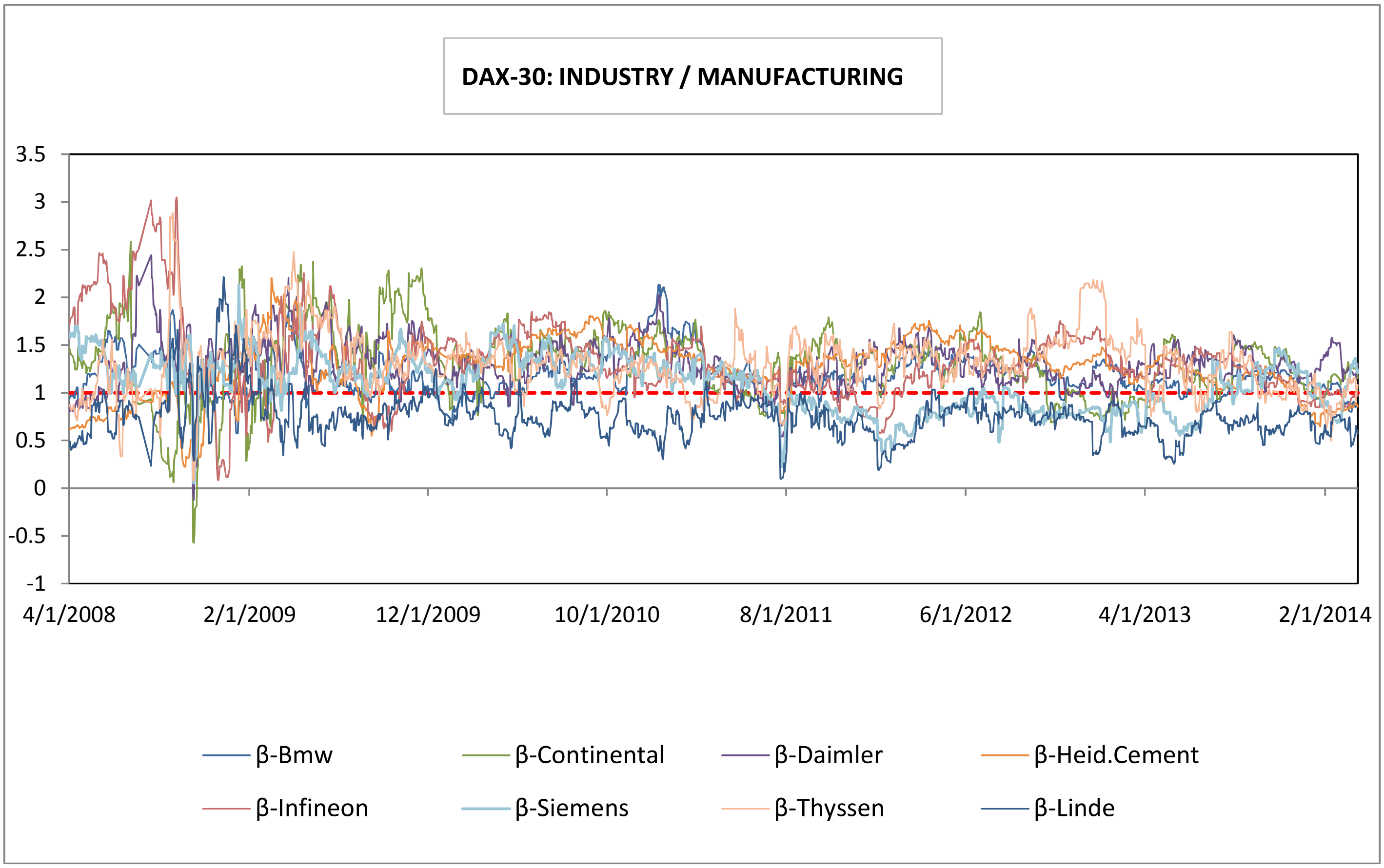
[24] http://de.statista.com/statistik/daten/studie/75495/umfrage/umsaetze-der-dax-konzerne/ (accessed in April 2014)

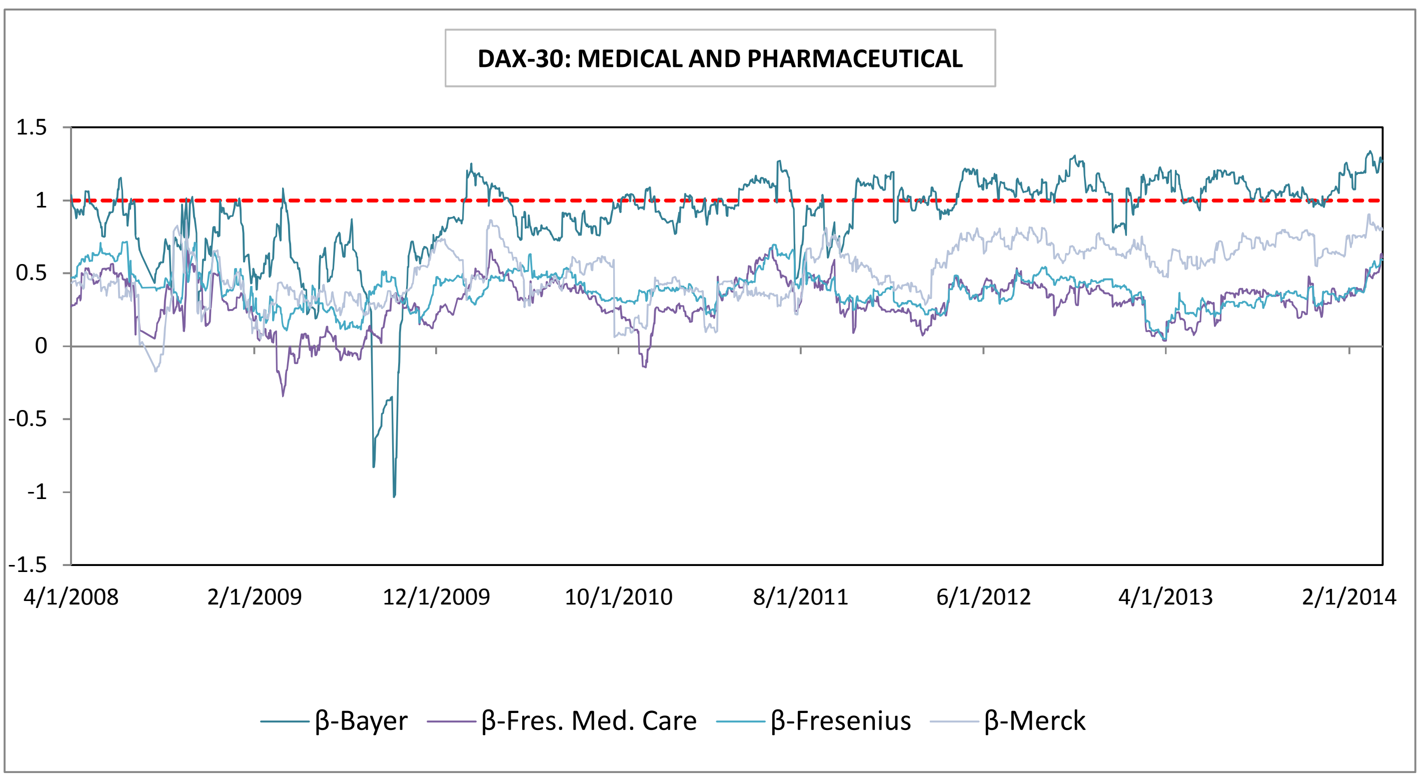
5.3 Data and beta calculation

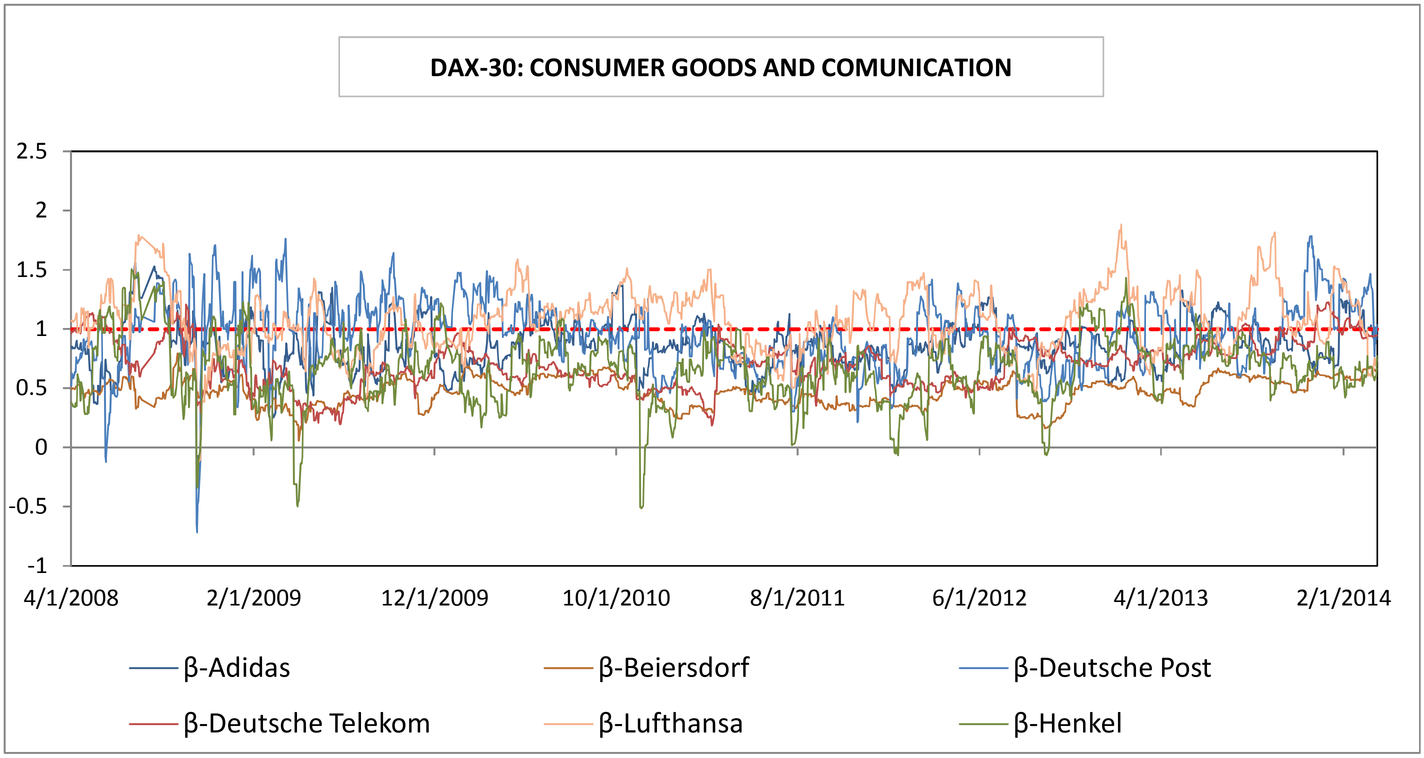
Historical data of the stock prices were downloaded from the Yahoo-Finance website and the daily return were calculated for the time period from April 2008 to March 2013 in order to have a time window of six years. The same was done with the historical data of the DAX index; this data is needed to calculate the risk-factors of the DAX companies. After consolidating the information in the corresponding excel sheet, each daily return, together with the DAX return, were used to estimate the evolution of alpha and beta by maximum likelihood method by using the Kalman filter. The next figures show the results of the beta time series[25]. To keep a clearly arranged breakdown of the data, the 29 company betas are distributed into five different figures, classified by industry.

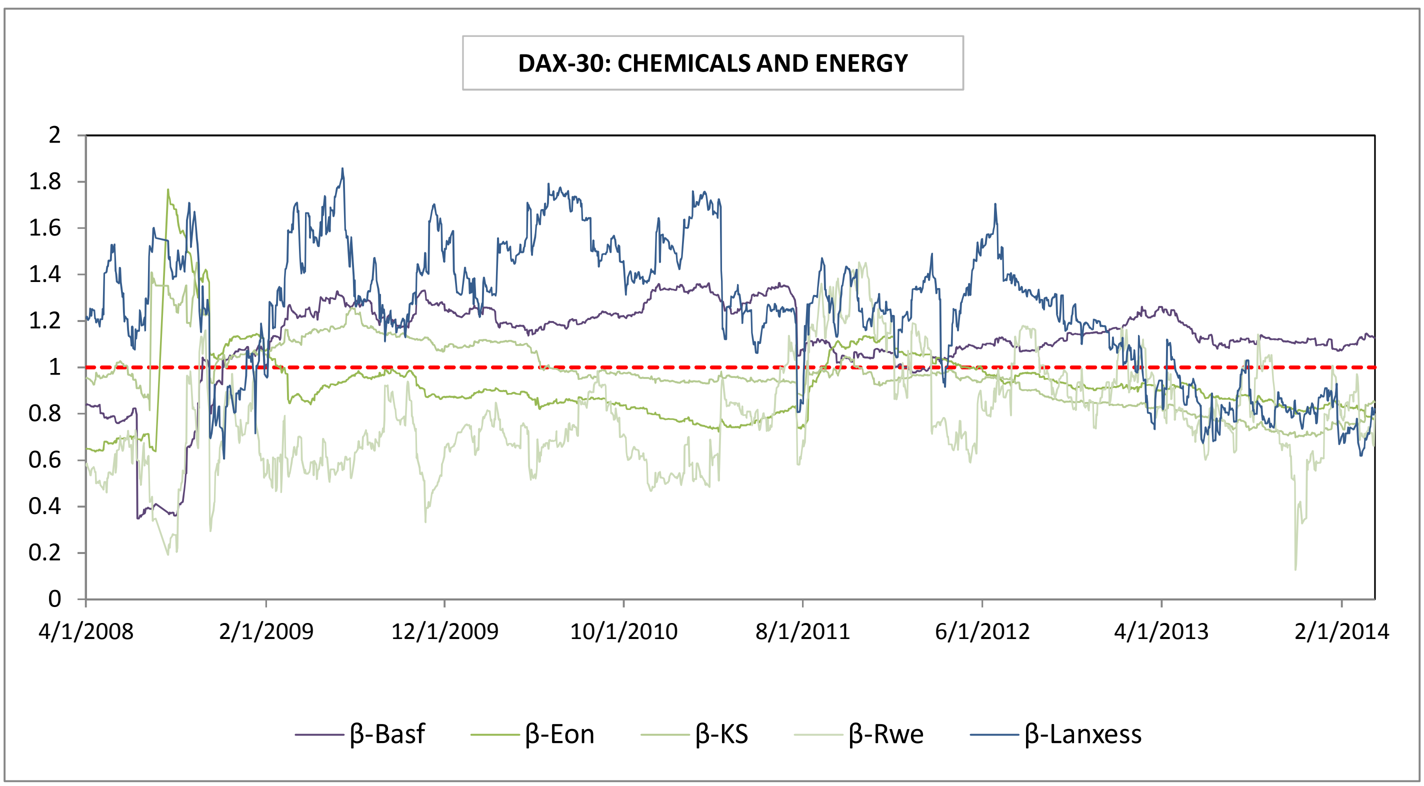


[25] Please note that the share of Volkswagen (VOW3.DE) is not considered since no complete historical data have been available on Yahoo Finance. Therefore, 29 DAX enterprises are studied in this work.


DAX-30: INDUSTRY / MANUFACTURING
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
4/1/2008
2/1/2009
12/1/2009
10/1/2010
8/1/2011
6/1/2012
4/1/2013
2/1/2014
β-Bmw
β-Continental
β-Daimler
β-Heid.Cement
β-Infineon
β-Siemens
β-Thyssen
β-Linde


DAX-30: MEDICAL AND PHARMACEUTICAL
1.5
1
0.5
0
-0.5
-1
-1.5
4/1/2008
2/1/2009
12/1/2009
10/1/2010
8/1/2011
6/1/2012
4/1/2013
2/1/2014
β-Bayer
β-Fres. Med. Care
β-Fresenius
β-Merck





By visualizing all beta-graphs into these illustrations, there is on the first view more confusion than observations to make. To answer the question what information we can get from that graphic, we need to focus on the dispersion, calculated with the variance formula, of all the stocks. It is quite obvious the collective behavior of the betas of all company stocks has certain relation to the market situation. The market situation is reflected by the DAX-30 values, which is illustrated last figures.

One can observe the higher dispersion of all beta-values until approximately April 2009. From this point on the deviation goes back in a continuously descending manner. Focusing now on the behavior of the DAX index, we observe a depreciating tendency until the same point of time, April 2009. Observe Figure 13.

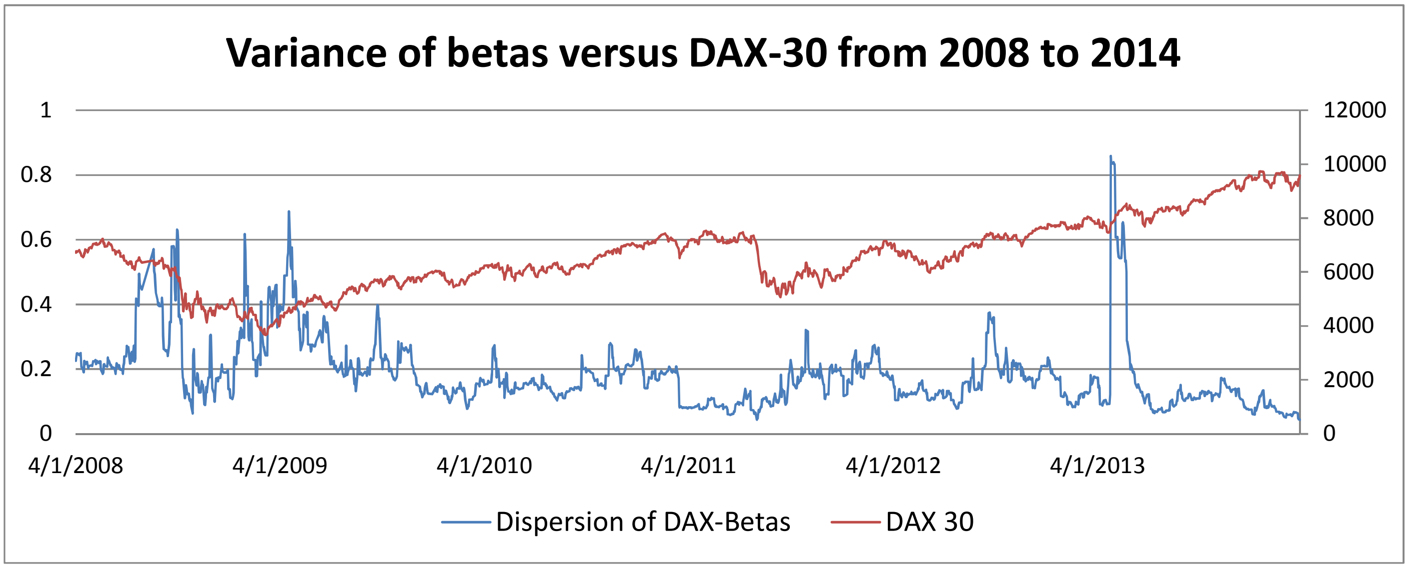


Figure 13: Variance/Dispersion of Betas

Analyzing this graph by simply observing the higher dispersion or variance of the DAX betas, it is recognizable that it ends with the U.S. subprime mortgage crisis of 2008. After the mentioned point, approximately middle of April 2009 the dispersion of the betas goes rapidly under the mark of 0.2 and the DAX value shows a tendency turn into a positive slope, which leads to an appreciation of the DAX values in general. The short-term peak of the beta dispersion in May 2013 is due to an extraordinary share price behavior of the Commerzbank stock, this can also be observed in the summary graph of the betas. This strong negative beta is due to a split event of the share price and led to extreme short term share depreciation. In the analysis of beta-dispersion and DAX index, this is treated as an outlier and not further considered.

With the two described graphs, the first conclusion can be made:

- In times of crisis, the dispersion of the beta-factors is much higher and more volatile than in times of constant growth.

In the following section, the focus will be on the single betas of each DAX company. It will be tried to identify certain pattern and behavior of the beta-graphs in order to understand what the “real” beta of a company is and to evaluate the consideration these patterns can help to calculate the cost of equity in a more precise and efficient way.

5.4 Characteristics and pattern of betas with time series analysis

As mentioned in an earlier moment of this work, when estimating the cost of equity with the CAPM, normally there is one beta factor used in order to define the sensibility of a stock-movement compared to the market. As we have seen in the illustrations in the last section, beta is not a constant factor and we cannot know how precise the cost of equity estimation really is. Before applying the CAPM on the companies listed in the German DAX index, it is necessary to identify certain behaviors and pattern of the DAX betas.

5.4.1. β which shows a high fluctuation over time

As we have already seen in Figure 13, there are certain different behaviors of beta during the time regarding their values. We discovered that during times of economic and financial uncertainty, e.g. crisis, the β of the stocks tend to move more than in times of constant growth like we experienced the last three years from 2011 to 2014. In order to understand better which companies and industries are more sensitive regarding β changes, times-series analysis lead to the following result, see Figure 14.

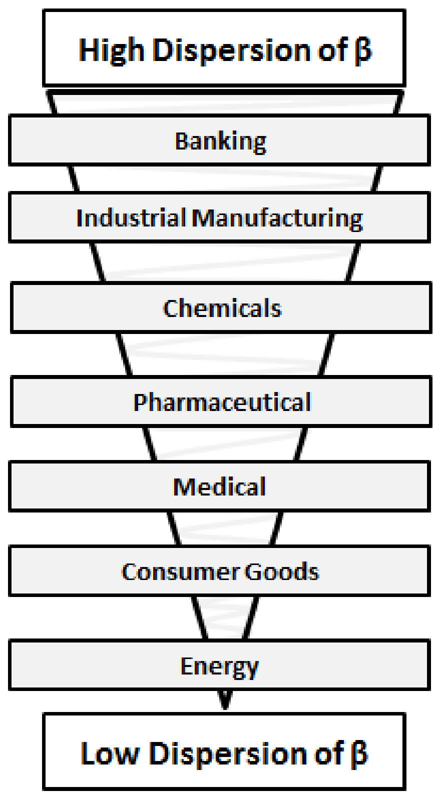


Figure 14: β-dispersion by industry

According to this figure, which is based on the calculated betas of the DAX stocks, we can see that companies of the energy and consumer goods sector have a more stable beta as e.g. banks or industrial manufacturing companies. This can be understood better by considering that companies which have an actual higher beta (>1.4) are also banks and industrial manufacturer and firms with a lower beta (<0.6) are the medical and consumer good companies.

5.4.2. β with particular continues positive or negative change

In the following paragraph, I analyze with more details specific stock-betas, in order to define time-series which can be used to obtain a may be more certain result when talking about cost of equity estimations or forecasting future returns. Two companies will be chosen for these analyses which are considered having this mentioned particular continues positive or negative change.

5.4.3. Example 1: Lanxess AG

The Lanxess AG is a publicly traded German chemical and polymer specialist enterprise. The company develops, produces and distributes plastics, natural rubber and special chemicals worldwide. The company has focused on sustainability and environmental responsibility. Lanxess employs approximately 17,000 professionals and has its headquarters in Cologne, located in the center of Germany. With its 14 business units, Lanxess has achieved 8.3 billion revenues in the financial year 2013.

For the visual analysis of the beta-time-series, the crisis of 2008 to 2009 will not be considered. Therefore, the time-window will be reduced from 1 November 2009 until the 29 March 2014. This example shows clearly that the beta factor of the stock of Lanxess is anything else then constant and by using the average of this beta time-window it's quite probable that impreciseness distorts the result significantly. See Figure 15.

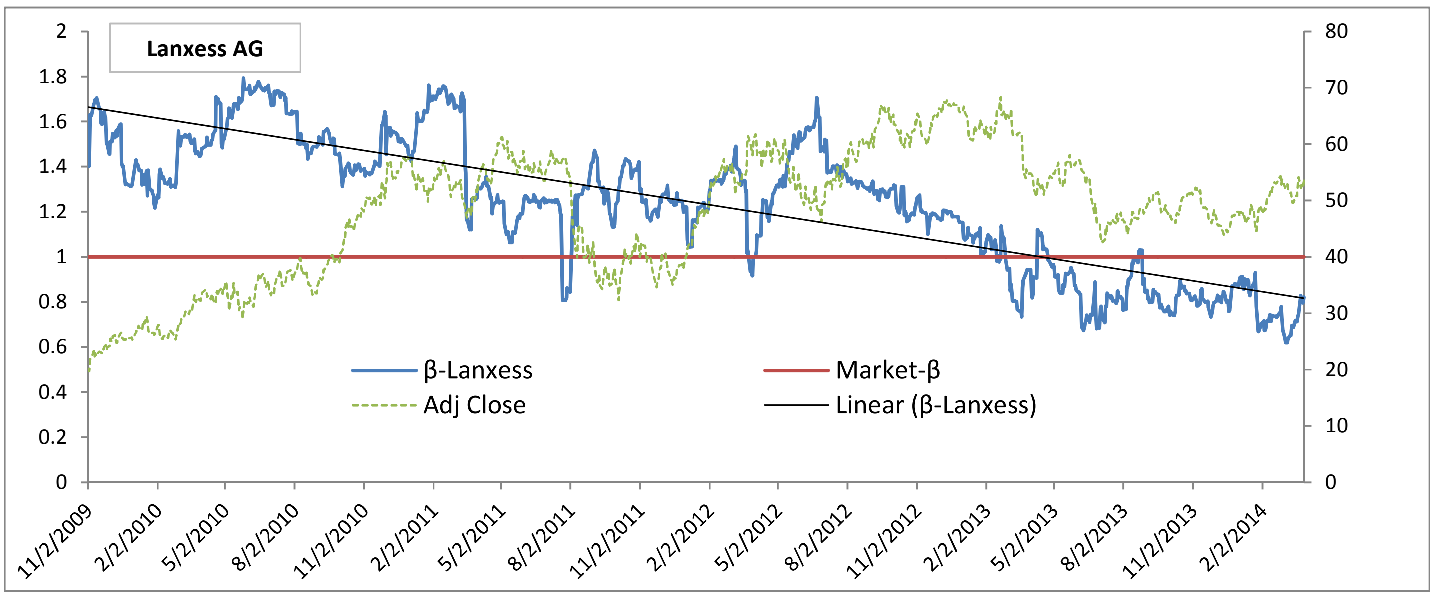


Figure 15: β-Factor and stock-price of Lanxess-AG

As we can see in the figure, the beta graph is illustrated in a four and a half year time window which gives relatively representative tendencies to be observed. While in the first three years the tendency is not that pronounced, from the middle of 2012 on until the end of March 2014, the beta decreases from 1.7 until 0.6. This enormous change cannot be overseen while calculating important financial indicators which decide if a million dollar investment is going to be made or not. Besides the blue graph, there is the dotted green graph which represents the daily adjusted close value of the Lanxess AG stock. Furthermore, there is the red horizontal line, which is the constant market-risk, which is always one. The black line with a decreasing slope shows the negative tendency of Lanxess's beta.

Until this point, no calculations have been made which could proof the functionality of this more or less visual analysis. These calculations are going to be made in the second part of the investigative sections of this work. Before doing that, a second real-life example should be mentioned first, in order to understand and realize the inconsistency of beta-factors.

5.4.4. Example 2: Merck KGaA[26]

If we observe the following graph in Figure 16, we recognize a similar behavior, just in the opposite direction; the beta-value is not decreasing over time, but increasing. Again, the linear tendency line indicates the direction. Starting in the area of 0.4 or even 0.1, the beta value growth over the years until it reaches its highest value 0.9. The assumption hereby is, that considering this beta-change, must lead to a more precise return-estimation. This statement will be approved or disproved in the following section.

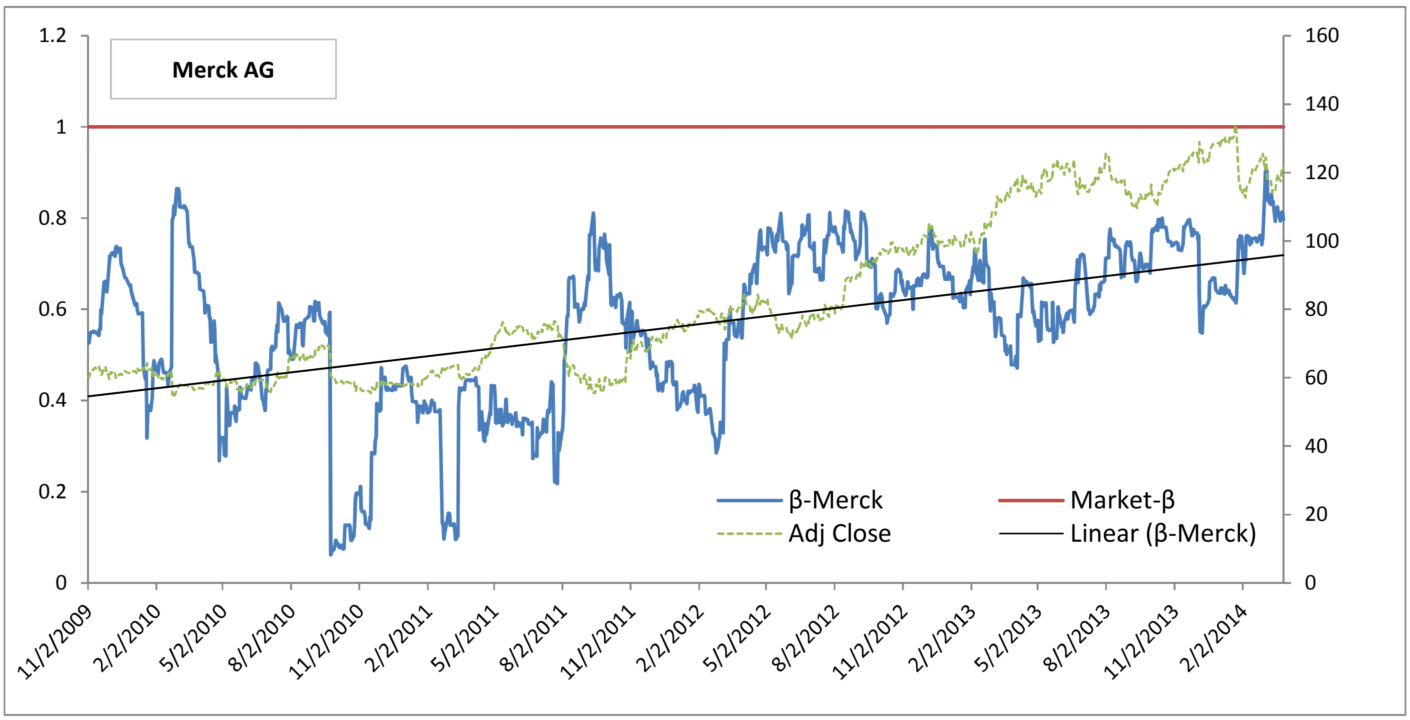


Figure 16: β-factor and stock-price of Merck AG

Merck AG is a leading German company in the pharmaceutical and chemical industry. The operations are divided in four areas, Merck Serono (Biopharmaceuticals), Consumer Health, Performance Materials (High-Tech Chemicals) and Merck Millipore (Life Science Tools). With its almost 40,000 employees worldwide Merck achieved in 2013 revenues of 10.7 billion of Euros. These two examples, Lanxess AG and Merck KGaA will be the base of the calculations of the following section.

[26] KGaA (Kommanditgesellschaft auf Aktien) is a German corporate designation standing for „partnership limited by shares".

6 Case studies: cost of equity estimation – time series analysis versus traditional beta calculation

6.1 Approach and objective

In this section, I point out the differences of possible betas by calculating them in the way introduced in the earlier section. First, I perform the traditional beta calculation with linear regression, where the covariance of both the observed security and the market return is divided to the variance of the market. Then, I do the same calculation with betas we defined analyzing the time-series. The main objective of these calculations and this section is the comparison of the two obtained results.

With that information, I will observe if using the information of the beta time series will help us to choose a beta factor, which will lead to a more precise result. The behaviors described in an earlier section are considered in this calculation by e.g. not using a specific period of time of the historical data. With that, possible significant beta change can be observed which might lead to a more precise beta and a more realistic and accurate rate of return.

6.2 Risk-free rate of Germany in 2013

For the further calculations and the cost of equity estimation, the risk-free rate is going to be required. The standard financial sources suggest using the rate of a 10-year government bond, particularly from an economic and political stable and established country like the Unites States of America or some central European countries, among them also Germany. Observing Figure 17, the graph of the 10-Year German Government bond, we discover an actual rate of about 1.5 %[27].

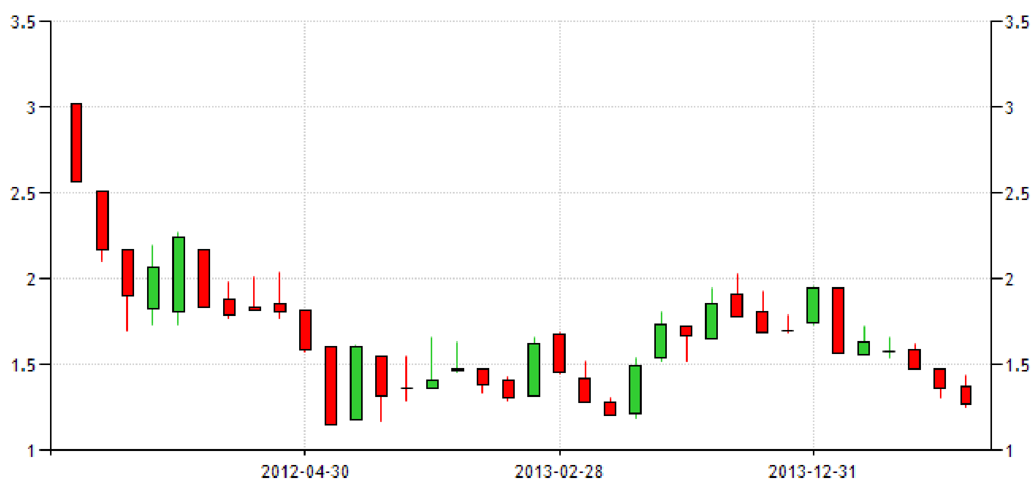


Figure 17: Germany 10 Year Government Bond[28]

[27] http://www.tradingeconomics.com/germany/government-bond-yield (accessed in April 2014)

[28] http://www.tradingeconomics.com/germany/government-bond-yield (accessed in April 2014)

Without any doubt, there exists a widespread range of different opinions and critics regarding the usage of first the risk-free rate in general and secondly the risk-free rate reflected by a government bond. The question of using the risk free rate or not should not be answered at this moment because it is a fundamental part of the CAPM which I will use in this actual section. But to concretize this number, I add another representative source which discloses the risk-free rate with another approach.

In 2013, Pablo Fernandez et al. collected within the scope of a statistical survey different financial parameters of financial and economic professors as well as company managers of 51 different countries. In this survey, realized by the IESE Business School of Spain, one focus was also the financial risk-free rate in these 51 countries. The result reflects the particular risk-free rates the participants have been using in this moment. See Figure 18.

RF	Number of answers	average	Median	St. Dev.	max	min	Av-Median
USA	2394	**2.4%**	2.2%	1.0%	6.0%	0.1%	0.2%
Spain	804	**4.4%**	4.6%	0.9%	6.0%	0.5%	-0.2%
Germany	343	**1.9%**	2.0%	0.6%	6.5%	0.1%	-0.1%
United Kingdom	247	**2.4%**	2.1%	1.0%	7.0%	0.2%	0.3%
Italy	205	**4.4%**	4.5%	0.6%	8.0%	1.5%	-0.1%
France	134	**2.0%**	2.0%	1.0%	5.0%	0.1%	0.0%
Switzerland	113	**1.3%**	1.3%	0.3%	3.0%	0.6%	0.0%
Brazil	112	**5.9%**	4.9%	2.4%	10.1%	3.0%	1.0%
Canada	110	**2.0%**	2.0%	0.5%	5.0%	1.0%	0.0%
China	95	**3.8%**	4.0%	0.6%	6.0%	1.7%	-0.2%

Figure 18: Survey of IESE Business School, Spain: Risk Free Rate[29]

Considering that this survey was published in June 2013 and the results were from around this moment in time, we can observe an approximately equal risk free rate of 1.9%. Based on this information, I decided to use 1.9% as the risk in the following case study.

6.3 Estimation of rate of return with static *β* and time-varying *β*

To do the following calculation, in order to estimate the rate of return of the two companies Lanxess and Merck, I will first consider the already mentioned 1.9% risk free rate. Then, I will apply the CAPM formula to get a first estimation. This is going to be the first step and the second will be the change of the beta regarding the tendency which we could observe in the time-series-graph on the earlier section.

[29] Fernandez, Aguirreamalloa and Linares (2013)

6.3.1 Lanxess AG

In order to estimate the rate of return, we need to have a time window as wide as possible so we chose the original data window with which the beta time series was calculated. The data-hold-back period with which the calculation is performed, is from 1 April 2008 until 29 March 2013. After this period, we have another year of data. The real average return of the Lanxess AG stock from 1 April 2013 until the 28 March 2014 will be compared with the results obtained from the first mentioned time window.

To calculate beta with the linear-regression model, I used the first mentioned time period, form 1 April 2008 until 29 March 2013, and applied the formula which can be found in every standard corporate finance literature, where R_i equals the actual daily average return of the observed stock and R_m equals the actual return of the market, in this case the daily average return of the DAX-30 index. Applying this formula the beta which we get is 1.3.

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$

Figure 19: Beta coefficient standard formula[30]

$$\bar{r}_a = r_f + \beta_a(\bar{r}_m - r_f)$$

Where:
r_f = Risk free rate
β_a = Beta of the security
$\bar{r}_m$ = Expected market return

Figure 20: CAPM standard formula[31]

[30] http://people.duke.edu/~charvey/classes/ba350/riskman/riskman.htm (accessed in April 2014)
[31] http://www.investopedia.com/terms/c/capm.asp (accessed in April 2014)

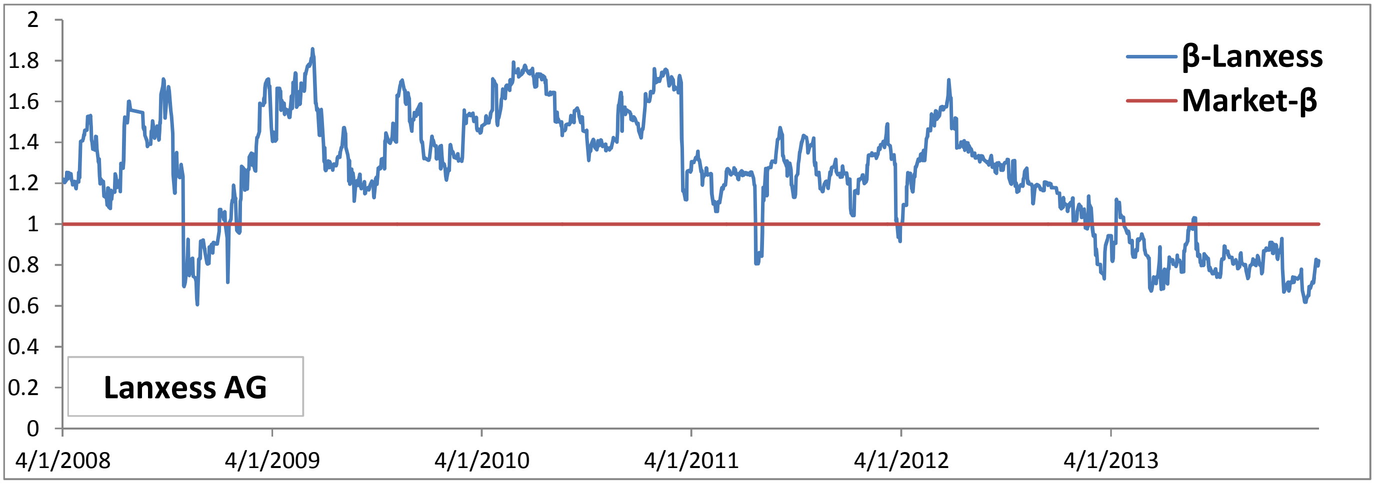


Figure 21: Time Series Beta of Lanxess AG from 01.04.2008 – 29.03.2014

Applying now the CAPM formula like explained in earlier sections of this work, considering the risk-free rate of 1.9%, the expected average daily return is 0.034%.Observing the data sheet of Lanxess AG and calculating the average of the real daily return if the time-window we used, we get 0.007%. So the actual return is about 4.8 times lower than the expected average daily return we obtained from the CAPM formula. Coming back to the beta time series analysis, we can see on the chart, that the beta o 1.3 is not reasonable at all to use. As the chart shows, the real beta of the moment we start to estimate the rate of return (1 April 2013), is already under one, with a strongly descending tendency.

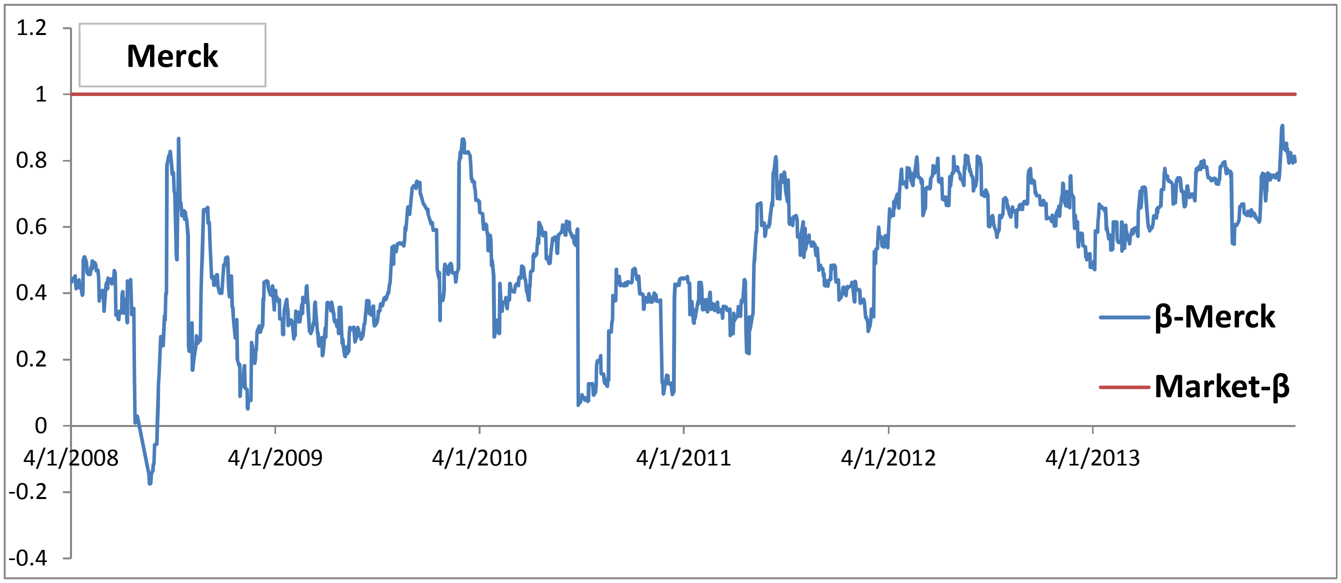
Considering the β-information obtained by the time-series-analysis, the CAPM will be applied again with a beta of 0.7. With this consideration, the expected return for 1 April 2013 until 28 March 2014 decreases from 0.007% (beta 1.3) to 0.002% (beta 0.7). This 0.002% estimated daily return is still three times higher than the actual return, nevertheless more precise than the return estimated with the first beta used.

Lanxess	
Static Beta (01.04.2008-29.03.2013):	1.3
Expected return daily with static beta:	0.0337%
Real avg. Daily return (01.04.2013-28.03.2014):	0.00713%
Accuracy:	473%
Expected return daily time series beta (0.7):	0.0211%
Accuracy:	296%

Figure 22: Calculations: Return Estimations Lanxess

6.3.2 Merck KGaA

Applying the same procedure as with Lanxess and considering the following already observed β-graph, we obtain similar results just with considering a higher beta than the static one obtained by linear regression analysis.



Merck	
Static Beta (01.04.2008-29.03.2013):	0.4828
Expected return daily with static beta:	0.0162%
Real avg. Daily return (01.04.2013-28.03.2014):	0.02593%
Accuracy:	62.29%
Expected return daily time series beta (0.08):	0.0233%
Accuracy:	90.02%

Figure 23: Calculations: Return Estimations Lanxess

7 Summary and conclusion

As shown in the present document, all existing methods to estimate any financial performance indicator have their challenge. The main difficulty and unchangeable fact is that all data to be used is from the past. The future is and always will be insecure. If financial data from the past is used in order to do forecasts, the same behavior as in the past will be expected.

The financial risk every investment has, no matter if it is a specific company risk which occurs because of particular firm events, or market risk, under which all companies are suffer because of economic uncertainty or macroeconomic events, is a key point to consider in all kind of strategic decisions. Because of this importance the market risk, which cannot be diversified, has to be as precise as possible. As proofed in the earlier sections, the market risk factor β is not constant over time. Although, in financial estimations of return on cost of equity usually an average value of the past is used.

The Kalman filter introduced in this document is an ideal tool and algorithm to calculate time-series of the β-factor in order to visualize its past-behavior. This information can be analyzed and used in order to find a more real and precise β-factor for further calculations.

The calculations of the last sections of this document have shown the functionality of the time-series evaluation. It has proven that considering the behavior observed during the time-series leads to another β-factor which further on leads to an improved result in rate of return estimation.

This method does not automatically improve all financial calculation, cause not all securities show a specific time-series behavior which can be used to modify the risk-factors. Nevertheless, the Kalman filter can be applied to any financial historic data of the stock market. This tool should be used considering any investment in the stock market because it illuminates the individual behaviors of each stock.

This information can help with important decisions and can be one significant factor deciding for or against an investment.

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Declaration

This manuscript is a presentation of my original research work. Wherever contributions of others are involved, every effort was made to indicate this clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions.

Guatemala, 18th July 2014

Place, Date

Signature