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Score-driven QAR-EGARCH-M model of risk premium and volatility for the Meixner probability distribution

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Abstract: Score-driven QAR-EGARCH-M (quasi-auto regressive exponential generalized autoregressive conditional heteroscedasticity in mean) models with long-run and short-run volatility components and leverage effects are presented. In-sample statistical performance and out-of-sample volatility predictive accuracy are studied for three alternatives: (a) Meixner (MXN) distribution; (b) exponential generalized beta distribution of the second kind (EGB2); (c) normal-inverse Gaussian (NIG) distribution. Conditions of the asymptotic properties of the maximum likelihood (ML) estimator are presented. Weekly log-returns are used for five international stock indices from 1992 to 2020. QAR within the risk premium improves statistical performance and predictive accuracy, and MXN is superior to EGB2 and NIG.

Keywords: Risk premium; Two-component conditional volatility; Score-driven time series; Statistical performance; Predictive accuracy; Maximum likelihood estimation.

JEL classification codes: C22, C51, C52, C58

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1. Introduction

In several works from the literature, ARMA (autoregressive moving average) (Box and Jenkins, 1970) models of the expected return are combined with GARCH-M (generalized autoregressive heteroscedasticity in mean) (Engle et al., 1987). Some recent works that use ARMA-GARCH-M models are Liu et al. (2011), Su et al. (2011) and Mohammadi (2017). Those papers show that dynamic specifications in the mean may improve statistical performance and out-of-sample forecast accuracy. Motivated by the results, this paper introduces the same type of model, named QAR-EGARCH-M, to the body of literature on score-driven time series models, and its statistical performance and predictive accuracy are studied in an econometric application.

The first contribution of this paper is to extend the recent EGARCH-M-2 (exponential GARCH-M with long-run and short-run volatility components) model from the work of Harvey and Lange (2018), by adding a score-driven dynamic parameter to the risk premium of returns on financial assets. The extended model is named QAR-EGARCH-M (quasi-AR, QAR) (Harvey, 2013). In QAR-EGARCH-M, the filters in risk premium and log-scale (the latter is a non-linear transformation of volatility) are updated by using the score function with respect to risk premium and the score function with respect to log scale, respectively. Both one-component and two-component versions of QAR-EGARCH-M are estimated, named QAR-EGARCH-M-1 and QAR-EGARCH-M-2, respectively. The statistical inference and conditions of the asymptotic properties of the maximum likelihood (ML) estimator are presented for QAR-EGARCH-M-2, by using results from the literature on score-driven models. In relation to this first contribution, the question to be answered in the empirical application case study of this paper is the following: Are the statistical performance and predictive accuracy of QAR-EGARCH-M superior to the statistical performance and predictive accuracy of EGARCH-M?

For the probability distribution of returns on financial assets, this paper considers the Meixner (MXN) distribution (Schoutens, 2002), the exponential generalized beta distribution of the second kind (EGB2) (Prentice, 1975), and the normal-inverse Gaussian (NIG) distribution (Barndorff-Nielsen and Halgreen, 1977) alternatives. For those distributions all moments exist, and for each of those distributions two shape parameters influence skewness and tail-thickness. The one-component volatility models for those distributions are MXN-EGARCH-M-1, EGB2-EGARCH-M-1 and NIG-EGARCH-M-1, respectively. The two-component volatility models for those distributions are MXN-EGARCH-M-2, EGB2-EGARCH-M-2 and NIG-EGARCH-M-2, respectively. The reason for the consideration of the MXN, EGB2 and NIG distributions for QAR-EGARCH-M is that the score functions transform unexpected returns in similar ways: (a) The score function with respect to risk premium that updates QAR, is an approximately linearly increasing and asymmetric transformation of unexpected returns. (b) The score function with respect to log scale that updates EGARCH in an asymmetric way, asymptotically Winsorizes (e.g. Caivano and Harvey, 2014) unexpected returns.

The second contribution of this paper is to extend the MXN-EGARCH-1 model from the work of Blazsek and Haddad (2020) to the QAR-MXN-EGARCH-M-2 model. This extension involves the use of the score function with respect to location (that is, risk premium in this paper) for the MXN distribution, which to the best of our knowledge is new in the literature on score-driven models. The MXN distribution is an infinitely divisible distribution (as is the NIG distribution), and for the MXN distribution the reader is referred to the following works: Schoutens (2002), Grigoletto and Provasi (2008), Madan and Yor (2008), Bozejko and Demni (2010) and Kawai (2012). In relation to this second contribution, the question to be answered in empirical application of this paper is the following: Are the statistical performance and predictive accuracy of QAR-MXN-EGARCH-M superior to the statistical performances and predictive accuracies of QAR-EGB2-EGARCH-M and QAR-NIG-EGARCH-M?

Statistical performance is studied for five major stock market indices: Dow Jones Industrial Average (DJIA), CAC 40 Index (FCHI), DAX Performance Index (GDAXI), Nikkei Index (N225) and Amsterdam Exchange Index (AEX). The estimation window includes weekly data for the period of 19 October 1992 to 20 January 2020 for all indices. The choice of these specific stock indices in the application case study is motivated by the literature on financial applications of the MXN distribution in the works of Schoutens (2002) and Grigoletto and Provasi (2008).

The use of weekly data frequency is motivated by the work of Harvey and Lange (2018) for the estimation of the EGARCH-M-2 model. The empirical results indicate that the use of QAR improves in-sample statistical performance for several indices and the improvement is the most significant for the MXN distribution. This latter finding indicates that the separation of risk premium and volatility dynamics is probably the most effective for the MXN distribution.

Predictive performance is studied for DJIA. The full data window is divided into an initial estimation window and the forecasting window. One-step ahead forecasts of volatility are estimated by using the rolling-window forecasting approach. Each rolling estimation window includes 1,000 observations (approximately 19.2 years) and the forecasting window includes weekly data for the period of 19 December 2011 to 20 January 2020. The same proxy of true volatility is used as in the work of Harvey and Lange (2018), that is, the square root of weekly realized variance. Six alternative loss functions are used in the analysis of predictive accuracy (Hansen and Lunde, 2005; Patton, 2011; Harvey and Lange, 2018), and predictive accuracies are compared by using the Diebold–Mariano test (Diebold and Mariano, 1995). The results on forecasting greatly support the use of QAR-MXN-EGARCH-M for financial investors and risk managers, since: (a) QAR-EGARCH-M is always superior to EGARCH-M; (b) QAR-MXN-EGARCH-M is always superior to QAR-EGB2-EGARCH-M and QAR-NIG-EGARCH-M.

The remainder of the paper is organized as follows: Section 2 reviews the relevant literature. Section 3 describes the dataset. Section 4 presents the QAR-EGARCH-M-2 model. Section 5 presents the statistical inference and the results on statistical performances. Section 6 presents the forecasting method and the results on predictive accuracies. Section 7 concludes.

2. Literature review

The relationship between risk premium and volatility of financial assets have been extensively studied in the literature for several decades. The econometric applications significantly improve after the work of Engle et al. (1987), in which the ARCH-M model is introduced. In ARCH-M, both risk premium and volatility are time-varying, and the model can represent the compensation (risk premium) required by risk averse investors for holding risky financial assets. ARCH-M is an extension of the ARCH model (Engle, 1982), as the risk premium is directly influenced by volatility in the return equation of ARCH-M. Subsequent dynamic models of volatility from the literature, for example, GARCH (Bollerslev, 1986), EGARCH (Nelson, 1991) and the stochastic volatility (SV) model (Harvey et al., 1994; Taylor, 1994), are also extended and include volatility in the risk premium of financial assets. Nevertheless, as reported by Harvey and Lange (2018), the results on the sign and the significance of the volatility parameter in risk premium are inconclusive in the literature (e.g. French et al., 1987; Turner et al., 1989; Nelson, 1991; Campbell and Hentschel, 1992; Glosten et al., 1993).

As the often experienced long memory behaviour of conditional volatility (e.g. Harvey, 2013; Janus et al., 2014) causes problems for effective statistical estimation, two-component models of volatility are suggested in the literature as a possible solution. For example, Engle and Lee (1999) introduce the two-component GARCH model and Alizadeh et al. (2002) suggest the practical use of two-component SV models. In those models, volatility is the sum of long-run and short-run components, in which the short-run component measures a temporal variation in volatility after an extreme observation. An advantage of the use of two-component volatility models is that they approximate long memory behaviour of time series (Harvey, 2013). Another advantage of the two-component volatility models is flexibility: (a) Different leverage effects (Black, 1976; Glosten et al., 1993) may be used in the long-run and short-run volatility components. (b) Both long-run and short-run volatility components may be included in the risk premium. According to the literature, the statistical performance and predictive accuracy of two-component volatility models may be superior to those of one-component volatility models.

The work of Harvey and Lange (2018) is extended in this paper, thus presenting the most direct relation between the present paper and the body of literature. The paper of Harvey and Lange (2018) is motivated by the inconclusive empirical results of ARCH-M-type volatility models and by the effective statistical opportunities that are offered by the two-component volatility models. The authors present score-driven EGARCH-M models with two-components and leverage effects (EGARCH-M-2) for the Student's t-distribution and the skewed t-distribution. For those models, the conditions of the asymptotic properties of the ML parameter estimates are also presented. Those authors also study the in-sample statistical performance and out-of-sample volatility predictive accuracy, by using weekly National Association of Securities Dealers Automated Quotations Index (NASDAQ), Nikkei Index (N225) and Standard & Poor's 500 Index (S&P 500) log-return data. Harvey and Lange (2018) compare volatility predictive accuracy to an extended version of the benchmark GARCH model of Engle and Lee (1999), for which the Student's t-distribution for the error term and leverage effects for both long-run and short-run volatility components are considered. The results of Harvey and Lange (2018) support the asymptotic properties of the ML estimates and indicate that the predictive performance of EGARCH-M-2 is superior to GARCH with two volatility components.

The score-driven EGARCH-M model of Harvey and Lange (2018) belongs to the recent and growing literature on score-driven time series models. That body of literature begins with the seminal works of Creal et al. (2008), Harvey and Chakravarty (2008), Creal et al. (2013) and Harvey (2013). Creal et al. (2013) name those models as generalized autoregressive score (GAS) models and Harvey (2013) names them as dynamic condition score (DCS) models. In the GAS/DCS framework, each dynamic parameter is driven by a partial derivative of the log conditional density of the dependent variables with respect to a time-varying parameter (hereinafter, score function). Thus, GAS/DCS models are observation-driven time series models (Cox et al., 1981). GAS/DCS models use optimal filters according to the Kullback–Leibler divergence measure (Blasques et al., 2015), and the conditions of the asymptotic properties of the ML estimator are incorporated into many GAS/DCS models (Harvey, 2013).

In relation to the present paper, the most relevant volatility models from the GAS/DCS literature are: Beta-t-EGARCH (Harvey and Chakravarty, 2008); GED-EGARCH (general error distribution EGARCH) (Harvey, 2013); Beta-Skew-t-EGARCH (skewed t-distribution EGARCH) (Harvey and Sucarrat, 2014); EGB2-EGARCH (exponential generalized beta distribution of the second kind EGARCH) (Caivano and Harvey, 2014); Beta-Skew-Gen-t-EGARCH (skewed generalized t-distribution EGARCH) (Harvey and Lange, 2017); NIG-EGARCH (normal-inverse

Gaussian distribution EGARCH) (Blazsek et al., 2018); MXN-EGARCH (Meixner distribution EGARCH) (Blazsek and Haddad, 2020).

3. Data and summary statistics

The empirical analyses in the present paper are based on weekly log-returns $y_t = 100 \times \ln(p_t/p_{t-1})$ for five major stock market indices: DJIA, FCHI, GDAXI, N225 and AEX (see the definitions of these tickers in Table 1). The choice of these specific stock indices is motivated by the literature on financial applications of the MXN distribution. The work of Schoutens (2002) uses daily Nikkei 225 log-return data for the period of 1997 to 1999 and daily S&P 500 log-return data for the period of 1970 to 2001. The work of Grigoletto and Provasi (2008) uses weekly AEX, DJIA, FCHI, GDAXI and S&P 500 log-return data for the period of January 2002 to December 2006. For the United States the use of DJIA is chosen in the present paper, instead of S&P 500. Thus, stock indices of the following five countries are studied: United States, France, Germany, Japan and Netherlands. The source of the dataset of this paper is Yahoo Finance. The use of the weekly data frequency is motivated by the work of Harvey and Lange (2018) for the estimation of score-driven EGARCH-M models. Furthermore, log-returns are used for all indices instead of log-excess returns on the risk-free rate, because risk-free rate data are not available for all countries of this paper.

For all indices the same data period is used in this paper, in order to make the statistical results more comparable. The start of the data period is determined by data availability. AEX has the shortest available data period from Yahoo Finance, which defines the sample period of this paper. Descriptive statistics are reported in Table 1. The skewness and excess kurtosis estimates indicate skewed and fat-tailed probability distributions for log-returns, respectively. The non-zero $\mathrm{corr}(y_t, y_{t-1})$ estimates indicate possible serial correlation in the mean. The negative $\mathrm{corr}(|y_t|, y_{t-1})$ estimates indicate leverage effects in volatility. The Shapiro–Wilk test (Shapiro and Wilk, 1965) of normal distribution rejects normal distribution of weekly log-return for all assets (the same results are obtained for alternative tests of the normal distribution), which motivates the consideration of skewed and fat-tailed probability distributions.

4. Model specification

The QAR-EGARCH-M-2 model for $y_t = 100 \times \ln(p_t/p_{t-1})$ of a financial asset is:

$$y_t = \mu_t + \exp(\lambda_t)\epsilon_t = c + \mu_t^* + \kappa_{\mathrm{L}} \exp(\lambda_{\mathrm{L},t}) + \kappa_{\mathrm{S}} \exp(\lambda_{\mathrm{S},t}) + \exp(\lambda_t)\epsilon_t \tag{1}$$

$$\mu_t^* = \phi\mu_{t-1}^* + \psi u_{\mu,t-1} \tag{2}$$

$$\lambda_t = \omega + \lambda_{\mathrm{L},t} + \lambda_{\mathrm{S},t} \tag{3}$$

$$\lambda_{\mathrm{L},t} = \beta_{\mathrm{L}}\lambda_{\mathrm{L},t-1} + \alpha_{\mathrm{L}} u_{\lambda,t-1} + \alpha_{\mathrm{L}}^* \mathrm{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1) \tag{4}$$

$$\lambda_{\mathrm{S},t} = \beta_{\mathrm{S}}\lambda_{\mathrm{S},t-1} + \alpha_{\mathrm{S}} u_{\lambda,t-1} + \alpha_{\mathrm{S}}^* \mathrm{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1) \tag{5}$$

for weeks $t = 1, \ldots, T$, where pre-sample data are used for p_0. This model is a direct extension of the two-component EGARCH-M model from the work of Harvey and Lange (2018). Dynamic parameters μ_t and λ_t represent the risk premium and log-scale of returns, respectively. Moreover, ϵ_t is specified according to the MXN, EGB2 and NIG distributions that are presented in the following sections. The mean of ϵ_t is not zero for these probability distributions, thus the mean of log-returns is not μ_t. In this paper, the unconditional mean of ϵ_t is not subtracted from ϵ_t; unlike in the work of Harvey and Lange (2018). Nevertheless, our approach does not cause any problems for the statistical inference, as expected returns are simultaneously determined by the parameters within μ_t and by the shape parameters of ϵ_t. The conditional mean of y_t is reported in the following sections for each probability distribution, in which the conditional volatilities σ_t are also reported. The shape parameters of MXN, EGB2 and NIG specify skewed and fat-tailed log-return distributions, as in the works of Harvey and Sucarrat (2014) and Harvey and Lange (2018), in which the skewed t-distribution is used, and in the work of Harvey and Lange (2017), in which the skewed generalized t-distribution is used.

Component μ_t^* from the risk premium is specified according to the QAR(1) model (Harvey, 2013) and it measures a particular serial correlation in the mean that is not captured by EGARCH-M-2. If this component is significant, then the overall model performance is improved. As a consequence, practitioners may obtain benefits from the improved volatility prediction accuracy. The score function $u_{\mu,t}$ that updates μ_t^* is defined in the following sections for each probability distribution. For this component it is required that $\psi \neq 0$ (Harvey, 2013).

In this model, $\lambda_{\mathrm{L},t}$ and $\lambda_{\mathrm{S},t}$ represent the long-run and short-run volatility components, respectively. For these components $\beta_\mathrm{L} > \beta_\mathrm{S}$, $\alpha_\mathrm{L} \neq 0$, $\alpha_\mathrm{S} \neq 0$, $\alpha_\mathrm{L}^* \neq 0$ and $\alpha_\mathrm{S}^* \neq 0$ are required (Harvey and Lange, 2018). Parameters κ_L and κ_S measure the effects of long-run and short-run volatility components, respectively, on the risk premium. Parameter κ_L is positive according to the inter-temporal capital asset pricing model (ICAPM) (Merton, 1973), and parameter κ_S might be negative due to a 'news effect' (e.g. Chou, 1988; Schwert, 1989). Parameters α_L^* and α_S^* measure leverage effects on long-run and short-run volatility components, respectively (Harvey, 2013). The score function $u_{\lambda,t}$ that updates both volatility components is defined in the following sections.

The following special cases of QAR-EGARCH-M-2 are also estimated in this paper: (a) Under the restriction $\lambda_{\mathrm{S},t} = 0$, the one-component QAR-EGARCH-M-1 model is obtained. (b) Under the restriction $\mu_t^* = 0$, the two-component EGARCH-M model of Harvey and Lange (2018) is obtained (EGARCH-M-2). (c) Under the restrictions $\mu_t^* = 0$ and $\lambda_{\mathrm{S},t} = 0$, the one-component EGARCH-M model of Harvey and Lange (2018) is obtained (EGARCH-M-1).

4.1. MXN distribution

For this alternative, the error term has the MXN distribution $\epsilon_t \sim \text{MXN}[0, 1, \pi\text{tanh}(\delta_1), \exp(\delta_2)]$, where δ_1 and δ_2 are shape parameters that take values from the set of real numbers, and influence tail-heaviness and skewness. The log conditional density is given by:

$$\ln f(y_t|y_1, \ldots, y_{t-1}; \Theta) = -\lambda_t + 2\exp(\delta_2)\ln\{2\text{cos}[\pi\text{tanh}(\delta_1)/2]\} - \ln(2\pi) \tag{6}$$

$$-\ln\Gamma\{2\exp(\delta_2)\} + \pi\text{tanh}(\delta_1)\epsilon_t + 2\ln|\Gamma[\exp(\delta_2) + i\epsilon_t]|$$

where Θ represents all time-constant model parameters, $\cos(x)$ is the cosine function and i is the imaginary unit. The following function is defined: $g(\lambda_t) = \Gamma[\exp(\delta_2) + i(y_t - \mu_t)\exp(-\lambda_t)]$, where λ_t is a real number. Under this notation $\partial \ln|g(\lambda_t)|/\partial\lambda_t = \text{Re}[g'(\lambda_t)/g(\lambda_t)]$, where $\text{Re}(z)$ is the real part of complex number z. Since $\Gamma'(x) = \Gamma(x)\Psi^{(0)}(x)$, where $\Psi^{(0)}(x)$ is the digamma function, the score functions $u_{\mu,t}$ and $u_{\lambda,t}$, respectively, are:

$$\frac{\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)}{\partial\mu_t} = -\frac{\pi\text{tanh}(\delta_1)}{\exp(\lambda_t)} + 2\text{Re}\left\{-\frac{i}{\exp(\lambda_t)}\epsilon_t\Psi^{(0)}[\exp(\delta_2) + i\epsilon_t]\right\} \tag{7}$$

$$u_{\mu,t} = \frac{\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)}{\partial\mu_t} \times \exp(2\lambda_t) \tag{8}$$

$$\frac{\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)}{\partial\lambda_t} = u_{\lambda,t} = 2\text{Re}\left\{-i\epsilon_t\Psi^{(0)}[\exp(\delta_2) + i\epsilon_t]\right\} - \pi\text{tanh}(\delta_1)\epsilon_t - 1 \tag{9}$$

The use of $u_{\lambda,t}$ for the MXN distribution has already been suggested in the work of Blazsek and Haddad (2020) for score-driven MS-EGARCH models with leverage effects, but to the best of or knowledge the use of $u_{\mu,t}$ for the MXN distribution is new in the literature on GAS/DCS models. The above definition of the updating term provides the framework of QAR-MXN-EGARCH-M-2.

In Figures 1(a-b), the non-linear transformation of ϵ_t is presented for DJIA. Score function $u_{\mu,t}$ uses an approximately linearly increasing transformation of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. Score function $u_{\lambda,t}$ uses an asymptotic Winsorizing of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. The conditional mean and conditional volatility of y_t are given by:

$$E(y_t|y_1,\ldots,y_{t-1}) = \mu_t + \exp(\lambda_t + \delta_2)\tan[\pi\tanh(\delta_1)/2] \tag{10}$$

$$\sigma_t = \sigma(y_t|y_1,\ldots,y_{t-1}) = \left\{\frac{\exp(\lambda_t + \delta_2)}{\cos[\pi\tanh(\delta_1)] + 1}\right\}^{1/2} \tag{11}$$

Equation (11) defines the one-step ahead volatility forecasting formula.

4.2. EGB2 distribution

For this alternative, the error term has the EGB2 distribution $\epsilon_t \sim \mathrm{EGB2}[0, 1, \exp(\delta_1), \exp(\delta_2)]$, where δ_1 and δ_2 are shape parameters that take values from the set of real numbers, and influence skewness and tail-heaviness. The log conditional density is given by:

$$\ln f(y_t|y_1,\ldots,y_{t-1};\Theta) = \exp(\delta_1)\epsilon_t - \lambda_t - \ln\Gamma[\exp(\delta_1)] \tag{12}$$

$$-\ln\Gamma[\exp(\delta_2)] + \ln\Gamma[\exp(\delta_1) + \exp(\delta_2)] - [\exp(\delta_1) + \exp(\delta_2)]\ln[1 + \exp(\epsilon_t)]$$

Score functions $u_{\mu,t}$ and $u_{\lambda,t}$ are, respectively, given by:

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)}{\partial \mu_t} = u_{\mu,t} \times \{\Psi^{(1)}[\exp(\delta_1)] + \Psi^{(1)}[\exp(\delta_2)]\}\exp(2\lambda_t) \tag{13}$$

$$u_{\mu,t} = \{\Psi^{(1)}[\exp(\delta_1)] + \Psi^{(1)}[\exp(\delta_2)]\}\exp(\lambda_t)\left\{[\exp(\delta_1) + \exp(\delta_2)]\frac{\exp(\epsilon_t)}{\exp(\epsilon_t) + 1} - \exp(\delta_1)\right\} \tag{14}$$

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)}{\partial \lambda_t} = u_{\lambda,t} = [\exp(\delta_1)+\exp(\delta_2)]\frac{\epsilon_t \exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\delta_1)\epsilon_t - 1 \quad (15)$$

The above definition of the updating term provides the QAR-EGB2-EGARCH-M-2 model. In Figures 1(c-d), the non-linear transformation of ϵ_t is presented for DJIA. Score function $u_{\mu,t}$ uses an approximately linearly increasing transformation of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. Score function $u_{\lambda,t}$ uses an asymptotic Winsorizing of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. The conditional mean and conditional volatility of y_t are given by:

$$E(y_t|y_1,\ldots,y_{t-1}) = \mu_t + \exp(\lambda_t)\left\{\Psi^{(0)}[\exp(\delta_1)] - \Psi^{(0)}[\exp(\delta_2)]\right\} \quad (16)$$

$$\sigma_t = \sigma(y_t|y_1,\ldots,y_{t-1}) = \exp(\lambda_t)\left\{\Psi^{(1)}[\exp(\delta_1)] + \Psi^{(1)}[\exp(\delta_2)]\right\}^{1/2} \quad (17)$$

Equation (15) defines the one-step ahead volatility forecasting formula.

4.3. NIG distribution

For this alternative, the error has the NIG distribution $\epsilon_t \sim \mathrm{NIG}[0, 1, \exp(\delta_1), \exp(\delta_1)\tanh(\delta_2)]$, where δ_1 and δ_2 are shape parameters that are real numbers, and they influence tail-heaviness and skewness; the skewness parameter is $\exp(\delta_1)\tanh(\delta_2)$. The log conditional density is:

$$\ln f(y_t|y_1,\ldots,y_{t-1};\Theta) = \delta_1 - \lambda_t - \ln(\pi) + \exp(\delta_1)[1-\tanh^2(\delta_2)]^{1/2} \quad (18)$$

$$+\exp(\delta_1)\tanh(\delta_2)\epsilon_t + \ln K^{(1)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right] - \frac{1}{2}\ln(1+\epsilon_t^2)$$

where $K^{(j)}(x)$ is the modified Bessel function of the second kind of order j. Score functions $u_{\mu,t}$

and $u_{\lambda,t}$ are, respectively, given by:

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)}{\partial \mu_t} = -\exp(\delta_1-\lambda_t)\tanh(\delta_2) + \frac{\epsilon_t}{\exp(\lambda_t)(1+\epsilon_t^2)} \tag{19}$$

$$+\frac{\exp(\delta_1-\lambda_t)\epsilon_t}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]+K^{(2)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}$$

$$u_{\mu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)}{\partial \mu_t} \times \exp(2\lambda_t) \tag{20}$$

$$u_{\lambda,t} = -1 - \exp(\delta_1)\tanh(\delta_2)\epsilon_t + \frac{\epsilon_t^2}{1+\epsilon_t^2} \tag{21}$$

$$+\frac{\exp(\delta_1)\epsilon_t^2}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]+K^{(2)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}$$

The above definition of the updating term provides the framework of QAR-NIG-EGARCH-M-2. In Figures 1(e-f), the non-linear transformation of ϵ_t is presented for DJIA. Score function $u_{\mu,t}$ uses an approximately linearly increasing transformation of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. Score function $u_{\lambda,t}$ uses an asymptotic Winsorizing of ϵ_t that is asymmetric as $|\epsilon_t| \to \infty$. The conditional mean and conditional volatility of y_t are given by:

$$E(y_t|y_1,\ldots,y_{t-1}) = \mu_t + \frac{\exp(\lambda_t)\tanh(\delta_2)}{[1-\tanh^2(\delta_2)]^{1/2}} \tag{22}$$

$$\sigma_t = \sigma(y_t|y_1,\ldots,y_{t-1}) = \left\{\frac{\exp(2\lambda_t-\delta_1)}{[1-\tanh^2(\delta_2)]^{3/2}}\right\}^{1/2} \tag{23}$$

Equation (23) defines the one-step ahead volatility forecasting formula.

5. In-sample estimation and results

5.1. Statistical inference

QAR-EGARCH-M-2 and its special cases are estimated by using the ML method, as follows:

$$\hat{\Theta}_{\mathrm{ML}} = \arg\max_{\Theta} \mathrm{LL}(y_1, \ldots, y_T; \Theta) = \arg\max_{\Theta} \sum_{t=1}^{T} \ln f(y_t | y_1, \ldots, y_{t-1}; \Theta) \tag{24}$$

All estimations are performed numerically, by using alternative start values of parameters to find a global maximum. In the maximization of the LL function, the convergence tolerance for gradient is 10^{-5} for all parameters. The numerically estimated inverse information matrix is used for the estimation of the standard errors of $\hat{\Theta}_{\mathrm{ML}}$. Standard errors of transformed parameters are computed by using the delta method (e.g. Davidson and MacKinnon, 2004).

The following derivatives are defined (Harvey, 2013):

$$X_{*,t} = \phi + \psi \frac{\partial u_{\mu,t}}{\partial \mu_t} \tag{25}$$

$$X_{\mathrm{L},t} = \beta_{\mathrm{L}} + [\alpha_{\mathrm{L}} + \alpha_{\mathrm{L}}^{*} \mathrm{sgn}(-\epsilon_t)] \frac{\partial u_{\lambda,t}}{\partial \lambda_t} \tag{26}$$

$$X_{\mathrm{S},t} = \beta_{\mathrm{S}} + [\alpha_{\mathrm{S}} + \alpha_{\mathrm{S}}^{*} \mathrm{sgn}(-\epsilon_t)] \frac{\partial u_{\lambda,t}}{\partial \lambda_t} \tag{27}$$

The following conditions of the consistency and asymptotic normality of ML are based on Harvey (2013), Blasques et al. (2018), Harvey and Lange (2018) and Ayala et al. (2019):

Condition 1 is that the unconditional variances of $X_{*,t}$, $X_{\mathrm{L},t}$, $X_{\mathrm{S},t}$, $u_{\mu,t}$ and $u_{\lambda,t}$ are finite, and that the unconditional mean of each product that is formed by all possible pairs of those variables is also finite. Condition 2 is that $C_{2,*} = |\phi| < 1$, $C_{2,\mathrm{L}} = |\beta_{\mathrm{L}}| < 1$ and $C_{2,\mathrm{S}} = |\beta_{\mathrm{S}}| < 1$.

Condition 3 is that $C_{3,*} = E(X_{*,t}^2) < 1$, $C_{3,\mathrm{L}} = E(X_{\mathrm{L},t}^2) < 1$, $C_{3,\mathrm{S}} = E(X_{\mathrm{S},t}^2) < 1$, $C_{3,*,\mathrm{L}} = |E(X_{*,t}X_{\mathrm{L},t})| < 1$, $C_{3,*,\mathrm{S}} = |E(X_{*,t}X_{\mathrm{S},t})| < 1$ and $C_{3,\mathrm{L},\mathrm{S}} = |E(X_{\mathrm{L},t}X_{\mathrm{S},t})| < 1$. Condition 4 is that $C_{4,*} = E(\ln |X_{*,t}|) < 0$, $C_{4,\mathrm{L}} = E(\ln |X_{\mathrm{L},t}|) < 0$ and $C_{4,\mathrm{S}} = E(\ln |X_{\mathrm{S},t}|) < 0$.

Conditions 2 to 4 are empirically estimated ex-post. For all conditions, partial derivatives are numerically computed and expected values are estimated by using the sample average.

5.2. Estimation results

Statistical performance of all models is measured by using the following likelihood-based model selection metrics: LL, Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan–Quinn criterion (HQC) (e.g. Hamilton, 1994; Davidson and MacKinnon, 2004). The estimates of these metrics for all indices and models are presented in Table 2. The results indicate that: (a) For some indices QAR-EGARCH-M is superior to EGARCH-M (this is shown by using bold numbers in Table 2). (b) For some indices the MXN distribution is superior to the EGB2 and NIG distributions (this is shown by using † in Table 2). The joint superiority of MXN and QAR is clear for DJIA, FCHI, GDAXI and AEX, but the opposite may be true for N225. These results suggest that the consideration of the MXN distribution and the QAR extension of EGARCH-M are likely to improve statistical performance.

The parameter estimates and estimates of the ML conditions for MXN-EGARCH-M-1 and MXN-EGARCH-M-2 are presented in Table 3. The same statistics for QAR-MXN-EGARCH-M-1 and QAR-MXN-EGARCH-M-2 are presented in Table 4. Furthermore, the same estimates for the EGB2 and NIG distributions are presented in the Appendix.

The effect of the long-run volatility component on the risk premium is positive for almost all of the cases, which supports ICAPM. The effect of the short-run volatility component on expected return is significantly negative in several cases, which supports the theories of Chou (1988) and Schwert (1989), and which is similar to the empirical findings of Harvey and Lange (2018) for NASDAQ and N225. Both long-run and short-run volatility components are significant for all cases (i.e. α_{L} or α_{L}^* is significant and α_{S} or α_{S}^* is significant for all cases). In Figure 2, the long-run and short-run volatility components are presented for all indices for QAR-MXN-EGARCH-M-2.

In most of the cases the leverage effect parameter is more important for the short-run volatility component than for the long-run volatility component, as suggested by the work of Engle and Lee (1999). The QAR component is significant for all indices besides N225. For some indices, only ψ is significantly different from zero within QAR and ϕ is non-significant. In Figure 3, the evolution of QAR is presented for all indices for QAR-MXN-EGARCH-M-2.

The estimates of Conditions 2 to 4 are always in accordance with the boundaries, indicating that those conditions are satisfied (Tables 3 and 4; Appendix).

6. Out-of-sample forecasting and results

6.1. Forecasting method

To further motivate the practical use of QAR-MXN-EGARCH-M, the following out-of-sample volatility forecasting exercise is performed. The full data window is divided into an initial estimation window (19 October 1992 to 12 December 2011; 1,000 weekly observations) and the forecasting window (19 December 2011 to 20 January 2020; $T_f = 423$ weekly observations). One-step ahead forecasts of volatility are computed, by using the rolling-window forecasting approach. For one-step ahead volatility forecasting, σ_t is computed by using Equations (11), (17) and (23) for the MXN, EGB2 and NIG distributions, respectively. As a model-independent proxy of true volatility σ_t^*, the square of the weekly realized variance is used, in the same way as in the work of Harvey and Lange (2018). The source of data for realized variance is the Oxford-Man Institute (OMI) of Quantitative Finance. For σ_t^*, variable 'rv5' is downloaded from: https://realized.oxford-man.ox.ac.uk. Motivated by the works of Hansen and Lunde (2005), Patton (2011) and Harvey and Lange (2018), the following six loss functions are used:

$$\begin{aligned}
&\text{MSE}_1 = T_f^{-1} \textstyle\sum_{t=1}^{T_f} (\sigma_t^* - \sigma_t)^2 && \text{MSE}_2 = T_f^{-1} \textstyle\sum_{t=1}^{T_f} [(\sigma_t^*)^2 - \sigma_t^2]^2 \\
&\text{QLIKE} = T_f^{-1} \textstyle\sum_{t=1}^{T_f} \left\{ \frac{(\sigma_t^*)^2}{\sigma_t^2} - \ln\left[\frac{(\sigma_t^*)^2}{\sigma_t^2}\right] - 1 \right\} && \text{R}^2\text{LOG} = T_f^{-1} \textstyle\sum_{t=1}^{T_f} \left\{ \ln\left[\frac{(\sigma_t^*)^2}{\sigma_t^2}\right] \right\}^2 \\
&\text{MAE}_1 = T_f^{-1} \textstyle\sum_{t=1}^{T_f} |\sigma_t^* - \sigma_t| && \text{MAE}_2 = T_f^{-1} \textstyle\sum_{t=1}^{T_f} |(\sigma_t^*)^2 - \sigma_t^2|
\end{aligned} \tag{28}$$

Comparison of predictive accuracy is performed by comparing the estimates of the loss functions and by using the Diebold–Mariano test of out-of-sample predictive accuracy.

6.2. Forecasting results

Tests of predictive accuracy are performed for EGARCH-M-1 and QAR-EGARCH-M-1, by focusing on DJIA (Table 5). Only one-component volatility models are considered for one stock market index, because of the reliability and speed of the rolling window-based forecasting approach for one-component models. Nevertheless, the differences between predictive accuracies of EGARCH-M and QAR-EGARCH-M for alternative probability distributions can be studied by using this exercise. In Table 5, the loss functions for the forecasting period and the Diebold–Mariano test results are presented. The results are striking: (a) The loss function of QAR-MXN-EGARCH-M-1 is always lower than the loss functions of QAR-EGB2-EGARCH-M-1 and QAR-NIG-EGARCH-M-1. (b) The loss functions of QAR-MXN-EGARCH-M-1, QAR-EGB2-EGARCH-M-1 and QAR-NIG-EGARCH-M-1 are always lower than the loss functions of MXN-EGARCH-M-1, EGB2-EGARCH-M-1 and NIG-EGARCH-M-1, respectively. The Diebold–Mariano test supports these conclusions for most of the loss function alternatives.

These results indicate that the use of the non-linear QAR component in the risk premium, not only significantly improves overall statistical performance, but it also significantly improves predictive accuracy of volatility. This is the main message of this paper for practitioners.

7. Conclusions

The recent work of Harvey and Lange (2018) is extended in the present paper, by adding a QAR dynamic parameter into the risk premium of financial assets. Three alternative probability distributions are used for the extended model: MXN, EGB2 and NIG. The use of the location score function for the MXN distribution is new in the GAS/DCS literature. The consideration of the MXN, EGB2 and NIG distributions is motivated by the fact that the score functions corresponding to these distributions transform unexpected returns in similar ways. Conditions of consistency and asymptotic normality of the ML estimator are presented, by using existing results from the literature on GAS/DCS models. The EGARCH-M-1 and EGARCH-M-2 models of Harvey and Lange (2018) are estimated and are compared to the new QAR-EGARCH-M-1 and QAR-EGARCH-M-2 models for the MXN, EGB2 and NIG probability distributions.

Control data are used for five major stock indices: DJIA, FCHI, GDAXI, N255 and AEX. For EGARCH-M-1 and EGARCH-M-2, our results are similar to the results from the work of Harvey and Lange (2018). Firstly, the short-run volatility component has a negative effect on the risk premium for several cases. Secondly, the long-run volatility component has a positive effect on the risk premium for almost all of the cases. The results are also in accordance with the relevant literature (e.g. Chou 1988; Schwert 1989). Thirdly, the leverage effects are more significant for the short-run volatility component than for the long-run volatility component. This result is also in accordance with the results of Engle and Lee (1999). Fourthly, some of the conditions of the ML estimator are estimated ex-post for all models and assets, and the estimates never indicate failures of those conditions.

For the MXN distribution, the statistical performance metrics for DJIA, FCHI, GDAXI and AEX suggest that QAR-EGARCH-M is superior to EGARCH-M, but the opposite seems to be true for N225. For the EGB2 and NIG distributions, the results are mixed. These findings suggest that the separation of risk premium and volatility dynamics is the most effective for the MXN distribution. The results on out-of-sample predictive accuracy for rolling windows are very similar for the MXN, EGB2 and NIG distributions. The focus of the forecasting exercise is DJIA and only specifications with one volatility component are used. The forecast accuracy of QAR-EGARCH-M-1 is significantly superior to the forecast accuracy of EGARCH-M-1 for all probability distributions and the forecast accuracy of QAR-MXN-EGARCH-1 is significantly superior to the forecast accuracies of QAR-EGB2-EGARCH-1 and QAR-NIG-EGARCH-1.

In summary, both the statistical performance and predictive accuracy results indicate that the QAR-EGARCH-M extension of EGARCH-M is useful for practitioners and that this extension is especially beneficial when the MXN probability distribution is assumed for stock returns.

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References

Alizadeh S, Brandt MW, Diebold FX. 2002. Range-based estimation of stochastic volatility models. *Journal of Finance* **57**(3): 1047–1091. doi: 10.1111/1540-6261.00454.

Ayala A, Blazsek S, Escribano A. 2019. Maximum likelihood estimation of score-driven models with dynamic shape parameters: an application to Monte Carlo value-at-risk. *Working Paper* 19-12, University Carlos III of Madrid, Department of Economics. https://e-archivo.uc3m.es/handle/10016/28638.

Barndorff-Nielsen O, Halgreen C. 1977. Infinite divisibility of the hyperbolic and generalized inverse Gaussian distributions. *Probability Theory and Related Fields* **38**(4): 309–311. doi: 10.1007/bf00533162.

Black F. 1976. Studies of stock market volatility changes. *1976 Proceedings of the American Statistical Association Business and Economic Statistics Section*.

Blasques F, Gorgi P, Koopman SJ. 2018. Feasible invertibility conditions and maximum likelihood estimation for observation-driven models. *Electronic Journal of Statistics* **12**: 1019–1052. doi: 10.1214/18-EJS1416.

Blasques F, Koopman SJ, Lucas A. 2015. Information-theoretic optimality of observation-driven time series models for continuous responses. *Biometrika* **102**(2): 325–343. doi: 10.1093/biomet/asu076.

Blazsek S, Haddad M. 2020. Estimation and statistical performance of Markov-switching score-driven volatility models: the case of G20 stock markets. *Discussion Paper* 1/2020, Francisco Marroquín University, School of Business. https://en.ufm.edu/wp-content/uploads/2020/02/Blazsek-Haddad-2020-GESG-Disussion-Paper-no.1-2020-1.pdf.

Blazsek S, Ho HC, Liu SP. 2018. Score-driven Markov-switching EGARCH models: an application to systematic risk analysis. *Applied Economics* **50**(56): 6047–6060. doi: 10.1080/00036846.2018.1488073.

Bollerslev T. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* **31**(3): 307–327. doi: 10.1016/0304-4076(86)90063-1.

Box GEP, Jenkins GM. 1970. *Time Series Analysis Forecasting and Control*. Holden-Day, San Francisco.

Bozejko M, Demni N. 2010. Topics on Meixner families. *Banach Center Publications* **89**(1): 61-74. ISSN: 0137-6934.

Caivano M, Harvey A. 2014. Time-series models with an EGB2 conditional distribution. *Journal of Time Series Analysis* **35**(6): 558–571. doi: 10.1111/jtsa.12081.

Campbell JY, Hentschel L. 1992. No news is good news: an asymmetric model of changing volatility in stock returns. *Journal of Financial Economics* **31**(3): 281–318. doi: 10.1016/0304-405X(92)90037-X.

Chou RY. 1988. Volatility persistence and stock valuations: some empirical evidence using GARCH. *Journal of Applied Econometrics* **3**(4): 279–294. doi: 10.1002/jae.3950030404.

Cox DR, Gudmundsson G, Lindgren G, Bondesson L, Harsaae E, Laake P, Lauritzen SL. 1981. Statistical analysis of time series: some recent developments [with discussion and reply]. *Scandinavian Journal of Statistics* **8**(2): 93–115.

Creal D, Koopman SJ, Lucas A. 2008. A general framework for observation driven time-varying parameter models. *Tinbergen Institute Discussion Paper* 08-108/4.

Creal D, Koopman SJ, Lucas A. 2011. A dynamic multivariate heavy-tailed model for time-varying volatilities and correlations. *Journal of Business & Economic Statistics* **29**(4): 552–563. doi: 10.1198/jbes.2011.10070.

Creal D, Koopman SJ, Lucas A. 2013. Generalized autoregressive score models with applications. *Journal of Applied Econometrics* **28**(5): 777–795. doi: 10.1002/jae.1279.

Davidson R, MacKinnon JG. 2004. *Econometric Theory and Methods.* Oxford University Press, New York.

Diebold FX, Mariano RS. 1995. Comparing predictive accuracy. *Journal of Business & Economic Statistics* **13**(3): 253–263. doi: 10.1080/07350015.1995.10524599.

Engle RF. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* **50**(4): 987–1007. doi: 10.2307/1912773.

Engle RF, Lee GGJ. 1999. A long-run and short-run component model of stock return volatility. In: Engle, R.F., White, H. (eds.), *Cointegration, Causality, and Forecasting: A Festschrift in Honour of Clive W.J. Granger.* Oxford University Press, Oxford.

Engle RF, Lilien DM, Robins PR. 1987. Estimating time varying risk premia in the term structure: the Arch-M model. *Econometrica* **55**(2): 391–407. doi: 10.2307/1913242.

French KR, Schwert GW, Stambaugh RF. 1987. Expected stock returns and volatility. *Journal of Financial Economics* **19**(1): 3–29. doi: 10.1016/0304-405X(87)90026-2.

Glosten LR, Jagannathan R, Runkle DE. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance* **48**(5): 1779–1801. doi: 10.1111/j.1540-6261.1993.tb05128.x.

Grigoletto M, Provasi C. 2008. Simulation and estimation of the Meixner distribution. *Communication in Statistics and Computation* **38**(1): 58–77. doi: 10.1080/03610910802395679.

Hamilton JD. 1994. *Time Series Analysis.* Princeton University Press, Princeton.

Hansen PR, Lunde A. 2005. A forecast comparison of volatility models: does anything beat a GARCH(1,1)? *Journal of Applied Econometrics* **20**(7), 873–889. doi: 10.1002/jae.800.

Harvey AC. 2013. *Dynamic Models for Volatility and Heavy Tails.* Cambridge University Press, Cambridge.

Harvey AC, Chakravarty T. 2008. Beta-t-(E)GARCH. *Cambridge Working Papers in Economics* 0840, Faculty of Economics, University of Cambridge.
http://www.econ.cam.ac.uk/research-files/repec/cam/pdf/cwpe0840.pdf.

Harvey A, Lange RJ. 2017. Volatility modeling with a generalized t distribution. *Journal of Time Series Analysis* **38**(2): 175–190. doi: 10.1111/jtsa.12224.

Harvey A, Lange RJ. 2018. Modeling the interactions between volatility and returns using EGARCH-M. *Journal of Time Series Analysis* **39**(6): 909–919. doi: 10.1111/jtsa.12224. doi: 10.1111/jtsa.12419.

Harvey AC, Ruiz E, Shephard N. 1994. Multivariate stochastic variance models. *The Review of Economic Studies* **61**(2): 247–264. doi: 10.2307/2297980.

Harvey A, Sucarrat G. 2014. EGARCH models with fat tails, skewness and leverage. *Computational Statistics & Data Analysis* **76**: 320–338. doi: 10.1016/j.csda.2013.09.022.

Janus P, Koopman SJ, Lucas A. 2014. Long memory dynamics for multivariate dependence under heavy tails. *Journal of Empirical Finance* **29**: 187–206. doi: 10.1016/j.jempfin.2014.09.007.

Kawai R. 2012. Likelihood ratio gradient estimation for Meixner distribution and Lévy processes. *Computational Statistics* **27**: 739–755. doi:10.1007/s00180-011-0288-7.

Liu H, Erdem E, Shi J. 2011. Comprehensive evaluation of ARMA–GARCH(-M) approaches for modeling the mean and volatility of wind speed. *Applied Energy* **88**(3): 724-732.
doi: 10.1016/j.apenergy.2010.09.028.

Merton R. 1973. An intertemporal capital asset pricing model. *Econometrica* **41**(5): 867–887.
doi: 10.2307/1913811.

Madan D, Yor M. 2008. Representing the CGMY and Meixner Levy processes as time changed Brownian motions. *Journal of Computational Finance* **12**(1): 27–47. doi: 10.21314/JCF.2008.181.

Mohammadi M., 2017. Prediction of α-stable GARCH and ARMA-GARCH-M models. *Journal of Forecasting.* **36**(7): 859–866. doi: 10.1002/for.2477.

Nelson DB. 1991. Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* **59**(2): 347–370. doi: 10.2307/2938260.

Patton AJ. 2011. Data-based ranking of realised volatility estimators. *Journal of Econometrics* **161**(2): 284–303. doi: 10.1016/j.jeconom.2010.12.010.

Prentice RL. 1975. Discrimination among some parametric models. *Biometrika* **62**(3): 607–614. doi: 10.1093/biomet/62.3.607.

Schoutens W. 2002. The Meixner process: theory and applications in finance. *Eurandom Report* 2001-004. Eurandom, Eindhoven.

Schwert GW. 1989. Why does stock market volatility change over time? *The Journal of Finance* **44**(5): 1115–1153. doi: 10.1111/j.1540-6261.1989.tb02647.x.

Shapiro SS, Wilk MB. 1965. An analysis of variance test for normality (complete samples). *Biometrika* **52**(3–4): 591–611. doi: 10.1093/biomet/52.3-4.591.

Su YC, Huang HC, Lin YJ. 2011. GJR-GARCH model in value-at-risk of financial holdings. *Applied Financial Economics* **21**(24): 1819–1829. doi: 10.1080/09603107.2011.595677.

Taylor SJ. 1994. Modelling stochastic volatility : a review and comparative study. *Mathematical Finance* **4**(2), 183–204. doi: 10.1111/j.1467-9965.1994.tb00057.x.

Turner CM, Startz R, Nelson CR. 1989. A Markov model of heteroskedasticity, risk, and learning in the stock market. *Journal of Financial Economics* **25**(1): 3–22. doi: 10.1016/0304-405X(89)90094-9.

Table 1. Descriptive statistics

	DJIA	FCHI	GDAXI	N225	AEX
Stock exchange	NYSE, NASDAQ	Euronext Paris	Frankfurt Stock Exchange	Tokyo Stock Exchange	Euronext Amsterdam
Location	United States	France	Germany	Japan	Netherlands
Start date	19 October 1992	19 October 1992	19 October 1992	19 October 1992	19 October 1992
End date	20 January 2020	20 January 2020	20 January 2020	20 January 2020	20 January 2020
Data frequency	Weekly	Weekly	Weekly	Weekly	Weekly
Minimum	−20.0298	−25.0504	−24.3470	−27.8844	−28.7546
Maximum	10.6977	12.4321	14.9422	11.4497	13.5816
Average	0.1554	0.0904	0.1568	0.0252	0.1118
Standard deviation	2.2163	2.8498	3.0114	2.9333	2.8306
Skewness	−0.8992	−0.6967	−0.6354	−0.8068	−1.1405
Excess kurtosis	7.6025	5.4041	5.1231	6.5671	9.6841
$\mathrm{Corr}(y_t, y_{t-1})$	−0.0739	−0.0287	−0.0369	−0.0058	−0.0791
$\mathrm{Corr}(\lvert y_t\rvert, y_{t-1})$	−0.1938	−0.2141	−0.1567	−0.2025	−0.2048
Shapiro–Wilk test, p-value	2.48E-23	1.79E-18	1.10E-20	3.85E-19	1.46E-25

Notes: Dow Jones Industrial Average (ticker: DJIA); CAC 40 (ticker: FCHI); DAX Performance Index (ticker: GDAXI); Nikkei 225 (ticker: N225); Amsterdam Exchange Index (ticker: AEX); New York Stock Exchange (NYSE); National Association of Securities Dealers Automated Quotations (NASDAQ). For the Shapiro–Wilk normality test statistic the p-values are reported, which always indicate the rejection of the normal distribution null hypothesis at the 1% level. *Source of data:* Yahoo Finance

Table 2(a). In-sample model performance for DJIA, FCHI and GDAXI

DJIA	MXN-EGARCH-M-1	QAR-MXN-EGARCH-M-1	MXN-EGARCH-M-2	QAR-MXN-EGARCH-M-2
LL	−2.041561	**−2.037623**†	−2.040030	**−2.034886**†
AIC	4.094366	**4.089301**†	4.096925	**4.089448**†
BIC	**4.123941**	4.126269†	4.141287	**4.141203**†
HQC	4.105413	**4.103109**†	4.113495	**4.108780**†

DJIA	EGB2-EGARCH-M-1	QAR-EGB2-EGARCH-M-1	EGB2-EGARCH-M-2	QAR-EGB2-EGARCH-M-2
LL	−2.040959	**−2.039835**	−2.037208	**−2.035059**
AIC	**4.093161**	4.096537	**4.088472**	4.089795
BIC	**4.122736**	4.140898	**4.125440**	4.141550
HQC	**4.104208**	4.113106	**4.102280**	4.109127

DJIA	NIG-EGARCH-M-1	QAR-NIG-EGARCH-M-1	NIG-EGARCH-M-2	QAR-NIG-EGARCH-M-2
LL	−2.041072	**−2.038717**	−2.037241	**−2.035060**
AIC	**4.093389**	4.094299	**4.088537**	4.089797
BIC	**4.122963**	4.138661	**4.125505**	4.141552
HQC	**4.104435**	4.110869	**4.102345**	4.109129

FCHI	MXN-EGARCH-M-1	QAR-MXN-EGARCH-M-1	MXN-EGARCH-M-2	QAR-MXN-EGARCH-M-2
LL	−2.336503	**−2.332632**	−2.336039	**−2.329142**
AIC	4.684249	**4.679319**†	4.688944	**4.677961**
BIC	**4.713824**	4.716287†	4.733305	**4.729716**
HQC	4.695296	**4.693127**†	4.705514	**4.697292**

FCHI	EGB2-EGARCH-M-1	QAR-EGB2-EGARCH-M-1	EGB2-EGARCH-M-2	QAR-EGB2-EGARCH-M-2
LL	−2.335963	**−2.331268**	−2.332220	**−2.328754**
AIC	4.683169	**4.679401**	4.678495	**4.677185**
BIC	**4.712743**	4.723762	**4.715463**	4.728940
HQC	**4.694216**	4.695971	**4.692303**	4.696516

FCHI	EGB2-EGARCH-M-1	QAR-EGB2-EGARCH-M-1	EGB2-EGARCH-M-2	QAR-EGB2-EGARCH-M-2
LL	−2.336118	**−2.335401**	−2.332284	**−2.328657**
AIC	**4.683479**	4.687667	4.678624	**4.676990**
BIC	**4.713054**	4.732029	**4.715591**	4.728745
HQC	**4.694526**	4.704237	**4.692432**	4.696322

GDAXI	MXN-EGARCH-M-1	QAR-MXN-EGARCH-M-1	MXN-EGARCH-M-2	QAR-MXN-EGARCH-M-2
LL	−2.359384	**−2.356852**	−2.351929†	**−2.347897**†
AIC	4.730012	**4.727759**	4.720724†	**4.715471**†
BIC	**4.759586**	4.764727†	**4.765085**	4.767226†
HQC	**4.741059**	4.741568	4.737293†	**4.734802**†

GDAXI	EGB2-EGARCH-M-1	QAR-EGB2-EGARCH-M-1	EGB2-EGARCH-M-2	QAR-EGB2-EGARCH-M-2
LL	−2.359144	**−2.352080**	−2.356970	**−2.348865**
AIC	4.729533	**4.721025**	4.727994	**4.717407**
BIC	**4.759107**	4.765386	**4.764962**	4.769162
HQC	4.740579	**4.737595**	4.741802	**4.736738**

GDAXI	NIG-EGARCH-M-1	QAR-NIG-EGARCH-M-1	NIG-EGARCH-M-2	QAR-NIG-EGARCH-M-2
LL	−2.359126	**−2.356171**	−2.356826	**−2.348361**
AIC	4.729496	**4.729208**	4.727707	**4.716399**
BIC	**4.759070**	4.773570	**4.764674**	4.768154
HQC	**4.740543**	4.745778	4.741515	**4.735730**

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics, by comparing EGARCH-M and QAR-EGARCH-M. † indicates that the MXN distribution is superior to the EGB2 and NIG distributions.

Table 2(b). In-sample model performance for N225 and AEX

N225	MXN-EGARCH-M-1	QAR-MXN-EGARCH-M-1	MXN-EGARCH-M-2	QAR-MXN-EGARCH-M-2
LL	−2.410722	**−2.409399**	−2.405449†	**−2.404829**
AIC	**4.832687**	4.832853	**4.827764**†	4.829334
BIC	**4.862261**	4.869821†	**4.872125**	4.881089
HQC	**4.843734**	4.846661	**4.844334**†	4.848665
N225	**EGB2-EGARCH-M-1**	**QAR-EGB2-EGARCH-M-1**	**EGB2-EGARCH-M-2**	**QAR-EGB2-EGARCH-M-2**
LL	−2.409820	**−2.404511**	−2.408484	**−2.403864**
AIC	4.830885	**4.825888**	4.831023	**4.827405**
BIC	**4.860459**	4.870249	**4.867991**	4.879160
HQC	**4.841931**	4.842458	**4.844831**	4.846736
N225	**NIG-EGARCH-M-1**	**QAR-NIG-EGARCH-M-1**	**NIG-EGARCH-M-2**	**QAR-NIG-EGARCH-M-2**
LL	−2.409960	**−2.404624**	−2.408652	**−2.403211**
AIC	4.831163	**4.826115**	4.831359	**4.826098**
BIC	**4.860737**	4.870476	**4.868327**	4.877853
HQC	**4.842209**	4.842685	**4.845167**	4.845429

AEX	MXN-EGARCH-M-1	QAR-MXN-EGARCH-M-1	MXN-EGARCH-M-2	QAR-MXN-EGARCH-M-2
LL	−2.244549†	**−2.241429**	−2.236156†	**−2.233126**†
AIC	4.500342†	**4.496912**	4.489178†	**4.485929**†
BIC	**4.529916**†	4.533880†	**4.533539**†	4.537684†
HQC	4.511388†	**4.510720**	4.505747†	**4.505260**†
AEX	**EGB2-EGARCH-M-1**	**QAR-EGB2-EGARCH-M-1**	**EGB2-EGARCH-M-2**	**QAR-EGB2-EGARCH-M-2**
LL	−2.244778	**−2.236892**	−2.241770	**−2.233774**
AIC	4.500799	**4.490649**	4.497594	**4.487225**
BIC	**4.530373**	4.535011	**4.534562**	4.538980
HQC	4.511845	**4.507219**	4.511402	**4.506556**
AEX	**NIG-EGARCH-M-1**	**QAR-NIG-EGARCH-M-1**	**NIG-EGARCH-M-2**	**QAR-NIG-EGARCH-M-2**
LL	−2.244598	**−2.241134**	−2.241555	**−2.233549**
AIC	4.500439	**4.499133**	4.497165	**4.486774**
BIC	**4.530013**	4.543495	**4.534133**	4.538529
HQC	**4.511485**	4.515703	4.510973	**4.506106**

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics, by comparing EGARCH-M and QAR-EGARCH-M. † indicates that the MXN distribution is superior to the EGB2 and NIG distributions.

Table 3. Parameter estimates and ML conditions; MXN-EGARCH-M-1 (A) and MXN-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	−0.0844(0.1546)	−0.1953(0.2198)	−0.0437(0.2116)	−0.4187(0.3494)	0.1561(0.1560)
κ_{L}	1.7961***(0.4099)	6.3973***(1.4972)	3.2755***(0.8230)	2.0340***(0.5125)	1.8404***(0.5660)
ω	0.3090*(0.1650)	0.7469***(0.1817)	0.7374***(0.1127)	1.0067***(0.1399)	0.5341***(0.1440)
β_{L}	0.9126***(0.0157)	0.9465***(0.0123)	0.9130***(0.0189)	0.8908***(0.0261)	0.9538***(0.0114)
α_{L}	0.0662***(0.0118)	0.0718***(0.0098)	0.0840***(0.0125)	0.0569***(0.0107)	0.0693***(0.0091)
α^*_{L}	0.0259**(0.0107)	−0.0213**(0.0093)	0.0002(0.0132)	0.0227***(0.0084)	0.0142(0.0100)
δ_1	−0.4067***(0.0546)	−0.4267***(0.0417)	−0.4094***(0.0472)	−0.2684***(0.0449)	−0.3959***(0.0485)
δ_2	0.5284**(0.2630)	1.3562***(0.2018)	0.7425***(0.2166)	0.3145(0.2443)	0.4859*(0.2645)
A. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,\mathrm{L}}$	0.9126	0.9465	0.9130	0.8908	0.9538
$C_{3,\mathrm{L}}$	0.6230	0.5400	0.5499	0.6430	0.6838
$C_{4,\mathrm{L}}$	−0.3621	−0.3017	−0.3853	−0.2789	−0.2643

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	−1.8771***(0.5237)	0.0210(8.4713)	−3.2526***(0.7477)	0.9073(2.0391)	−1.2204***(0.4578)
κ_{L}	1.2267***(0.3515)	6.1106**(2.4822)	3.8888***(1.1228)	1.6278**(0.6522)	1.7759***(0.6144)
κ_{S}	1.8750***(0.5131)	0.1578(6.5876)	3.1693***(0.7617)	−1.0582(1.8859)	1.2523***(0.3775)
ω	0.7846***(0.1776)	0.6720***(0.2162)	1.0054***(0.2052)	1.0190***(0.1494)	0.7898***(0.1973)
β_{L}	0.9574***(0.0179)	0.9468***(0.0123)	0.9705***(0.0110)	0.9117***(0.0248)	0.9779***(0.0085)
β_{S}	0.7050***(0.0983)	0.9263***(0.0620)	0.5550***(0.1354)	0.7237***(0.1367)	0.6529***(0.0833)
α_{L}	0.0603***(0.0182)	0.0785***(0.0165)	0.0543***(0.0118)	0.0628***(0.0217)	0.0626***(0.0100)
α_{S}	0.0057(0.0221)	−0.0146(0.0128)	0.0289(0.0191)	−0.0344(0.0248)	−0.0351*(0.0183)
α^*_{L}	−0.0036(0.0120)	−0.0236**(0.0116)	−0.0123(0.0075)	0.0180(0.0145)	−0.0030(0.0076)
α^*_{S}	0.0600***(0.0167)	0.0091(0.0090)	0.0469***(0.0179)	0.0335*(0.0173)	0.0858***(0.0149)
δ_1	−0.3023***(0.0463)	−0.4330***(0.0423)	−0.4253***(0.0425)	−0.2375***(0.0439)	−0.3470***(0.0484)
δ_2	0.0337(0.2350)	1.3524***(0.1971)	0.5756***(0.2123)	0.2359(0.2325)	0.2330(0.2286)
B. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,\mathrm{L}}$	0.9574	0.9468	0.9705	0.9117	0.9779
$C_{2,\mathrm{S}}$	0.7050	0.9263	0.5550	0.7237	0.6529
$C_{3,\mathrm{L}}$	0.7510	0.5180	0.7396	0.6665	0.7660
$C_{3,\mathrm{S}}$	0.5020	0.9642	0.2658	0.6184	0.5580
$C_{3,\mathrm{L,S}}$	0.5891	0.7012	0.3657	0.5954	0.5387
$C_{4,\mathrm{L}}$	−0.1513	−0.3316	−0.1406	−0.2475	−0.1451
$C_{4,\mathrm{S}}$	−0.4648	−0.0542	−1.0277	−0.2889	−0.5208

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table 4. Parameter estimates and ML conditions; QAR-MXN-EGARCH-M-1 (A) and QAR-MXN-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	0.0049(0.1502)	−0.0610(0.1997)	0.0604(0.2033)	−0.9163*(0.5301)	0.1678(0.1521)
ϕ	−0.1958(0.2965)	0.0811(0.2849)	0.0949(0.3396)	0.8882***(0.0873)	−0.8438***(0.1002)
ψ	−0.0742**(0.0322)	−0.2055***(0.0703)	−0.0853**(0.0365)	0.0246(0.0195)	−0.0367*(0.0212)
κ_{L}	1.5232***(0.3523)	5.5561***(1.3320)	2.9058***(0.7606)	2.5200***(0.6166)	1.6787***(0.4901)
ω	0.4448***(0.1665)	0.8686***(0.1877)	0.8612***(0.1238)	0.9503***(0.1560)	0.5800***(0.1532)
β_{L}	0.9190***(0.0159)	0.9518***(0.0117)	0.9236***(0.0185)	0.8794***(0.0283)	0.9539***(0.0116)
α_{L}	0.0715***(0.0120)	0.0728***(0.0097)	0.0872***(0.0123)	0.0518***(0.0113)	0.0716***(0.0094)
α^*_{L}	0.0250**(0.0103)	−0.0201**(0.0088)	−0.0017(0.0123)	0.0289***(0.0093)	0.0161*(0.0096)
δ_1	−0.3818***(0.0525)	−0.4270***(0.0401)	−0.4014***(0.0473)	−0.2784***(0.0459)	−0.3924***(0.0477)
δ_2	0.3279(0.2550)	1.1180***(0.1956)	0.5484**(0.2200)	0.3674(0.2543)	0.3643(0.2598)
A. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,\dagger}$	0.1958	0.0811	0.0949	0.8882	0.8438
$C_{2,\mathrm{L}}$	0.9190	0.9518	0.9236	0.8794	0.9539
$C_{3,\dagger}$	0.0137	0.0330	0.0311	0.7342	0.6438
$C_{3,\mathrm{L}}$	0.6395	0.5847	0.5823	0.6466	0.6898
$C_{3,\dagger,\mathrm{L}}$	0.0737	0.1439	0.1343	0.6360	0.6345
$C_{4,\dagger}$	−2.3347	−1.7568	−1.7842	−0.1547	−0.2209
$C_{4,\mathrm{L}}$	−0.3208	−0.2597	−0.3314	−0.2998	−0.2562

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	1.0574(1.5037)	−1.4475(0.9200)	0.6055(0.9961)	1.1947(2.1775)	0.9396(0.7394)
ϕ	0.5684***(0.1246)	0.2587(0.2172)	0.4454***(0.1283)	−0.5222(0.6104)	0.4475***(0.1568)
ψ	−0.0788**(0.0328)	−0.1092**(0.0452)	−0.1020***(0.0377)	−0.0195(0.0207)	−0.0860***(0.0329)
κ_{L}	1.3018***(0.3176)	2.7549***(0.7567)	2.3199***(0.5879)	1.6435***(0.6307)	1.3057***(0.4242)
κ_{S}	−0.9972(1.4876)	1.3301*(0.8021)	−0.5923(0.9382)	−1.3591(2.0518)	−0.6633(0.6757)
ω	0.6446***(0.1625)	0.9937***(0.1752)	1.0600***(0.1594)	1.0229***(0.1473)	0.6131***(0.2069)
β_{L}	0.9420***(0.0156)	0.9752***(0.0101)	0.9662***(0.0122)	0.9177***(0.0239)	0.9775***(0.0082)
β_{S}	0.5922***(0.1815)	0.6723***(0.1403)	0.5173***(0.1294)	0.7093***(0.1368)	0.4953***(0.1186)
α_{L}	0.0707***(0.0116)	0.0545***(0.0113)	0.0633***(0.0106)	0.0597***(0.0190)	0.0601***(0.0088)
α_{S}	−0.0094(0.0148)	−0.0237(0.0200)	−0.0072(0.0169)	−0.0319(0.0225)	−0.0408**(0.0169)
α^*_{L}	0.0135(0.0114)	−0.0078(0.0066)	−0.0042(0.0079)	0.0168(0.0131)	0.0081(0.0081)
α^*_{S}	0.0412**(0.0178)	0.0621***(0.0159)	0.0747***(0.0163)	0.0353**(0.0166)	0.0873***(0.0160)
δ_1	−0.3224***(0.0463)	−0.3391***(0.0486)	−0.3240***(0.0412)	−0.2358***(0.0440)	−0.3193***(0.0442)
δ_2	0.0718(0.2361)	0.4516**(0.2089)	0.1999(0.2071)	0.2301(0.2292)	0.2203(0.2305)
B. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,*}$	0.5684	0.2587	0.4454	0.5222	0.4475
$C_{2,\mathrm{L}}$	0.9420	0.9752	0.9662	0.9177	0.9775
$C_{2,\mathrm{S}}$	0.5922	0.6723	0.5173	0.7093	0.4953
$C_{3,*}$	0.4870	0.1368	0.3266	0.2369	0.3084
$C_{3,\mathrm{L}}$	0.6724	0.7718	0.7460	0.6836	0.7688
$C_{3,\mathrm{S}}$	0.4407	0.5420	0.3478	0.5912	0.4185
$C_{3,*,\mathrm{L}}$	0.5796	0.3380	0.5200	−0.3906	0.5012
$C_{3,*,\mathrm{S}}$	0.4010	0.2416	0.2739	−0.3565	0.2719
$C_{3,\mathrm{L},\mathrm{S}}$	0.4682	0.5605	0.3949	0.5904	0.4128
$C_{4,*}$	−0.3607	−0.9855	−0.5295	−0.7090	−0.5667
$C_{4,\mathrm{L}}$	−0.2203	−0.1296	−0.1534	−0.2301	−0.1539
$C_{4,\mathrm{S}}$	−0.5750	−0.4792	−0.8690	−0.3159	−0.8064

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table 5. Out-of-sample volatility forecast performance for DJIA

A. Forecasting method:		Start dates				End dates
One-step ahead forecasts						
Rolling-window approach						
First estimation window		19 October 1992				12 December 2011
Forecasting window		19 December 2011				20 January 2020
B. Loss functions:	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
(M1) MXN-EGARCH-M-1	0.4511	18.5322	0.2810	0.7610	0.5370	1.8752
(M2) QAR-MXN-EGARCH-M-1	**0.4380**†	**18.4124**†	**0.2742**†	**0.7371**†	**0.5262**†	**1.8359**†
(M3) EGB2-EGARCH-M-1	0.4623	18.9177	0.2844	0.7696	0.5428	1.9003
(M4) QAR-EGB2-EGARCH-M-1	**0.4462**	**18.8165**	**0.2763**	**0.7396**	**0.5292**	**1.8508**
(M5) NIG-EGARCH-M-1	0.4560	18.7399	0.2825	0.7636	0.5395	1.8865
(M6) QAR-NIG-EGARCH-M-1	**0.4444**	**18.6357**	**0.2763**	**0.7423**	**0.5300**	**1.8526**
C. Diebold–Mariano test:	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
DM(M1-M2)	4.0635***	1.5733	3.9464***	3.2371***	5.7074***	5.3788***
DM(M3-M4)	3.7335***	1.5999	3.9340***	3.1371***	4.6950***	5.1584***
DM(M5-M6)	4.4711***	2.1781**	4.3166***	3.3870***	5.6752***	5.2745***
DM(M4-M2)	1.3211	1.2467	1.7940*	2.4450**	2.0116*	1.6926*
DM(M6-M2)	1.5205	1.2747	2.4140**	3.4161***	2.9444***	2.1806**

Notes: † indicates that the loss function estimate for MXN is lower than the loss function estimates for EGB2 and NIG. Bold numbers indicate that the loss function estimate for QAR-EGARCH-M-1 is lower than the loss function estimate for EGARCH-M-1. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Fig. 1(a). $u_{\mu,t}$ for QAR-MXN-EGARCH-M-2

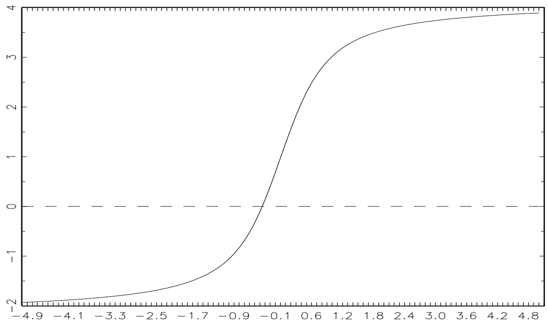

Fig. 1(b). $u_{\lambda,t}$ for QAR-MXN-EGARCH-M-2

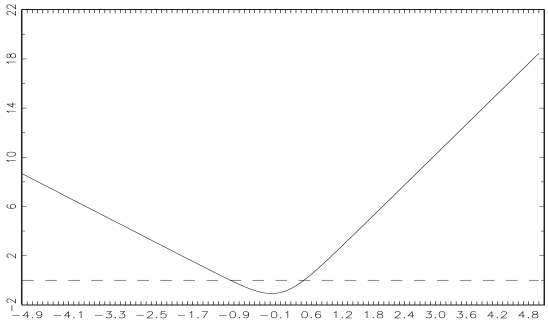

Fig. 1(c). $u_{\mu,t}$ for QAR-EGB2-EGARCH-M-2

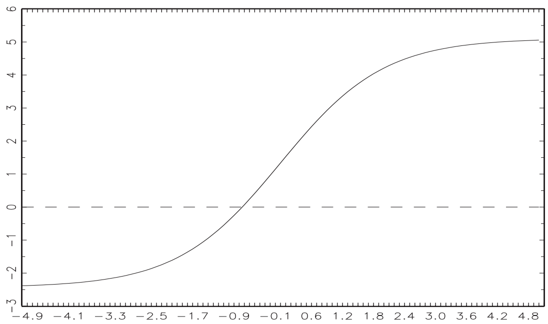

Fig. 1(d). $u_{\lambda,t}$ for QAR-EGB2-EGARCH-M-2

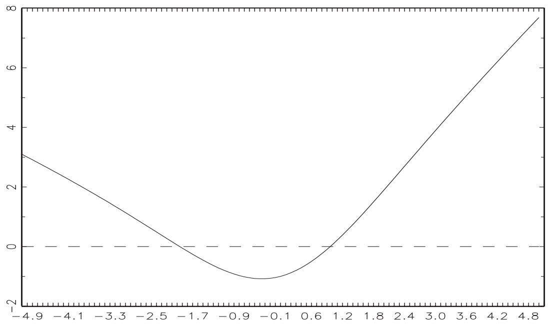

Fig. 1(c). $u_{\mu,t}$ for QAR-NIG-EGARCH-M-2

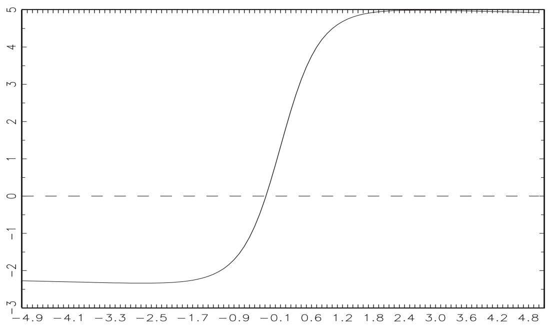

Fig. 1(d). $u_{\lambda,t}$ for QAR-NIG-EGARCH-M-2

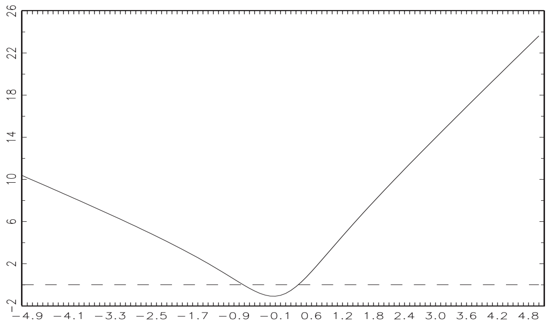

Fig. 1. Score functions as functions of ϵ_t; full sample estimates of shape parameters and $\lambda_t = 0$ are used for DJIA

Fig. 2(a). $\lambda_{\mathrm{L},t}$ for DJIA

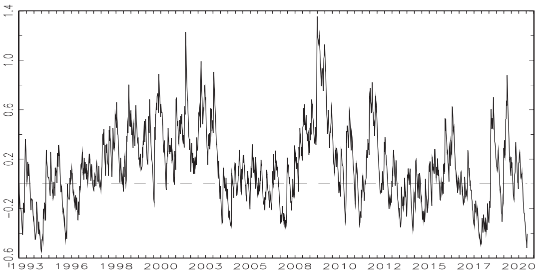

Fig. 2(b). $\lambda_{\mathrm{S},t}$ for DJIA

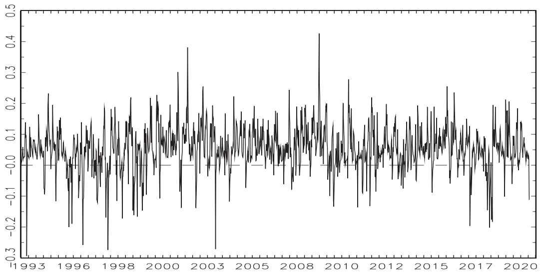

Fig. 2(c). $\lambda_{\mathrm{L},t}$ for FCHI

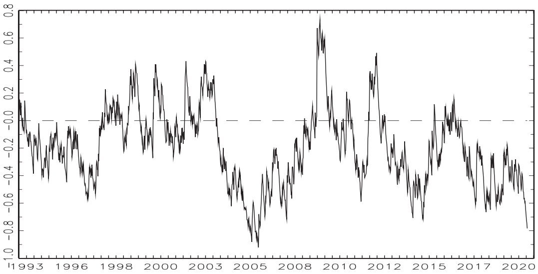

Fig. 2(d). $\lambda_{\mathrm{S},t}$ for FCHI

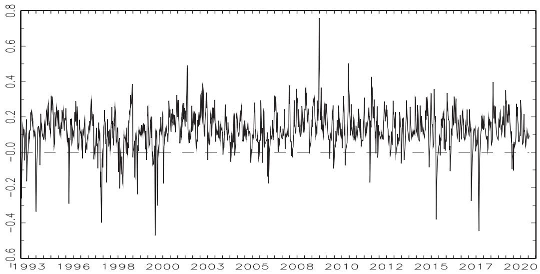

Fig. 2(e). $\lambda_{\mathrm{L},t}$ for GDAXI

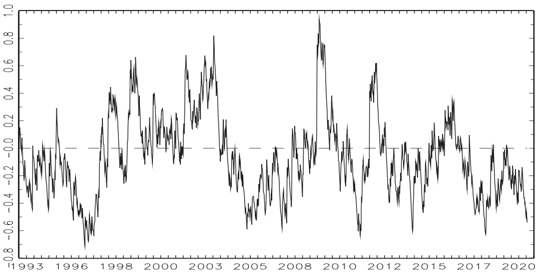

Fig. 2(f). $\lambda_{\mathrm{S},t}$ for GDAXI

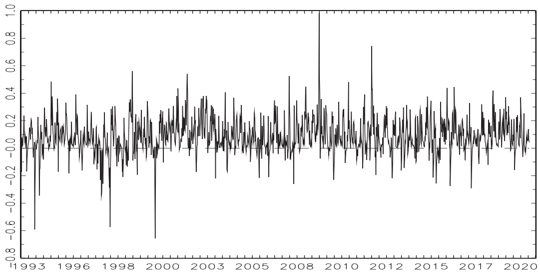

Fig. 2(g). $\lambda_{\mathrm{L},t}$ for N225

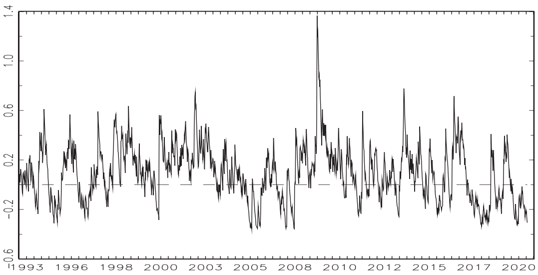

Fig. 2(h). $\lambda_{\mathrm{S},t}$ for N225

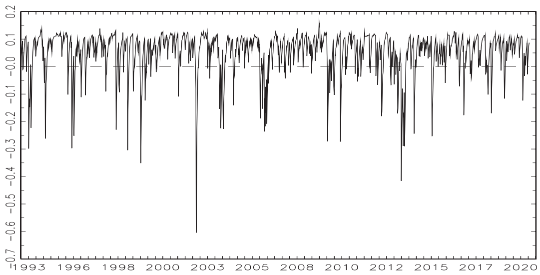

Fig. 2(i). $\lambda_{\mathrm{L},t}$ for AEX

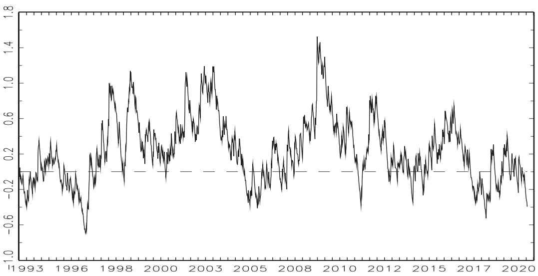

Fig. 2(j). $\lambda_{\mathrm{S},t}$ for AEX

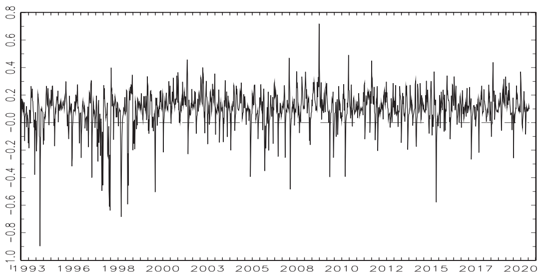

Fig. 2. Long-run and short-run volatility components; QAR-MXN-EGARCH-M-2

Fig. 3(a). μ_t^* for DJIA

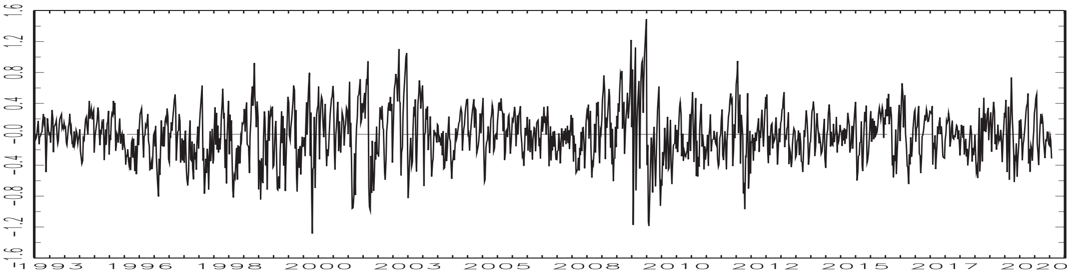

Fig. 3(b). μ_t^* for FCHI

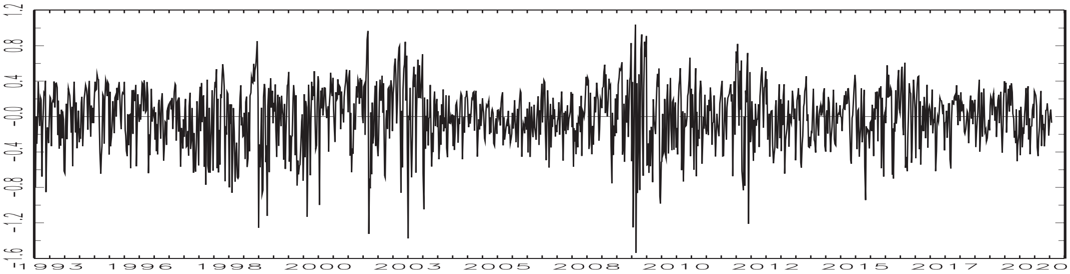

Fig. 3(c). μ_t^* for GDAXI

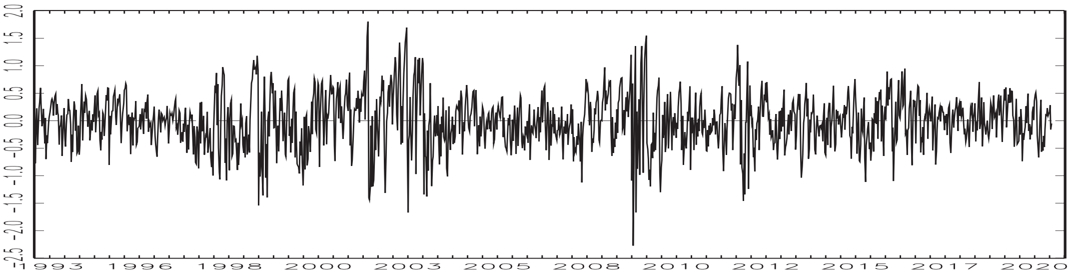

Fig. 3(d). μ_t^* for N225

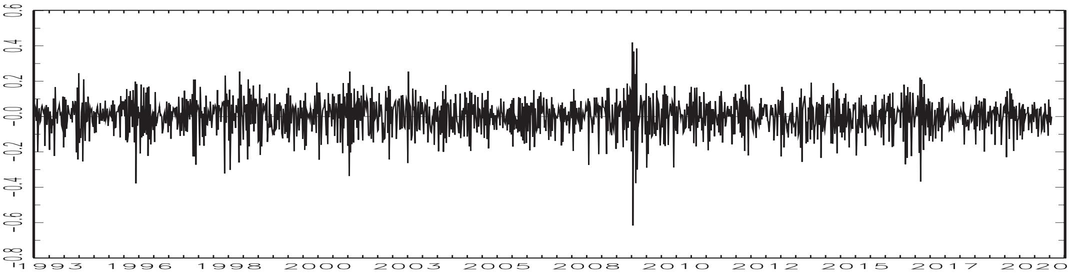

Fig. 3(e). μ_t^* for AEX

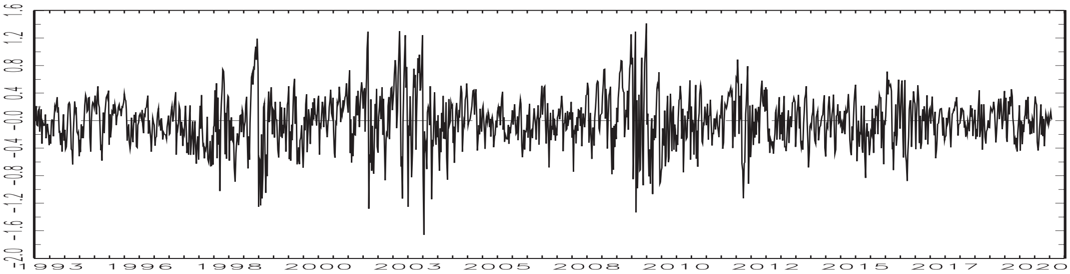

Fig. 3. QAR component in the risk premium; QAR-MXN-EGARCH-M-2

Appendix

Table A1. Parameter estimates and ML conditions; EGB2-EGARCH-M-1 (A) and EGB2-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	−0.0908(0.1521)	−0.1837(0.2167)	−0.0224(0.2047)	−0.3876(0.3400)	0.1625(0.1513)
κ_{L}	2.0315***(0.5365)	6.5564***(1.6426)	3.5713***(0.9609)	2.0201***(0.5548)	2.1969***(0.8241)
ω	0.2773(0.2296)	1.5666***(0.2524)	0.9370***(0.2374)	0.6030***(0.1877)	0.5260(0.3288)
β_{L}	0.9120***(0.0156)	0.9466***(0.0123)	0.9110***(0.0192)	0.8973***(0.0248)	0.9537***(0.0112)
α_{L}	0.0657***(0.0124)	0.0730***(0.0100)	0.0889***(0.0132)	0.0565***(0.0108)	0.0702***(0.0096)
α^*_{L}	0.0216*(0.0127)	−0.0226**(0.0099)	−0.0051(0.0148)	0.0217**(0.0087)	0.0089(0.0123)
δ_1	0.2774(0.2439)	0.9466***(0.2077)	0.4304**(0.2080)	0.1800(0.2515)	0.3023(0.2511)
δ_2	1.3711***(0.3319)	2.0914***(0.1998)	1.5089***(0.2475)	0.8432***(0.3213)	1.3412***(0.3367)
A. Conditions:	**DJIA**	**FCHI**	**GDAXI**	**N225**	**AEX**
$C_{2,\mathrm{L}}$	0.9120	0.9466	0.9110	0.8973	0.9537
$C_{3,\mathrm{L}}$	0.6016	0.5290	0.5194	0.6514	0.6579
$C_{4,\mathrm{L}}$	−0.3702	−0.3003	−0.3978	−0.2704	−0.2738

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	−1.8718***(0.5466)	−6.7009***(1.6519)	−3.3241***(0.7194)	0.9791(1.9666)	−1.1734**(0.4851)
κ_{L}	1.2084***(0.3758)	6.9944***(1.7627)	4.1795***(1.2507)	1.5138**(0.6279)	1.7329**(0.6824)
κ_{S}	1.8886***(0.5342)	6.5227***(1.6666)	3.2595***(0.7436)	−1.0519(1.8230)	1.2214***(0.4033)
ω	0.0280(0.2253)	1.5465***(0.2595)	1.0111***(0.2958)	0.4994***(0.1696)	0.3202(0.3070)
β_{L}	0.9590***(0.0173)	0.9663***(0.0093)	0.9693***(0.0113)	0.9143***(0.0243)	0.9786***(0.0084)
β_{S}	0.6994***(0.1002)	0.4194**(0.2088)	0.5428***(0.1437)	0.7259***(0.1356)	0.6520***(0.0869)
α_{L}	0.0598***(0.0177)	0.0608***(0.0098)	0.0559***(0.0124)	0.0645***(0.0228)	0.0619***(0.0101)
α_{S}	0.0066(0.0218)	0.0020(0.0203)	0.0299(0.0197)	−0.0372(0.0257)	−0.0334*(0.0188)
α^*_{L}	−0.0032(0.0117)	−0.0220***(0.0075)	−0.0137*(0.0081)	0.0184(0.0142)	−0.0029(0.0080)
α^*_{S}	0.0588***(0.0166)	0.0277(0.0202)	0.0437**(0.0188)	0.0327*(0.0170)	0.0846***(0.0153)
δ_1	−0.1760(0.2465)	0.6751***(0.2007)	0.2488(0.2083)	0.1201(0.2389)	0.0145(0.2390)
δ_2	0.5363*(0.3162)	1.9018***(0.1869)	1.3455***(0.2332)	0.6835**(0.2918)	0.8288***(0.3189)
B. Conditions:	**DJIA**	**FCHI**	**GDAXI**	**N225**	**AEX**
$C_{2,\mathrm{L}}$	0.9590	0.9663	0.9693	0.9143	0.9786
$C_{2,\mathrm{S}}$	0.6994	0.4194	0.5428	0.7259	0.6520
$C_{3,\mathrm{L}}$	0.7542	0.6242	0.7240	0.6657	0.7683
$C_{3,\mathrm{S}}$	0.4907	0.1848	0.2469	0.6289	0.5486
$C_{3,\mathrm{L,S}}$	0.5572	0.2792	0.3485	0.6009	0.5376
$C_{4,\mathrm{L}}$	−0.1481	−0.1884	−0.1459	−0.2495	−0.1417
$C_{4,\mathrm{S}}$	−0.4688	−1.1684	−1.0577	−0.2783	−0.5204

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table A2. Parameter estimates and ML conditions; QAR-EGB2-EGARCH-M-1 (A) and QAR-EGB2-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	0.0062(0.1478)	−0.0487(0.1981)	0.0804(0.1994)	−0.9296*(0.5245)	0.1775(0.1475)
ϕ	−0.2883(0.2904)	0.0655(0.2937)	0.0649(0.3715)	0.8916***(0.0828)	−0.8506***(0.0975)
ψ	−0.0732***(0.0240)	−0.0847***(0.0263)	−0.0628**(0.0254)	0.0290(0.0195)	−0.0347**(0.0170)
κ_{L}	1.6244***(0.4359)	5.9995***(1.5592)	3.2505***(0.9343)	2.5752***(0.6640)	1.9746***(0.7174)
ω	0.1301(0.2227)	1.4869***(0.2636)	0.8655***(0.2486)	0.6172***(0.1831)	0.4213(0.3150)
β_{L}	0.9199***(0.0157)	0.9520***(0.0117)	0.9205***(0.0188)	0.8848***(0.0270)	0.9542***(0.0113)
α_{L}	0.0702***(0.0123)	0.0745***(0.0101)	0.0920***(0.0129)	0.0516***(0.0115)	0.0718***(0.0096)
α^*_{L}	0.0229*(0.0118)	−0.0227**(0.0095)	−0.0072(0.0142)	0.0276***(0.0095)	0.0110(0.0119)
δ_1	0.0891(0.2492)	0.7590***(0.2050)	0.2889(0.2133)	0.2263(0.2596)	0.1959(0.2552)
δ_2	1.0645***(0.3446)	1.8946***(0.1944)	1.3350***(0.2649)	0.9295***(0.3376)	1.2101***(0.3465)
A. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,*}$	0.2883	0.0655	0.0649	0.8916	0.8506
$C_{2,\mathrm{L}}$	0.9199	0.9520	0.9205	0.8848	0.9542
$C_{3,*}$	0.1129	0.0664	0.0460	0.8017	0.7343
$C_{3,\mathrm{L}}$	0.6334	0.5667	0.5489	0.6863	0.6687
$C_{3,*,\mathrm{L}}$	0.2345	0.0236	0.0261	0.6413	0.6798
$C_{4,*}$	−1.4412	−1.9818	−2.1414	−0.1202	−0.1692
$C_{4,\mathrm{L}}$	−0.3177	−0.2620	−0.3500	−0.2946	−0.2643

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	1.0574(1.5037)	−3.6375***(1.3261)	−1.0303(0.8381)	1.2436(2.0931)	1.1708(0.8018)
ϕ	0.5684***(0.1246)	0.0809(0.2873)	0.4081**(0.1790)	−0.5087(0.6109)	0.3963**(0.1618)
ψ	−0.0788**(0.0328)	−0.0861***(0.0316)	−0.0926***(0.0308)	−0.0280(0.0281)	−0.1169***(0.0335)
κ_{L}	1.3018***(0.3176)	6.0476***(1.5585)	3.1776***(0.9292)	1.5494**(0.6118)	1.2529***(0.4339)
κ_{S}	−0.9972(1.4876)	3.5498***(1.3422)	0.9507(0.7980)	−1.3369(1.9728)	−0.8858(0.7434)
ω	0.6446***(0.1625)	1.4132***(0.2704)	0.7485***(0.2826)	0.5023***(0.1695)	0.0966(0.2688)
β_{L}	0.9420***(0.0156)	0.9678***(0.0096)	0.9679***(0.0118)	0.9209***(0.0232)	0.9770***(0.0082)
β_{S}	0.5922***(0.1815)	0.5269**(0.2450)	0.5437***(0.1372)	0.7102***(0.1383)	0.4696***(0.1214)
α_{L}	0.0707***(0.0116)	0.0605***(0.0107)	0.0620***(0.0116)	0.0607***(0.0195)	0.0597***(0.0090)
α_{S}	−0.0094(0.0148)	0.0137(0.0219)	0.0163(0.0197)	−0.0341(0.0228)	−0.0391**(0.0166)
α^*_{L}	0.0135(0.0114)	−0.0197***(0.0075)	−0.0110(0.0078)	0.0167(0.0128)	0.0095(0.0085)
α^*_{S}	0.0412**(0.0178)	0.0240(0.0203)	0.0626***(0.0178)	0.0349**(0.0162)	0.0859***(0.0163)
δ_1	−0.3224***(0.0463)	0.5555***(0.2045)	0.0690(0.2231)	0.1251(0.2359)	0.0263(0.2400)
δ_2	0.0718(0.2361)	1.7238***(0.1957)	0.9967***(0.2649)	0.6873**(0.2889)	0.7738**(0.3097)
B. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,*}$	0.5684	0.0809	0.4081	0.5087	0.3963
$C_{2,\mathrm{L}}$	0.9420	0.9678	0.9679	0.9209	0.9770
$C_{2,\mathrm{S}}$	0.5922	0.5269	0.5437	0.7102	0.4696
$C_{3,*}$	0.4966	0.0752	0.2630	0.2662	0.2798
$C_{3,\mathrm{L}}$	0.6990	0.6593	0.7386	0.6852	0.7774
$C_{3,\mathrm{S}}$	0.3778	0.2472	0.2897	0.5971	0.3317
$C_{3,*,\mathrm{L}}$	0.4631	0.0492	0.3437	0.4163	0.3198
$C_{3,*,\mathrm{S}}$	0.3106	0.0119	0.1337	0.3794	0.1312
$C_{3,\mathrm{L},\mathrm{S}}$	0.4724	0.3670	0.3771	0.5944	0.3898
$C_{4,*}$	−0.3607	−1.9026	−1.0468	−0.6899	−1.0844
$C_{4,\mathrm{L}}$	−0.2203	−0.1748	−0.1484	−0.2278	−0.1529
$C_{4,\mathrm{S}}$	−0.5750	−0.8951	−0.9425	−0.3088	−0.8654

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table A3. Parameter estimates and ML conditions; NIG-EGARCH-M-1 (A) and NIG-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	$-0.0851(0.1532)$	$-0.1890(0.2178)$	$-0.0308(0.2072)$	$-0.3982(0.3416)$	$0.1615(0.1528)$
κ_{L}	$1.9130^{***}(0.4771)$	$6.4528^{***}(1.5298)$	$3.4666^{***}(0.8980)$	$1.9890^{***}(0.5231)$	$2.1001^{***}(0.7261)$
ω	$1.1246^{***}(0.1887)$	$2.3860^{***}(0.2358)$	$1.7833^{***}(0.2069)$	$1.5241^{***}(0.1392)$	$1.3735^{***}(0.2716)$
β_{L}	$0.9130^{***}(0.0156)$	$0.9465^{***}(0.0123)$	$0.9115^{***}(0.0191)$	$0.8962^{***}(0.0252)$	$0.9538^{***}(0.0112)$
α_{L}	$0.0659^{***}(0.0121)$	$0.0726^{***}(0.0099)$	$0.0873^{***}(0.0129)$	$0.0572^{***}(0.0108)$	$0.0700^{***}(0.0094)$
α^*_{L}	$0.0235^{**}(0.0118)$	$-0.0219^{**}(0.0095)$	$-0.0031(0.0141)$	$0.0224^{***}(0.0085)$	$0.0103(0.0113)$
δ_1	$1.7883^{***}(0.2981)$	$2.5747^{***}(0.1993)$	$1.9811^{***}(0.2271)$	$1.4209^{***}(0.2724)$	$1.8000^{***}(0.2926)$
δ_2	$-0.5320^{***}(0.0684)$	$-0.5324^{***}(0.0496)$	$-0.5245^{***}(0.0567)$	$-0.3446^{***}(0.0591)$	$-0.5156^{***}(0.0586)$
A. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,\mathrm{L}}$	0.9130	0.9465	0.9115	0.8962	0.9538
$C_{3,\mathrm{L}}$	0.7905	0.8587	0.7794	0.7623	0.8631
$C_{4,\mathrm{L}}$	−0.1268	−0.0702	−0.1245	−0.1430	−0.0778

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	$-1.5365^{**}(0.6290)$	$4.1311^{***}(1.2528)$	$3.5328^{***}(0.5098)$	$0.9558(1.9329)$	$1.7776(1.7020)$
κ_{L}	$1.4408^{***}(0.4798)$	$4.0614^{*}(2.0830)$	$-0.7163^{**}(0.2938)$	$1.5059^{**}(0.6224)$	$-0.6258(0.8028)$
κ_{S}	$1.7311^{***}(0.4741)$	$-1.3268(0.8657)$	$0.8967^{*}(0.4926)$	$-1.0424(1.7893)$	$1.4796(1.1212)$
ω	$0.9208^{***}(0.1841)$	$2.3338^{***}(0.2652)$	$1.6721^{***}(0.2025)$	$1.4452^{***}(0.1278)$	$1.2361^{***}(0.2431)$
β_{L}	$0.9720^{***}(0.0128)$	$0.9442^{***}(0.0122)$	$0.9279^{***}(0.0152)$	$0.9142^{***}(0.0244)$	$0.9577^{***}(0.0124)$
β_{S}	$0.9273^{***}(0.0127)$	$0.9411^{***}(0.0128)$	$0.9224^{***}(0.0162)$	$0.7257^{***}(0.1333)$	$0.9531^{***}(0.0101)$
α_{L}	$-0.0187^{**}(0.0091)$	$0.1163^{***}(0.0403)$	$-0.1139^{***}(0.0357)$	$0.0650^{***}(0.0229)$	$-0.0304(0.0264)$
α_{S}	$0.0777^{***}(0.0164)$	$-0.0497(0.0384)$	$0.1922^{***}(0.0417)$	$-0.0373(0.0259)$	$0.0901^{***}(0.0276)$
α^*_{L}	$0.0050(0.0051)$	$-0.0385^{*}(0.0201)$	$0.0157(0.0171)$	$0.0185(0.0143)$	$0.0052(0.0067)$
α^*_{S}	$0.0242^{*}(0.0127)$	$0.0203(0.0143)$	$-0.0106(0.0294)$	$0.0332^{*}(0.0171)$	$0.0122(0.0146)$
δ_1	$1.8442^{***}(0.2619)$	$2.6124^{***}(0.1812)$	$2.0011^{***}(0.2011)$	$1.3080^{***}(0.2539)$	$1.9045^{***}(0.2571)$
δ_2	$-0.5791^{***}(0.0651)$	$-0.5562^{***}(0.0495)$	$-0.5579^{***}(0.0516)$	$-0.2999^{***}(0.0577)$	$-0.5718^{***}(0.0565)$
B. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,\mathrm{L}}$	0.9720	0.9442	0.9279	0.9142	0.9577
$C_{2,\mathrm{S}}$	0.9273	0.9411	0.9224	0.7257	0.9531
$C_{3,\mathrm{L}}$	0.9583	0.8317	0.9358	0.7852	0.9382
$C_{3,\mathrm{S}}$	0.8083	0.9125	0.7381	0.5513	0.8490
$C_{3,\mathrm{L,S}}$	0.8703	0.8754	0.8258	0.6477	0.8864
$C_{4,\mathrm{L}}$	−0.0230	−0.0814	−0.0394	−0.1268	−0.0337
$C_{4,\mathrm{S}}$	−0.1164	−0.0518	−0.1526	−0.3085	−0.0873

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table A4. Parameter estimates and ML conditions; QAR-NIG-EGARCH-M-1 (A) and QAR-NIG-EGARCH-M-2 (B)

A. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	0.0070(0.1480)	−0.0525(0.1985)	0.0739(0.2006)	−0.9177*(0.5232)	0.1766(0.1489)
ϕ	−0.2508(0.2937)	0.0711(0.2887)	0.0720(0.3616)	0.8918***(0.0840)	−0.8486***(0.0983)
ψ	−0.0225**(0.0092)	−0.0123**(0.0048)	−0.0154**(0.0073)	0.0077(0.0052)	−0.0095*(0.0053)
κ_{L}	1.5822***(0.3924)	5.7959***(1.4458)	3.1738***(0.8705)	2.5062***(0.6255)	1.8903***(0.6377)
ω	1.0322***(0.1736)	2.2971***(0.2469)	1.7326***(0.2116)	1.5264***(0.1363)	1.2852***(0.2556)
β_{L}	0.9199***(0.0157)	0.9520***(0.0117)	0.9211***(0.0187)	0.8840***(0.0274)	0.9541***(0.0114)
α_{L}	0.0706***(0.0121)	0.0739***(0.0099)	0.0907***(0.0127)	0.0523***(0.0114)	0.0718***(0.0095)
α^*_{L}	0.0237**(0.0112)	−0.0215**(0.0091)	−0.0058(0.0135)	0.0284***(0.0093)	0.0124(0.0111)
δ_1	1.5488***(0.2917)	2.3557***(0.1948)	1.8166***(0.2349)	1.4860***(0.2829)	1.6732***(0.3007)
δ_2	−0.4981***(0.0673)	−0.5362***(0.0480)	−0.5196***(0.0574)	−0.3586***(0.0602)	−0.5110***(0.0601)
A. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,*}$	0.2508	0.0711	0.0720	0.8918	0.8486
$C_{2,\mathrm{L}}$	0.9199	0.9520	0.9211	0.8840	0.9541
$C_{3,*}$	0.0287	0.0288	0.0224	0.7405	0.6531
$C_{3,\mathrm{L}}$	0.7953	0.8659	0.7908	0.7364	0.8597
$C_{3,*,\mathrm{L}}$	0.1437	0.1555	0.1306	0.7460	0.7451
$C_{4,*}$	−1.8404	−1.8263	−1.9573	−0.1505	−0.2136
$C_{4,\mathrm{L}}$	−0.1242	−0.0659	−0.1165	−0.1558	−0.0806

B. Parameters:	DJIA	FCHI	GDAXI	N225	AEX
c	1.0156(1.4707)	−3.6009***(1.2992)	0.3488(0.9635)	3.0275(2.5146)	1.1085(0.7845)
ϕ	0.5727***(0.1263)	0.0939(0.2843)	0.4463***(0.1346)	0.7996***(0.1320)	0.4091**(0.1605)
ψ	−0.0432***(0.0156)	−0.0157**(0.0066)	−0.0402***(0.0132)	−0.0193(0.0124)	−0.0365***(0.0140)
κ_{L}	1.2973***(0.3397)	5.8680***(1.5461)	2.4472***(0.6605)	1.0934*(0.5936)	1.2715***(0.4343)
κ_{S}	−0.9393(1.4526)	3.4970***(1.3121)	−0.3281(0.9030)	−2.6795(2.4803)	−0.8218(0.7243)
ω	0.9447***(0.1593)	2.2348***(0.2673)	1.5300***(0.2117)	1.4283***(0.1293)	1.0632***(0.2352)
β_{L}	0.9440***(0.0154)	0.9685***(0.0095)	0.9670***(0.0121)	0.9330***(0.0226)	0.9772***(0.0082)
β_{S}	0.5896***(0.1812)	0.5388**(0.2347)	0.5165***(0.1299)	0.7571***(0.1056)	0.4767***(0.1206)
α_{L}	0.0701***(0.0116)	0.0599***(0.0106)	0.0631***(0.0108)	0.0601***(0.0194)	0.0596***(0.0089)
α_{S}	−0.0092(0.0149)	0.0130(0.0212)	−0.0042(0.0174)	−0.0321(0.0215)	−0.0395**(0.0167)
α^*_{L}	0.0124(0.0116)	−0.0190**(0.0074)	−0.0059(0.0080)	0.0121(0.0142)	0.0089(0.0083)
α^*_{S}	0.0420**(0.0179)	0.0258(0.0196)	0.0743***(0.0164)	0.0426**(0.0176)	0.0862***(0.0162)
δ_1	1.2156***(0.2787)	2.1584***(0.1929)	1.3875***(0.2318)	1.2348***(0.2432)	1.3827***(0.2655)
δ_2	−0.4154***(0.0619)	−0.5602***(0.0498)	−0.4249***(0.0530)	−0.2676***(0.0560)	−0.3989***(0.0574)
B. Conditions:	DJIA	FCHI	GDAXI	N225	AEX
$C_{2,*}$	0.5727	0.0939	0.4463	0.7996	0.4091
$C_{2,\mathrm{L}}$	0.9440	0.9685	0.9670	0.9330	0.9772
$C_{2,\mathrm{S}}$	0.5896	0.5388	0.5165	0.7571	0.4767
$C_{3,*}$	0.5006	0.0384	0.3504	0.7492	0.2949
$C_{3,\mathrm{L}}$	0.8298	0.9014	0.8843	0.8204	0.9065
$C_{3,\mathrm{S}}$	0.3536	0.2860	0.2706	0.5968	0.2465
$C_{3,*,\mathrm{L}}$	0.6410	0.1831	0.5552	0.7819	0.5138
$C_{3,*,\mathrm{S}}$	0.4108	0.1012	0.2941	0.6604	0.2547
$C_{3,\mathrm{L},\mathrm{S}}$	0.5282	0.5032	0.4685	0.6893	0.4465
$C_{4,*}$	−0.3551	−1.6820	−0.5380	−0.1454	−0.6239
$C_{4,\mathrm{L}}$	−0.0985	−0.0462	−0.0604	−0.1027	−0.0524
$C_{4,\mathrm{S}}$	−0.5420	−0.6411	−0.7019	−0.2705	−0.7584

Notes: Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.