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Robust score-driven inference of stochastic seasonality of the Russian rouble for different currency exchange rate regimes from 1999 to 2020

Szabolcs Blazsek* and Adrian Licht

School of Business, Universidad Francisco Marroquín, Ciudad de Guatemala, Guatemala

Abstract: The current account of the Russian Federation involves an annual seasonality component, which creates an annual seasonality for Russian rouble exchange rates. The practical use of a score-driven state-space model is suggested, in order to measure in a robust way the seasonality of the Russian rouble for the period of 1999 to 2020. The motivation for the practical use of score-driven models is that those models implement an optimal filtering mechanism, according to the Kullback-Leibler divergence in favour of the true data generating process. It is shown that the score-driven model is robust to structural changes that are in relation to different currency exchange regimes implemented by the Bank of Russia. The amplitude of the stochastic seasonality component is significant, which is approximately in the range of +/- 2% for the period of 1999 to 2020. The determinants of exchange rate seasonality are presented, by referring to the seasonality of (i) exports, imports, and primary income of Russia; (ii) crude oil production, natural gas exports and production, and coal exports, which are in relation to the most important export products of Russia; (iii) seasonal interventions of the Bank of Russia on the foreign exchange market for the sample period.

Keywords: Russian rouble; currency exchange rate regimes; stochastic local level; score-driven stochastic seasonality; dynamic conditional score; generalized autoregressive score

JEL classification codes: C22, C52, F31

*Corresponding author. Postal address: Escuela de Negocios, Universidad Francisco Marroquín, Calle Manuel F. Ayau, Zona 10, Ciudad de Guatemala 01010, Guatemala. E-mails: sblazsek@ufm.edu (S. Blazsek), adrian.licht@gmail.com (A. Licht)

1. Introduction

The importance of the annual seasonality component of the current account of the balance of payments of the Russian Federation has been expressed by Elvira Nabiulina, Governor of the Bank of Russia in the recent press event statement of the Bank of Russia (2017). Seasonal variations in the current account influence the Russian rouble (RUB) exchange rate movements on the foreign exchange market. In this paper, the practical use of a new score-driven state-space model for the Russian rouble is suggested, and robust measurements of the annual seasonality components of the RUB to US dollar (USD) and RUB to Euro (EUR) currency exchange rates for the period of 1999 to 2020 are analysed. The motivation for developing the econometric method is that robust estimates of the seasonality component of Russian rouble exchange rates are not reported in the body of academic literature, to the best of our knowledge. The robust score-driven stochastic seasonality method of this paper can be used in practice, for example, to deseasonalize the Russian rouble exchange rates for economic analyses, or to study the dynamic amplitude of seasonality of the Russian rouble exchange rates for financing, investment and policy decisions.

Score-driven state-space models are introduced in the works of Creal et al. (2013) and Harvey (2013). The latent dynamic parameters (i.e. filters) of those models are updated by the conditional score of the log conditional density of the dependent variables (hereinafter, score function). Score-driven models implement an optimal filtering mechanism, according to the Kullback–Leibler divergence with respect to the true data generating process. In the work of Blasques et al. (2015), it is shown that a score-driven update of the time series model reduces the Kullback–Leibler divergence in expectation and at every step, and they also show that only score-driven updates can have this property. An alternative to the score-driven model of this paper is the frequently used multiplicative seasonal ARIMA (autoregressive integrated moving average) model (e.g. Brockwell and Davis 1996), for which filter-updating is not optimal with respect to the Kullback–Leibler divergence. By using the results of Blasques et al. (2015), the benefits of statistical inference of score-driven state-space models are incorporated into the literature on the Russian rouble.

In Section 2, the score-driven state-space model is presented, which includes: (i) a score-driven $I(1)$ stochastic local level filter that is able to capture structural changes in currency exchange rate levels; (ii) a score-driven annual stochastic seasonality filter that measures time-varying amplitude of currency exchange rates; (iii) a score-driven EGARCH filter that identifies structural changes in the volatility of currency exchange rates. An advantage of the use of the score-driven state-space model is that it can be estimated in a robust way for structural changes in the Russian rouble exchange rates for the period of 1999 to 2020, which are due to the different exchange rate policies of the Bank of Russia for the sample period.

In Section 3, the statistical inference of the score-driven state-space model is presented, and conditions of asymptotic properties of the maximum likelihood (ML) estimator are reported. The true data generating process is approximated by using the next alternatives: the Student's t-distribution, skewed generalized t-distribution (Skew-Gen-t) (McDonald and Michelfelder 2017), exponential generalized beta distribution of the second kind (EGB2) (Prentice 1975), normal-inverse Gaussian (NIG) distribution (Barndorff-Nielsen and Halgreen 1977), and the Meixner (MXN) distribution (Schoutens 2002). These distributions are flexible due to the shape parameters, which may control tail-thickness, peakedness and asymmetry. The use of the MXN distribution for score-driven seasonality is new in the literature. In Section 3, the score functions that update the filters of the score-driven state-space model and their asymptotic properties are presented. Moreover, in Section 3, the one-step ahead forecasting formulae of (i) the currency exchange rates, and (ii) the volatilities of the currency exchange rates are also presented.

In Section 4, the dataset is described and the empirical results are presented. The similar results for alternative probability distributions show that the statistical inference procedures are robust for the Russian rouble. The ex-post estimates of the ML conditions do not indicate failures for the asymptotic properties of ML. The parameter estimates support the specifications of the filters in all score-driven state-space specifications. In Section 4, it is presented that the score-driven state-space model identifies the structural changes in the RUB to USD and RUB to

EUR exchange rates. The structural changes are in relation to different exchange rate regimes of the Bank of Russia, which are reviewed for the pre-sample period of 1991 to 1998 and for the sample period of 1999 to 2020. In Section 4, it is shown that the amplitude of the annual stochastic seasonality component is significant, which is approximately in the range of $\pm 2\%$ for the period of 1999 to 2020. The determinants of currency exchange rate seasonality are presented by referring to the annual seasonality of (i) the exports of goods and services, imports of goods and services, and the receivable and payable components of primary income from the current account of the Russian Federation; (ii) crude oil production, natural gas export and production, and coal exports, which are in relation to the most important export products of the Russian Federation. The last section of this paper (Section 5) concludes.

2. Score-driven state-space model for currency exchange rates

The RUB to USD and RUB to EUR currency exchange rates, both denoted by p_t, are decomposed into three time series components, by using the following model:

$$p_t = \mu_t + s_t + v_t = \mu_t + s_t + \exp(\lambda_t)\epsilon_t \tag{1}$$

for $t = 1, \ldots, T$ weekly observations in the sample, and v_t is defined by the second equality.

Firstly, the local level μ_t uses the following $I(1)$ specification (Harvey 2013):

$$\mu_t = \mu_{t-1} + \kappa u_{\mu,t-1} \tag{2}$$

where $u_{\mu,t}$ is the score function with respect to μ_t, which is defined according to the probability distribution of the error term ϵ_t, and its properties are presented in Section 3. Filter μ_t is initialized by using the first observation p_1. The local level component μ_t provides a flexible time series model, and it is robust to structural changes in the currency exchange rate time series due to the currency exchange policies of Bank of Russia (2013).

Secondly, the annual stochastic seasonality component s_t is given by:

$$s_t = D_t' \rho_t \tag{3}$$

where D_t is a 12×1 vector of dummy variables $D_t' = (D_{\text{Jan},t}, \ldots, D_{\text{Dec},t})$. For example, $D_{\text{Jan},t} = 1$ if week t is in January, zero otherwise. Moreover, ρ_t is a 12×1 vector of time-varying parameters, which allows a stochastic amplitude of seasonality, and is updated as follows:

$$\rho_t = \rho_{t-1} + \gamma_t u_{\mu,t-1} \tag{4}$$

The same score function updates both μ_t and s_t, as in the works of Harvey (2013), Harvey and Luati (2014), Blazsek and Hernandez (2018), Ayala and Blazsek (2019a, 2019b), and Blazsek and Licht (2020). Filter ρ_t includes the time-varying 12×1 parameter vector γ_t, for which each element is $\gamma_{j,t} = \gamma_j$ if $D_{j,t} = 1$, otherwise $\gamma_{j,t} = -\gamma_j/(12-1)$. This parametrization ensures that $\sum_{j=\text{Jan}}^{\text{Dec}} \gamma_{j,t} = 0$. Therefore, s_t is centred at zero. For this parametrization of γ_t, parameters γ_j for $j \in \{\text{Jan}, \ldots, \text{Dec}\}$ are jointly estimated with the rest of the parameters. Filter ρ_t is initialized by using a non-linear regression model, for which the dependent variable p_t is regressed on a constant, a deterministic linear time trend, and twelve dummies that indicate the months of the year. This regression model is estimated by using the non-linear least squares (NLS) method, because the sum of the parameters of the dummy variables is restricted to zero. The NLS estimation is performed by using data for the first year of the full sample period. This initialization procedure is suggested in the works of Harvey (2013) and Harvey and Luati (2014). The stochastic annual seasonality component s_t is significant, because of the seasonal exports, imports and primary income of Russia (e.g. Mironov 2015; Bozhechkova et al. 2017), and because of the seasonal interventions of the Bank of Russia on the currency exchange rate market (Bank of Russia 2013).

Thirdly, the conditional volatility of the error term v_t is updated by using the following score-driven EGARCH model (Harvey and Chakravarty 2008; Harvey 2013):

$$\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} \tag{5}$$

where $u_{\lambda,t}$ is the score function with respect to λ_t, which is defined according to the probability distribution of the error term ϵ_t, and its properties are defined in Section 3. Filter λ_t is initialized by using parameter λ_0. In the relevant literature, there are works which suggest the use of dynamic volatility models for Russian rouble exchange rate volatility (e.g. Kutu and Ngalawa 2016; Borotshkyn 2017; Zerihun et al. 2020). In the present paper, score-driven alternatives are used, which involve optimal updating mechanisms according to the Kullback–Leiber divergence (Blasques et al. 2015). By using a dynamic volatility model for p_t, the dispersion of the currency exchange rate around its location is robust to different currency exchange regimes of the Bank of Russia (Bank of Russia 2013).

As alternatives to the score-driven state-space specification of this section, the following models are also estimated: (i) two-component μ_t specification for local level (Harvey 2013); (ii) two-component λ_t specification for EGARCH (Harvey 2013; Harvey and Lange 2018); (iii) EGARCH with leverage effects (Harvey and Chakravarty 2008; Harvey 2013); (iv) annual stochastic seasonality for λ_t, in addition to the annual stochastic seasonality for the location component s_t (Harvey 2013; Ayala and Blazsek 2019a). The statistical performances of those alternatives are inferior to the statistical performance of the score-driven state-space model of this section.

3. Statistical inference

Score-driven models are estimated by using the ML method (e.g. Harvey 2013). The likelihood function is maximized numerically, and the ML estimates of parameters are given by:

$$\hat{\Theta}_{\mathrm{ML}} = \arg\max_{\Theta} \sum_{t=1}^{T} \ln f(p_t|p_1,\ldots,p_{t-1}) \tag{6}$$

where Θ is the vector of parameters in the conditional density function $f(p_t|p_1,\ldots,p_{t-1})$, for which the standard errors of parameters are estimated by using the Huber–White robust sandwich estimator (e.g. Davidson and MacKinnon 2004).

For the consistency and asymptotic normality of the ML estimates of parameters, it is required that $C_1 = |\beta| < 1$ and $C_2 = \beta^2 + 2\beta\alpha E(\partial u_{\lambda,t}/\partial\lambda_t) + \alpha^2 E[(\partial u_{\lambda,t}/\partial\lambda_t)^2] < 1$ (Harvey 2013). $C_1 < 1$ indicates that filter λ_t is covariance stationarity. $C_2 < 1$ indicates that those elements of the information matrix that correspond to the parameters of λ_t are finite. In addition, for the invertibility of filter λ_t, the following condition is required: $C_3 = E\{\ln|\partial\lambda_t/\partial\lambda_{t-1}|\} = E\{\ln|\beta + \alpha\partial u_{\lambda,t}/\partial\lambda_t|\} < 0$ (Blasques et al. 2018). The sample estimates of C_1, C_2 and C_3 are analysed to study possible failures of ML, for which the partial derivatives are estimated numerically and the expectations are estimated by using sample averages. For the parameters of the local level component μ_t, the asymptotic properties of ML hold because the parameter of μ_{t-1} is set to one, and it is also required that $\kappa \neq 0$ (Harvey 2013). For the parameters of the stochastic seasonality component s_t, the asymptotic properties of ML hold, since the parameter of ρ_{t-1} is set to one, and it is also required that at least one of the seasonality parameters γ_j for $j \in \{\text{Jan}, \dots, \text{Dec}\}$ is not zero. Components μ_t and s_t are effectively separated, because $E(s_t) = 0$ due to the parametrization of s_t.

The score functions, which update filters μ_t, ρ_t and λ_t, are defined according to the conditional density function of the ML estimator. Score function $u_{\mu,t}$ is the scaled partial derivative of the log conditional density of p_t with respect to μ_t (Harvey 2013). The use of the same score function in filters μ_t and s_t is motivated by the fact that the conditional scores with respect to μ_t and s_t are identical. Score function $u_{\lambda,t}$ is the partial derivative of the log conditional density of p_t with respect to λ_t (Harvey 2013). By using the results of Harvey (2013), $E(u_{\mu,t}) = E(u_{\mu,t}|y_1, \dots, y_{t-1}) = E(u_{\mu,t}|u_{\mu,1}, \dots, u_{\mu,t-1}) = 0$ and $E(u_{\lambda,t}) = E(u_{\lambda,t}|y_1, \dots, y_{t-1}) = E(u_{\lambda,t}|u_{\lambda,1}, \dots, u_{\lambda,t-1}) = 0$. Thus, both score functions are martingale difference sequences. The conditional density of p_t is determined by the probability distribution of ϵ_t, for which the Student's t, Skew-Gen-t, EGB2, NIG and MXN distributions are considered. These alternatives provide robustness for the ML estimation results.

In the reminder of this section, for each probability distribution, technical details are presented for (i) the log conditional density of p_t, (ii) the score functions $u_{\mu,t}$ and $u_{\lambda,t}$, and (iii) the one-step ahead forecasting formulae for p_t and for the volatility of p_t.

3.1. Student's t-distribution

For the Student's t-distribution $\epsilon_t \sim t[0, 1, \exp(\nu) + 2]$, where $\nu \in \mathbb{R}$ is a shape parameter. The parameter specification $[\exp(\nu) + 2]$ ensures that the first two moments of p_t exist.

(i) The log conditional density of p_t is

$$\ln f(p_t|p_1, \ldots, p_{t-1}) = \ln \Gamma \left[\frac{\exp(\nu) + 3}{2} \right] - \ln \Gamma \left[\frac{\exp(\nu) + 2}{2} \right] \tag{7}$$

$$- \frac{\ln(\pi) + \ln[\exp(\nu) + 2]}{2} - \lambda_t - \frac{\exp(\nu) + 3}{2} \ln \left\{ 1 + \frac{\epsilon_t^2}{\exp(\nu) + 2} \right\}$$

where $\Gamma(\cdot)$ is the gamma function.

(ii) The score function with respect to μ_t is given by (Harvey 2013):

$$\frac{\partial \ln f(p_t|p_1, \ldots, p_{t-1})}{\partial \mu_t} = \frac{[\exp(\nu) + 2] \exp(\lambda_t) \epsilon_t}{\epsilon_t^2 + \exp(\nu) + 2} \times \frac{\exp(\nu) + 3}{[\exp(\nu) + 2] \exp(2\lambda_t)} = \tag{8}$$

$$= u_{\mu,t} \times \frac{\exp(\nu) + 3}{[\exp(\nu) + 2] \exp(2\lambda_t)}$$

where the scaled score function $u_{\mu,t}$ is defined according to the last equality. The $u_{\mu,t}$ term trims outliers, because $u_{\mu,t} \to_p 0$ when $|\epsilon_t| \to \infty$ (Figures 1(a) and 2(a)). The discounting that is undertaken by $u_{\mu,t}$ is identical for the positive and negative sides of the distribution. The score function with respect to λ_t is given by (Harvey and Chakravarty 2008; Harvey 2013):

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1, \ldots, p_{t-1})}{\partial \lambda_t} = \frac{[\exp(\nu) + 3] \epsilon_t^2}{\exp(\nu) + 2 + \epsilon_t^2} - 1 \tag{9}$$

The updating term $u_{\lambda,t}$ Winsorizes extreme observations, because $u_{\lambda,t} \to_p c$ ($c > 0$ is a real number) when $|\epsilon_t| \to \infty$ (Figures 1(b) and 2(b)). The discounting that is undertaken by $u_{\lambda,t}$ is identical for the positive and negative sides of the probability distribution.

(iii) The conditional mean and conditional standard deviation of p_t, respectively, are:

$$E(p_t|p_1,\ldots,p_{t-1}) = \mu_t + s_t \tag{10}$$

$$\sigma_t = \sigma(p_t|p_1,\ldots,p_{t-1}) = \exp(\lambda_t)\left[\frac{\exp(\nu)+2}{\exp(\nu)}\right]^{1/2} \tag{11}$$

3.2. Skewed generalized t-distribution (Skew-Gen-t distribution)

For the Skew-Gen-t distribution $\epsilon_t \sim$ Skew-Gen-$t[0, 1, \tanh(\tau), \exp(\nu)+2, \exp(\eta)]$, where $\tanh(\cdot)$ is the hyperbolic tangent function, and $\tau \in \mathbb{R}$, $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ are shape parameters. For $\tanh(\tau) = 0$ and $\exp(\eta) = 2$, the Skew-Gen-t distribution is the Student's t-distribution. The parameter specification $[\exp(\nu) + 2]$ ensures that the first two moments of p_t exist.

(i) The log-density of p_t is (Ayala et al., 2019):

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = \eta - \lambda_t - \ln(2) - \frac{\ln[\exp(\nu)+2]}{\exp(\eta)} - \ln\Gamma\left[\frac{\exp(\nu)+2}{\exp(\eta)}\right] \tag{12}$$

$$-\ln\Gamma[\exp(-\eta)] + \ln\Gamma\left[\frac{\exp(\nu)+3}{\exp(\eta)}\right]$$

$$-\frac{\exp(\nu)+3}{\exp(\eta)}\ln\left\{1+\frac{|\epsilon_t|^{\exp(\eta)}}{[1+\tanh(\tau)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta)}\times[\exp(\nu)+2]}\right\}$$

where $\mathrm{sgn}(\cdot)$ is the signum function.

(ii) The score function with respect to μ_t is given by (Ayala et al., 2019):

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = \tag{13}$$

$$= \frac{[\exp(\nu)+2]\exp(\lambda_t)\epsilon_t|\epsilon_t|^{\exp(\eta)-2}}{|\epsilon_t|^{\exp(\eta)}+[1+\tanh(\tau)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta)}[\exp(\nu)+2]} \times \frac{\exp(\nu)+3}{[\exp(\nu)+2]\exp(2\lambda_t)} =$$

$$= u_{\mu,t} \times \frac{\exp(\nu) + 3}{[\exp(\nu) + 2] \exp(2\lambda_t)}$$

where the scaled score function $u_{\mu,t}$ is defined according to the second equality. The $u_{\mu,t}$ term trims extreme observations, because $u_{\mu,t} \rightarrow_p 0$ when $|\epsilon_t| \rightarrow \infty$ (Figures 1(c) and 2(c)). The discounting that is undertaken by $u_{\mu,t}$ is not identical for the positive and negative sides of the probability distribution. The score function with respect to λ_t is given by (Ayala et al., 2019):

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1, \ldots, p_{t-1})}{\partial \lambda_t} = \frac{|\epsilon_t|^{\exp(\eta)}[\exp(\nu) + 3]}{|\epsilon_t|^{\exp(\eta)} + [1 + \tanh(\tau)\text{sgn}(\epsilon_t)]^{\exp(\eta)}[\exp(\nu) + 2]} - 1 \tag{14}$$

The updating term $u_{\lambda,t}$ Winsorizes outliers, because $u_{\lambda,t} \rightarrow_p c_1$ when $\epsilon_t \rightarrow -\infty$ and $u_{\lambda,t} \rightarrow_p c_2$ when $\epsilon_t \rightarrow +\infty$ ($c_1 > 0$ and $c_2 > 0$ are real numbers) (Figures 1(d) and 2(d)). The Winsorizing that is undertaken by $u_{\lambda,t}$ is not identical for the positive and negative sides of the distribution.

(iii) The conditional mean and conditional standard deviation of p_t, respectively, are (Ayala et al., 2019):

$$E(p_t|p_1, \ldots, p_{t-1}) = \mu_t + s_t + 2\exp(\lambda_t)\tanh(\tau)[\exp(\nu) + 2]^{\exp(-\eta)} \times \frac{B\left\{\frac{2}{\exp(\eta)}, \frac{\exp(\nu)+1}{\exp(\eta)}\right\}}{B\left\{\frac{1}{\exp(\eta)}, \frac{\exp(\nu)+2}{\exp(\eta)}\right\}} \tag{15}$$

$$\sigma_t = \sigma(p_t|p_1, \ldots, p_{t-1}) = \exp(\lambda_t)[\exp(\nu) + 2]^{\exp(-\eta)} \times \tag{16}$$

$$\times \left\{ \frac{[3\tanh^2(\tau) + 1]B\left[\frac{3}{\exp(\eta)}, \frac{\exp(\nu)}{\exp(\eta)}\right]}{B\left[\frac{1}{\exp(\eta)}, \frac{\exp(\nu)+2}{\exp(\eta)}\right]} - \frac{4\tanh^2(\tau)B^2\left[\frac{2}{\exp(\eta)}, \frac{\exp(\nu)+1}{\exp(\eta)}\right]}{B^2\left[\frac{1}{\exp(\eta)}, \frac{\exp(\nu)+2}{\exp(\eta)}\right]} \right\}^{1/2}$$

where $B(\cdot, \cdot)$ is the beta function.

3.3. Exponential generalized beta distribution of the second kind (EGB2 distribution)

For the EGB2 distribution $\epsilon_t \sim \text{EGB2}[0, 1, \exp(\nu), \exp(\eta)]$, where $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ are shape parameters. For the EGB2 distribution all moments exist.

(i) The log conditional density is (Caivano and Harvey 2014):

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = \exp(\nu)\epsilon_t - \lambda_t - \ln\Gamma[\exp(\nu)] \tag{17}$$

$$-\ln\Gamma[\exp(\eta)] + \ln\Gamma[\exp(\nu)+\exp(\eta)] - [\exp(\nu)+\exp(\eta)]\ln[1+\exp(\epsilon_t)]$$

(ii) The score function with respect to μ_t is given by (Caivano and Harvey 2014):

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = \exp(\lambda_t)[\exp(\nu)+\exp(\eta)]\frac{\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\lambda_t)\exp(\nu) \tag{18}$$

$$u_{\mu,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} \times \{\Psi^{(1)}[\exp(\nu)] + \Psi^{(1)}[\exp(\eta)]\}\exp(2\lambda_t) \tag{19}$$

The updating term $u_{\mu,t}$ Winsorizes outliers, because $u_{\mu,t} \to_p c_1$ when $\epsilon_t \to -\infty$ and $u_{\mu,t} \to_p c_2$ when $\epsilon_t \to +\infty$ ($c_1 > 0$ and $c_2 > 0$ are real numbers) (Figures 1(e) and 2(e)). The Winsorizing that is undertaken by $u_{\mu,t}$ is not identical for the positive and negative sides of the probability distribution. The score function with respect to λ_t is given by (Caivano and Harvey 2014):

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \lambda_t} = u_{\lambda,t} = [\exp(\nu)+\exp(\eta)]\frac{\epsilon_t\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\nu)\epsilon_t - 1 \tag{20}$$

The updating term $u_{\lambda,t}$ performs a linearly increasing and asymmetric transformation of ϵ_t, as $|\epsilon_t| \to \infty$ (Figures 1(f) and 2(f)).

(iii) The conditional mean and conditional standard deviation of p_t, respectively, are (Caivano and Harvey 2014):

$$E(p_t|p_1,\ldots,p_{t-1}) = \mu_t + s_t + \exp(\lambda_t)\left\{\Psi^{(0)}[\exp(\nu)] - \Psi^{(0)}[\exp(\eta)]\right\} \tag{21}$$

$$\sigma_t = \sigma(p_t|p_1,\ldots,p_{t-1}) = \exp(\lambda_t)\left\{\Psi^{(1)}[\exp(\nu)] + \Psi^{(1)}[\exp(\eta)]\right\}^{1/2} \tag{22}$$

3.4. Normal-inverse Gaussian (NIG) distribution

For the NIG distribution $\epsilon_t \sim \text{NIG}[0, 1, \exp(\nu), \exp(\nu)\tanh(\eta)]$, where $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ are shape parameters. For the NIG distribution all moments exist.

(i) The log conditional density is (Blazsek et al. 2018):

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = \nu - \lambda_t - \ln(\pi) + \exp(\nu)[1-\tanh^2(\eta)]^{1/2} \tag{23}$$

$$+\exp(\nu)\tanh(\eta)\epsilon_t + \ln K^{(1)}\left\{\exp(\nu)\sqrt{1+\left[\frac{y_t-\mu_t}{\exp(\lambda_t)}\right]^2}\right\} - \frac{1}{2}\ln\left\{1+\epsilon_t^2\right\}$$

where $K^{(j)}(\cdot)$ is the modified Bessel function of the second kind of order j.

(ii) The score function with respect to μ_t is given by (Blazsek et al. 2018):

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = -\exp(\nu-\lambda_t)\tanh(\eta) + \frac{\epsilon_t}{\exp(\lambda_t)(1+\epsilon_t^2)} \tag{24}$$

$$+\frac{\exp(\nu-\lambda_t)\epsilon_t}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}$$

$$u_{\mu,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} \times \exp(2\lambda_t) \tag{25}$$

The updating term $u_{\mu,t}$ Winsorizes outliers, because $u_{\mu,t} \to_p c_1$ when $\epsilon_t \to -\infty$ and $u_{\mu,t} \to_p c_2$ when $\epsilon_t \to +\infty$ ($c_1 > 0$ and $c_2 > 0$ are real numbers) (Figures 1(g) and 2(g)). The Winsorizing that is undertaken by $u_{\mu,t}$ is not identical for the positive and negative sides of the probability distribution. The score function with respect to λ_t is given by (Blazsek et al. 2018):

$$u_{\lambda,t} = -1 - \exp(\nu)\tanh(\eta)\epsilon_t + \frac{\epsilon_t^2}{1+\epsilon_t^2} \tag{26}$$

$$+\frac{\exp(\nu)\epsilon_t^2}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}$$

The updating term $u_{\lambda,t}$ performs a linearly increasing and asymmetric transformation of ϵ_t, as $|\epsilon_t| \to \infty$ (Figures 1(h) and 2(h)).

(iii) The conditional mean and conditional standard deviation of p_t, respectively, are (Blazsek et al. 2018):

$$E(p_t|p_1,\ldots,p_{t-1}) = \mu_t + s_t + \frac{\exp(\lambda_t)\tanh(\eta)}{[1-\tanh^2(\eta)]^{1/2}} \tag{27}$$

$$\sigma_t = \sigma(p_t|p_1,\ldots,p_{t-1}) = \left\{\frac{\exp(2\lambda_t - \nu)}{[1-\tanh^2(\eta)]^{3/2}}\right\}^{1/2} \tag{28}$$

3.5. Meixner (MXN) distribution

For the MXN distribution $\epsilon_t \sim \text{MXN}[0, 1, \pi\tanh(\nu), \exp(\eta)]$, where $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ are shape parameters. For the MXN distribution all moments exist.

(i) The log conditional density is (Schoutens 2002):

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = -\lambda_t + 2\exp(\eta)\ln\{2\cos[\pi\tanh(\nu)/2]\} - \ln(2\pi) \tag{29}$$

$$-\ln\Gamma\{2\exp(\eta)\} + \pi\tanh(\nu)\epsilon_t + 2\ln|\Gamma[\exp(\eta) + i\epsilon_t]|$$

where $\cos(\cdot)$ is the cosine function, $\tanh(\cdot)$ is the hyperbolic tangent function, $\Gamma(\cdot)$ is the gamma function, and i is the imaginary unit. We define $g(\lambda_t) = \Gamma[\exp(\eta) + i(y_t - \mu_t)\exp(-\lambda_t)]$, for which $\partial \ln |g(\lambda_t)|/\partial \lambda_t = \mathrm{Re}[g'(\lambda_t)/g(\lambda_t)]$, where $\lambda_t \in \mathbb{R}$ and $\mathrm{Re}(\cdot)$ is the real part of a complex number. Since $\Gamma'(\cdot) = \Gamma(\cdot)\Psi^{(0)}(\cdot)$, where $\Psi^{(j)}(\cdot)$ is the polygamma function of order j.

(ii) The score function with respect to μ_t is given by:

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = -\frac{\pi \tanh(\nu)}{\exp(\lambda_t)} + 2\mathrm{Re}\left\{-\frac{i}{\exp(\lambda_t)}\Psi^{(0)}[\exp(\eta) + i\epsilon_t]\right\} \tag{30}$$

$$u_{\mu,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} \times \exp(2\lambda_t) \tag{31}$$

The updating term $u_{\mu,t}$ Winsorizes outliers, because $u_{\mu,t} \to_p c_1$ when $\epsilon_t \to -\infty$ and $u_{\mu,t} \to_p c_2$ when $\epsilon_t \to +\infty$ ($c_1 > 0$ and $c_2 > 0$) (Figures 1(i) and 2(i)). The Winsorizing of $u_{\mu,t}$ is not identical for the positive and negative sides of the probability distribution. The use of the MXN distribution for $u_{\mu,t}$ in score-driven models of seasonality is new in the literature.

The score function with respect to λ_t is given by:

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \lambda_t} = u_{\lambda,t} = 2\mathrm{Re}\left\{-i\epsilon_t\Psi^{(0)}[\exp(\eta) + i\epsilon_t]\right\} - \pi\tanh(\nu)\epsilon_t - 1 \tag{32}$$

The use of $u_{\lambda,t}$ for the MXN distribution is from the work of Blazsek and Haddad (2020). The updating term $u_{\lambda,t}$ performs a linearly increasing and asymmetric transformation of ϵ_t, as $|\epsilon_t| \to \infty$ (Figures 1(j) and 2(j)).

(iii) The conditional mean and conditional standard deviation of p_t, respectively, are:

$$E(p_t|p_1,\ldots,p_{t-1}) = \mu_t + s_t + \exp(\lambda_t + \eta)\tan[\pi\tanh(\nu)/2] \tag{33}$$

$$\sigma_t = \sigma(p_t|p_1,\ldots,p_{t-1}) = \left\{\frac{\exp(\lambda_t + \eta)}{\cos[\pi\tanh(\nu)] + 1}\right\}^{1/2} \tag{34}$$

It is noteworthy that the conditional mean of v_t is not zero for the Skew-Gen-t, EGB2, NIG and MXN distributions, which is indicated by the one-step ahead forecasting formulae for p_t (i.e. by the term in addition to $\mu_t + s_t$). The non-zero expected value of v_t is not problematic for the statistical inference of the score-driven state-space models. The one-step ahead forecasting formulae of this paper may be used in future works on the Russian rouble exchange rates, in which the predictive accuracies of alternative score-driven and classical time series models are compared for the period of the fully floating exchange rate regime from 2015.

[APPROXIMATE LOCATION OF FIGURES 1 AND 2]

4. Empirical results

4.1. Data

RUB to USD and RUB to EUR weekly currency exchange rate data are used for the period of 4 January 1999 to 27 January 2020. In Table 1(a), data sources, observation periods, sample sizes, and several descriptive statistics are reported for the currency exchange rate levels p_t and percentage changes $(p_t - p_{t-1})/p_{t-1}$. The augmented Dickey–Fuller tests (Dickey and Fuller 1979) results and the estimates of the degree of integration (Geweke and Porter-Hudak 1983; Robinson 1995) indicate that p_t is integrated of order one, which supports the $I(1)$ stochastic local level specification for μ_t. The ARCH test (Engle 1982) indicates heteroscedasticity for all time series, which supports the use of the EGARCH model for v_t. For the Shapiro–Wilk test (Shapiro and Wilk 1965), the normal distribution null hypothesis is always rejected. In Figure 3, the evolution of the currency exchange rates is presented.

[APPROXIMATE LOCATION OF TABLE 1 AND FIGURE 3]

4.2. Multiplicative seasonal ARIMA

In Table 1(b), the estimates for the multiplicative seasonal ARIMA model for the Russian rouble exchange rates are presented. For the multiplicative seasonal ARIMA model, estimates for the following lag-orders are presented: $(1, d, 1)(1, 0, 1)_{12}$ for $d = 0$ and $d = 1$. Thus, the model is specified as: $(1 - \phi L)(1 - \Phi L^{12})(1 - L)^d p_t = c + (1 + \theta L)(1 + \Theta L^{12})v_t$, where ϕ and θ are the level parameters, Φ and Θ are the seasonality parameters, and L is the lag operator.

The results on seasonality are robust to alternative AR and MA lag-orders. For $d = 0$ unit root is found for p_t (Table 1), which motivates $d = 1$. According to the estimates, the seasonality parameters are not significantly different from zero (Table 1). The specification for $d = 0$ with $\phi = 1$ is also estimated, and robust results on seasonality are obtained (the corresponding results are not reported in Table 1). The results for ARIMA are in contrast to the estimation results for score-driven models of the following section, for which seasonality effects are significantly different from zero. Furthermore, the log-likelihood (LL), Akaike information criterion (AIC), Bayesian information criterion (BIC), and Hannan–Quinn criterion (HQC) metrics for multiplicative seasonal ARIMA are also reported in Table 1, and they are compared to the same metrics for the score-driven state-space specifications in the following section.

4.3. Model diagnostics and parameter estimates for score-driven models

The parameter estimates and model diagnostics, for RUB to USD and RUB to EUR exchange rates, are presented in Tables 2 and 3, respectively. The statistical performances of different models are compared by using the LL, AIC, BIC and HQC metrics, which is motivated by the work of Harvey (2013). The score-driven model with EGB2 probability distribution has the best statistical performance for both Russian rouble exchange rates. Nevertheless, the ML estimates of the seasonality components are very similar for all the probability distributions of this paper (Appendix), which indicate robust results for the score-driven state-space models of the Russian rouble exchange rates. The results also indicate that the likelihood-based statistical performance of all score-driven state-space models is superior to the statistical performance of the multiplicative seasonal ARIMA model. In addition, the sample estimates of the C_1, C_2 and C_3 metrics never indicate failures of the asymptotic properties of the ML estimates.

With respect to parameter significance, κ is significantly different from zero for all models, which supports the score-driven stochastic local level specification for μ_t (Harvey 2013). For all cases, α and β are significantly different from zero, which supports the use of the EGARCH specification of heteroscedasticity. Some of the seasonality parameters (i.e. $\gamma_{\text{Jan}}, \ldots, \gamma_{\text{Dec}}$) are significant for all score-driven state-space specifications (Tables 2 and 3), which supports the use of the annual stochastic seasonality component for Russian rouble exchange rates.

[APPROXIMATE LOCATION OF TABLES 2 AND 3]

4.4. Structural changes

An advantage of the use of the score-driven state-space model is that it can be estimated in a robust way for structural changes in the Russian rouble exchange rates, which is provided by the $I(1)$ local level filter, the stochastic seasonality filter, and the EGARCH-based volatility filter.

In Figures 4(a) and 5(a), the estimates of the error term v_t are presented for RUB to USD and RUB to EUR, respectively. Those figures indicate three regimes for the full sample period with different levels of volatility: (i) January 1999 to January 2009, (ii) February 2009 to October 2014, and (iii) November 2014 to January 2020.

The volatility dynamics in Figures 4(a) and 5(a) motivate the use of EGARCH for the RUB to USD and RUB to EUR currency exchange rates. Furthermore, in Figures 4(b) and 5(b), the estimates of the standardized error term ϵ_t are presented for RUB to USD and RUB to EUR, respectively. Those figures indicate that heteroscedasticity is effectively controlled by EGARCH, since the standardized error term ϵ_t appears to be homoscedastic. The observations for which relatively large variance is inferred from Figures 4(b) and 5(b) are the outliers, which are in the standardized error term ϵ_t due to the outlier-robust updating mechanisms of the filters.

The different regimes for RUB to USD and RUB to EUR (Figures 4(a) and 5(a)) are in relation to different currency exchange regimes that are implemented by the Bank of Russia. In the following, the exchange rate policies of Russia for the period of 1991 to 2020 are reviewed.

4.4.1. Exchange rate policies for the pre-sample period of 1991 to 1998

When the Soviet Union dissolved in 1991, the Russian rouble exchange rate was determined in a multiple exchange rate system (Baliño et al. 1997). In 1992, the Bank of Russia began to intervene on the Moscow Interbank Currency Exchange (MICEX) with the objectives of smoothing the volatility of the the Russian rouble nominal exchange rates and ensuring a steady nominal depreciation of the Russian rouble. The multiple exchange rate system was unified in

July 1992. Since July 1993, exporters may sell foreign currencies on the exchange rate market through an authorized bank (Baliño et al. 1997).

In December 1994, as a part of new economic policies that aimed to achieve macroeconomic stability, a 10% limit on daily Russian rouble currency exchange rate movements was established (Baliño et al. 1997). In the second quarter of 1995, the Bank of Russia intervened by purchasing foreign currencies, in order to prevent an appreciation of the Russian rouble due to foreign capital inflows (Baliño et al. 1997). In July 1995, the Bank of Russia established a RUB to USD exchange rate band, according to which RUB to USD was permitted to fluctuate freely within a band, and the Bank of Russia intervened outside of the band (Baliño et al. 1997). In July 1995, the Bank of Russia fixed the 4,600 RUB to USD exchange rate for the midpoint with the ±6.5% band (Baliño et al. 1997). In July 1996, a sliding devaluation system was introduced, in which a sliding band with a 1.5% monthly depreciation rate was established with an initial minimum value of 5,000 RUB to USD and an initial maximum value of 5,600 RUB to USD (Baliño et al. 1997). The sliding band was slightly narrowed over time (Baliño et al. 1997).

4.4.2. Exchange rate policies for the sample period of 1999 to 2020

(i) After the 1998 Russian financial crisis, the Bank of Russia abandoned the sliding devaluation system and implemented a managed floating currency exchange rate regime (Bank of Russia 2013). For the period of 1999 to 2005, the Bank of Russia intervened only by performing RUB to USD operations. In 2005, the Bank of Russia introduced a USD and EUR basket (i.e. dual basket), as the operational indicator of its currency exchange rate policy (Bank of Russia 2020). Since 2005, the Bank of Russia intervened by performing operations in RUB to USD and RUB to EUR. The operational borders of the currency exchange rate band are defined based on balance of payments dynamics and currency exchange rate market developments (Bank of Russia 2020). For example, in February 2007, the basket composition was established at 55% for USD and 45% for EUR (Bank of Russia 2020).

(ii) During the 2008 US financial crisis, due to a substantial drop of oil prices and a strong outflow of foreign capital from Russia, the Bank of Russia established a fixed band for the

Russian rouble exchange rate between 26 and 41 RUB to USD in January 2009. Within those limits a floating interval was established, initially with a width of 2 RUB to USD, which later was increased (Bank of Russia 2009, 2020; Tabata 2011). Moreover, a procedure that regulates the modification of that band was also established in January 2009 (Bank of Russia 2009, 2020; Tabata 2011). In October 2010, the Bank of Russia abandoned the fixed band (Bank of Russia 2020). In December 2011, MICEX merged with the Russian Trading System (the Moscow stock market, founded in 1995), and created the Moscow Exchange, the largest exchange group in Russia. For the period of October 2010 to November 2014, the Bank of Russia implemented a managed floating exchange rate regime, which smoothed the volatility of the exchange rates without modifying its trends (Bank of Russia 2020). Smoothing of the volatility of the exchange rates was performed by selling or buying USD or EUR inside and outside the band. On July 2012, a floating operational band was established with a width of 7 RUB to USD. In August 2014, this operational band was widened to 9 RUB to USD from 7 RUB to USD (Bank of Russia 2020).

(iii) In November 2014, the Bank of Russia abolished the use of the operational band for exchange rate policies, and it only intervenes on the foreign exchange market on and outside the operational band of the Russian rouble exchanges rates (Bank of Russia 2020).

[APPROXIMATE LOCATION OF FIGURES 4 AND 5]

4.5. Stochastic seasonality

For the score-driven state-space model of the EGB2 probability distribution, the seasonality components of RUB to USD and RUB to EUR are presented in Figures 4(c) and 5(c), respectively. Those estimates indicate a significant magnitude of annual seasonality in the currency exchange rates. Very similar seasonality estimates are obtained for the rest of the probability distributions (Appendix). Thus, the score-driven models for the Russian rouble exchange rates of this paper provide robust results. In the following, the annual stochastic seasonality components of the Russian rouble exchange rates are explained, by using macroeconomic variables from the current account of balance of payments of the Russian Federation.

Turuntseva et al. (2018) argue that, in order to forecast several economic indicators for the Russian Federation, seasonality should be included. According to Elvira Nabiulina, Governor of the Bank of Russia, seasonal fluctuations are typical of the current account in Russia (Bank of Russia 2017). In relation to this, we also refer to the seasonal interventions of the Bank of Russia on the foreign currency market (Bank of Russia 2013).

In Figure 6, the evolution of exports of goods and services from Russia, imports of goods and services to Russia, and the primary income of Russia (receivable and payable components) are presented (source: Bank of Russia). The figure indicates significant annual seasonality components for those variables. Significant relationships between imports to Russia and the Russian rouble exchange rates are reported, for example, in the works of Sosunov and Zamulin (2006), Ivanova (2007), and Tyll et al. (2018). Gusev and Shirov (2009) use foreign trade forecasting models that are developed by the Institute of Economic Forecasting of the Russian Academy of Sciences (IEF RAS), as a basis to conduct a comparative analysis of changes in key foreign trade indicators (e.g. changes in world oil prices) under various hypotheses. Seasonal factors are included by these authors as an exogenous variable into the regression equations for imports and exports, to account for seasonal fluctuations. Oil prices are considered in the export equation, and the RUB to USD exchange rate is included in the import equation.

In the body of literature on crude oil production, several works emphasize the relationship between the crude oil exports from Russia and the Russian rouble currency exchange rates (e.g. Mironov 2015; Sosunov and Zamulin 2006; Alekhin 2016; Menash et al. 2017; Tyll et al. 2018). Oil production in Russia has been steadily increasing since 1998 (source: US Energy Information Administration). The volume of crude oil exports from Russia increased steadily for the period of 2000 to 2004, and the same exports have been approximately constant, at the 250 million tons per year level, for the period of 2004 to 2020.

In addition to crude oil exports, annual seasonality of the Russian rouble exchange rates is also due to the refined petroleum, natural gas and coal exports. Crude oil, refined petroleum, natural gas, and coal exports contribute to a significant portion of total exports from Russia in every year. For example, the exports of those products were approximately 55.5% of the total exports in 2017 (source: Observatory of Economic Complexity). In Table 4, annual seasonality for crude oil production (ticker: RUSOTTDY Index), natural gas exports (ticker: RUCUNGAS Index), natural gas production (ticker: RUSGTOMA Index), and coal exports (ticker: RUCUCOLE Index) are presented (source of monthly data: Bloomberg). In Table 4, the OLS-HAC (ordinary least squares, heteroscedasticity and autocorrelation consistent) estimates (Newey and West 1978) for the following regression model are presented: $y_t = c + \delta_{\text{Jan}} D_{\text{Jan},t} + \ldots + \delta_{\text{Dec}} D_{\text{Dec},t} + \theta_1 t + \theta_2 t^2 + v_t$. The OLS-HAC estimates suggest significant annual seasonality for the crude oil production, natural gas exports and production, and coal exports. Therefore, seasonal variations in the current account balance (Bozhechkova and Trunin 2018) and in the Russian rouble exchange rates are expected.

[APPROXIMATE LOCATION OF TABLE 4 AND FIGURE 6]

5. Conclusions

In this paper, the practical use of a score-driven state-space model has been suggested, to measure the annual seasonality components in a robust way for Russian rouble currency exchange rates for the period of 1999 to 2020. The motivation of the use of the score-driven models is that those models implement an optimal filtering mechanism, according to the Kullback–Leibler divergence in favour of the true data generating process. The same is not true for the frequently used multiplicative seasonal ARIMA model, for which the annual seasonality effects are not significant for the Russian rouble exchange rates. The estimation of seasonality effects for Russian rouble currency exchange rate data is motivated by the significant annual seasonality of the current account in the balance of payments of the Russian Federation.

An advantage of the use of the score-driven state-space model is that it can be estimated in a robust way for structural changes in the Russian rouble exchange rates. The structural changes are in relation to different currency exchange regimes that have been implemented by the Bank of Russia, and the exchange rate policies of Russia are reviewed for the pre-sample period of 1991 to 1998 and for the sample period of 1999 to 2020.

The statistical inference procedures have indicated that the score-driven state-space model for the Russian rouble provides robust estimates of the annual stochastic seasonality components of the Russian rouble exchange rates, which is approximately in the range of $\pm 2\%$ for the period of 1999 to 2020. These findings motivate the practical use of the robust score-driven stochastic seasonality method of this paper, in order to deseasonalize the Russian rouble exchange rates for economic analyses, or to study the dynamic amplitude of seasonality of the Russian rouble exchange rates for financing, investment and policy decisions.

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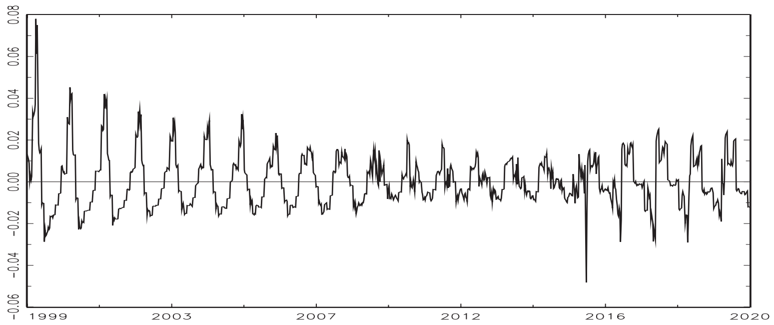
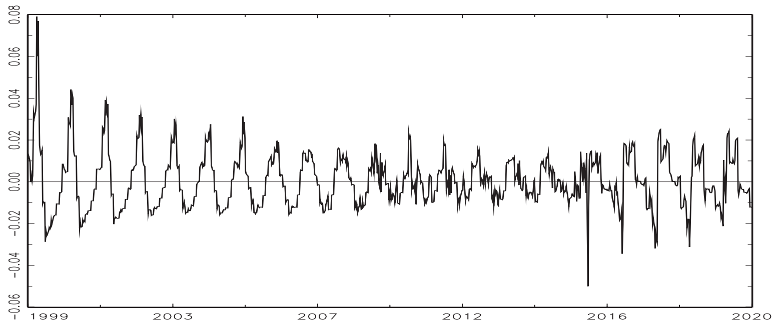
Appendix

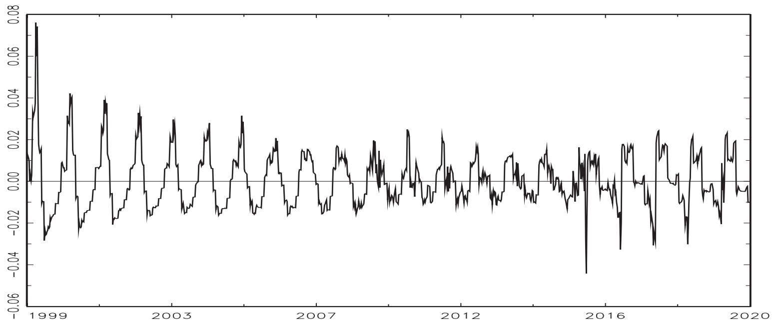
(a) Student's t for s_t/μ_t (0.99433)



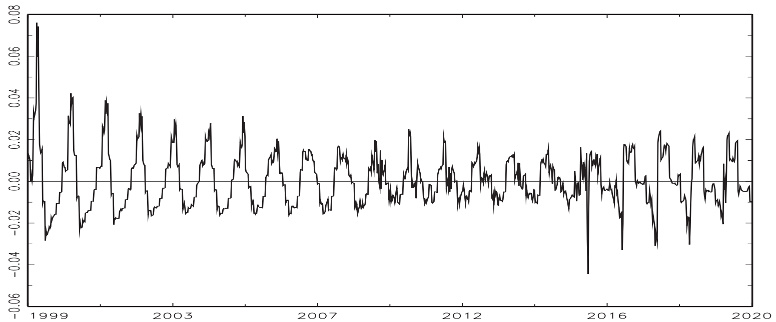
(b)Skew-Gen-t for s_t/μ_t (0.99698)



(c) EGB2 for s_t/μ_t (1.00000)



(d) NIG for s_t/μ_t (0.99996)



(e) MXN for s_t/μ_t (0.99972)

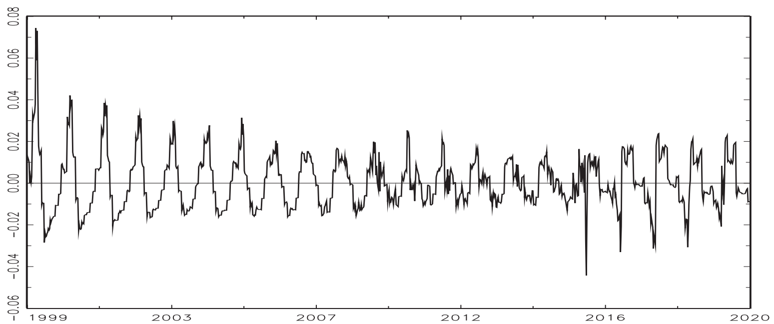
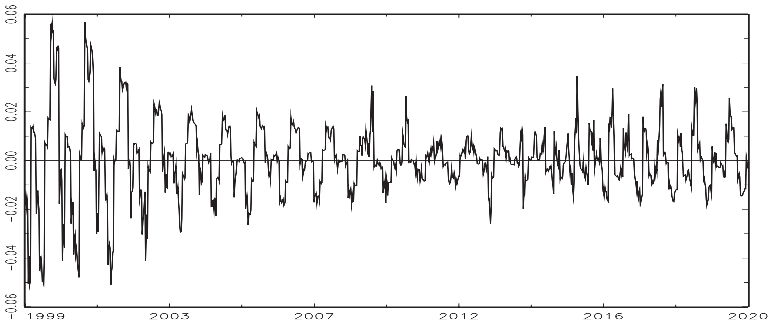


Figure A1. s_t/μ_t for RUB to USD for the period of 4 January 1999 to 27 January 2020 (weekly data).

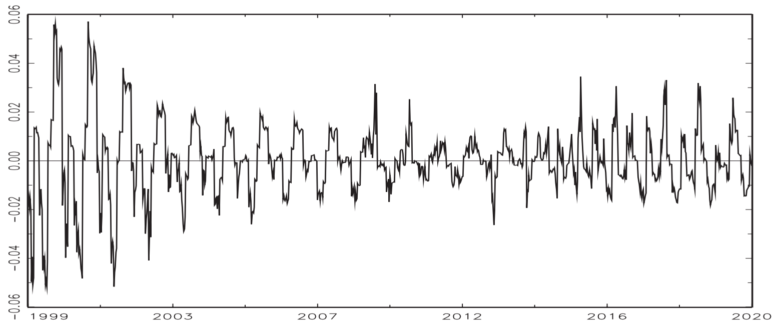
Notes: The correlation coefficients with the estimates of s_t/μ_t for EGB2 are presented in parentheses.

Appendix

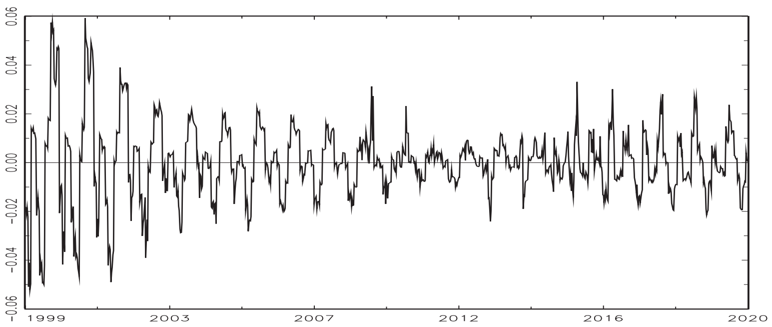
(a) Student's t for s_t/μ_t (0.99504)



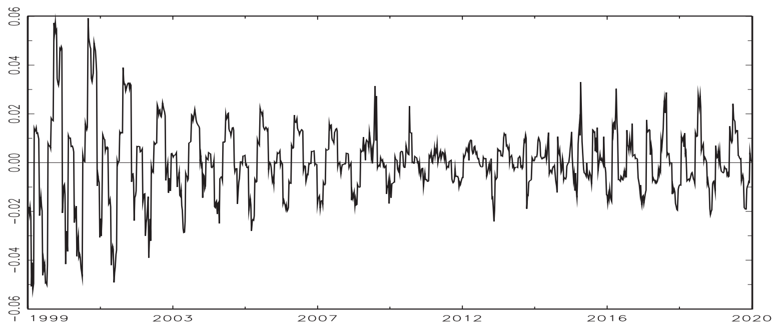
(b)Skew-Gen-t for s_t/μ_t (0.99397)



(c) EGB2 for s_t/μ_t (1.00000)



(d) NIG for s_t/μ_t (0.99993)



(e) MXN for s_t/μ_t (0.99971)

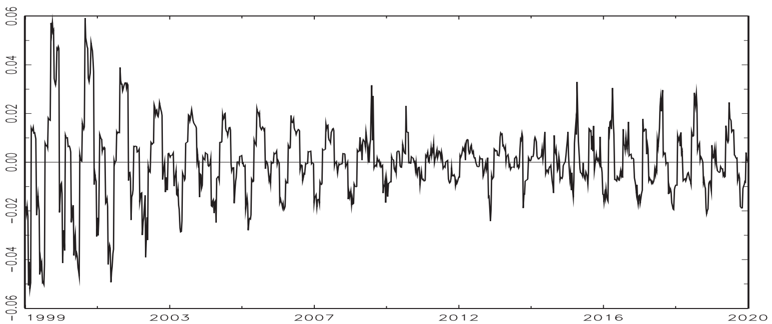


Figure A2. s_t/μ_t for RUB to EUR for the period of 4 January 1999 to 27 January 2020 (weekly data).
Notes: The correlation coefficients with the estimates of s_t/μ_t for EGB2 are presented in parentheses.

Table 1. Descriptive statistics.

(a) Descriptive statistics	RUB/USD	RUB/USD %	RUB/EUR	RUB/EUR %
Data source	Bloomberg	Bloomberg	Investing.com	Investing.com
Start date	4 January 1999	4 January 1999	4 January 1999	4 January 1999
End date	27 January 2020	27 January 2020	27 January 2020	27 January 2020
T	$1,100$	$1,100$	$1,100$	$1,100$
Minimum	22.4666	−0.1427	23.3660	−0.0893
Maximum	79.9695	0.1386	88.2196	0.1188
Average	37.5408	0.0010	44.5378	0.0010
Standard deviation	14.8219	0.0147	16.2848	0.0179
Skewness	1.1684	1.1747	0.8756	1.0604
Excess kurtosis	−0.3191	22.0984	−0.5799	7.1783
ADF test (constant) p-value	0.7867	0.0000	0.8601	0.0000
ADF test (constant, trend) p-value	0.5430	0.0000	0.3650	0.0000
Degree of integration (local Whittle)	0.9945	−0.0409	0.9964	−0.1907
Degree of integration (GPH)	0.9939	−0.0499	1.0325	−0.1636
ARCH test p-value	0.0000	0.0000	0.0000	0.0000
Shapiro–Wilk test p-value	0.0000	0.0000	0.0000	0.0000
(b) Multiplicative seasonal ARIMA	RUB/USD	RUB/USD	EUR/USD EUR/USD	
Lag-order specification	$(1,0,1)(1,0,1)_{12}$	$(1,1,1)(1,0,1)_{12}$	$(1,0,1)(1,0,1)_{12}$	$(1,1,1)(1,0,1)_{12}$
c	0.0691(0.1462)	0.0065(0.0117)	0.0915(0.1247)	0.0437(0.0400)
ϕ	0.9992***(0.0023)	0.7906***(0.0327)	0.9984***(0.0022)	−0.3350(0.4208)
Φ	−0.0574(0.2556)	−0.0358(0.3182)	0.1890(0.3325)	0.1897(0.3415)
θ	0.0634***(0.0071)	−0.6782***(0.0378)	−0.0128(0.0145)	0.3120(0.4246)
Θ	0.1707(0.2551)	0.1295(0.3180)	−0.1287(0.3361)	−0.1302(0.3453)
LL	−1.1377	−1.1243	−1.4128	−1.4131
AIC	2.2868	2.2601	2.8370	2.8377
BIC	2.3152	2.2885	2.8654	2.8661
HQC	2.2976	2.2709	2.8478	2.8484

Notes: For the Augmented Dickey–Fuller test (Dickey and Fuller, 1979), the optimal lag-order is selected by using the Bayesian information criterion (BIC). For the degree of fractional integration, the local Whittle estimator (Robinson 1995) and the Geweke and Porter-Hudak estimator (Geweke and Porter-Hudak 1983) with lag-order $33 \approx T^{1/2}$ are used. For the ARCH (autoregressive conditional heteroscedasticity) test (Engle 1982), lag order 4 is used. For normal distribution test, the Shapiro–Wilk test (Shapiro and Wilk 1965) is used. The seasonal ARIMA$(1,d,1)(1,0,1)_{12}$ model is $(1-\phi L)(1-\Phi L^{12})(1-L)^d p_t = c + (1+\theta L)(1+\Theta L^{12})v_t$, where L is the lag operator, and $d=0$ or $d=1$ is used. *** indicates parameter significance at the 1% level. According to the log-likelihood (LL), Akaike information criterion (AIC), Bayesian information critetion (BIC), and Hannan–Quinn criterion (HQC) metrics, the multiplicative seasonal ARIMA model is inferior to the score-driven state space models (see Tables 2 and 3).

Table 2. Parameter estimates and model diagnostics for RUB to USD, 4 January 1999 to 27 January 2020 (weekly data).

	Student's t	Skew-Gen-t	EGB2	NIG	MXN
κ	1.3256***(0.0688)	1.3583***(0.0513)	1.0232***(0.0420)	0.2422***(0.0634)	0.8198***(0.1971)
γ_{Jan}	−0.0780***(0.0180)	−0.0757***(0.0053)	−0.0545***(0.0196)	−0.0126***(0.0045)	−0.0413**(0.0204)
γ_{Feb}	0.1295+(0.0875)	0.1644***(0.0607)	0.1366*(0.0714)	0.0325*(0.0193)	0.1115+(0.0737)
γ_{Mar}	0.1785***(0.0525)	0.2058***(0.0297)	0.1537***(0.0297)	0.0367***(0.0122)	0.1269***(0.0370)
γ_{Apr}	0.6590***(0.0771)	0.7074***(0.0411)	0.5129***(0.0340)	0.1211***(0.0328)	0.4029***(0.0974)
γ_{May}	−0.1065***(0.0202)	−0.1105***(0.0102)	−0.0823***(0.0126)	−0.0195***(0.0058)	−0.0656***(0.0185)
γ_{Jun}	0.0419(0.0408)	0.0496+(0.0309)	0.0430+(0.0278)	0.0104+(0.0071)	0.0366*(0.0209)
γ_{Jul}	0.2422***(0.0602)	0.2376***(0.0398)	0.1633***(0.0362)	0.0384***(0.0106)	0.1294***(0.0427)
γ_{Aug}	0.0958**(0.0375)	0.0980***(0.0133)	0.0696***(0.0120)	0.0166***(0.0055)	0.0565***(0.0133)
γ_{Sep}	−0.0579(0.2899)	−0.2238***(0.0847)	−0.1122(0.1116)	−0.0281(0.0247)	−0.1018*(0.0577)
γ_{Oct}	0.0591(0.0456)	0.0153(0.0697)	0.0269(0.0543)	0.0058(0.0123)	0.0156(0.0386)
γ_{Nov}	−0.0258+(0.0165)	−0.0273***(0.0101)	−0.0189+(0.0116)	−0.0044+(0.0029)	−0.0150+(0.0094)
γ_{Dec}	−0.0792***(0.0222)	−0.0921***(0.0207)	−0.0749***(0.0154)	−0.0176***(0.0057)	−0.0595***(0.0201)
ω	−0.0118(0.0086)	−0.0127+(0.0081)	−0.0154**(0.0075)	−0.0018(0.0042)	−0.0111+(0.0072)
β	0.9855***(0.0075)	0.9854***(0.0067)	0.9853***(0.0062)	0.9855***(0.0062)	0.9853***(0.0064)
α	0.0958***(0.0236)	0.1014***(0.0204)	0.0860***(0.0189)	0.0863***(0.0192)	0.0835***(0.0210)
λ_0	−0.9802**(0.3859)	−1.0249***(0.3810)	−1.3724***(0.4341)	−0.4557(0.4067)	−1.0157**(0.4610)
ν	2.3331***(0.3052)	2.3648***(0.3127)	0.8288***(0.3187)	1.5091***(0.2794)	0.1631***(0.0382)
η	NA	0.6518***(0.0297)	0.3984(0.2920)	0.2172***(0.0476)	0.4043*(0.2152)
τ	NA	0.0927***(0.0097)	NA	NA	NA
LL	−0.4555	−0.4492	**−0.4468**	−0.4471	−0.4485
AIC	0.9437	0.9347	**0.9281**	0.9288	0.9316
BIC	1.0256	1.0257	**1.0146**	1.0152	1.0180
HQC	0.9747	0.9691	**0.9608**	0.9615	0.9643
C_1	0.9855	0.9854	0.9853	0.9855	0.9853
C_2	0.7377	0.7287	0.7395	0.7418	0.7498
C_3	−0.1966	−0.2051	−0.1943	−0.1929	−0.1859

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn criterion (HQC). Bold numbers indicate superior statistical performance. $C_1 < 1$ indicates that filter λ_t is covariance stationarity. $C_2 < 1$ indicates that those elements of the information matrix that correspond to the parameters of λ_t are finite. $C_3 < 0$ indicates that filter λ_t is invertible. $^+$, *, ** and *** indicate significance at the 15%, 10%, 5% and 1% levels, respectively. Robust standard errors, estimated by using the Huber–White sandwich estimator, are presented in parentheses.

Table 3. Parameter estimates and model diagnostics for RUB to EUR, 4 January 1999 to 27 January 2020 (weekly data).

	Student's t	Skew-Gen-t	EGB2	NIG	MXN
κ	1.0103***(0.0750)	0.9806***(0.0799)	0.8030***(0.0482)	0.0776**(0.0380)	1.5909+(1.1018)
γ_{Jan}	0.1525*(0.0821)	0.1482**(0.0649)	0.1008+(0.0617)	0.0099(0.0083)	0.2076(0.1542)
γ_{Feb}	0.3899***(0.0793)	0.3935***(0.0688)	0.3356***(0.0787)	0.0325*(0.0172)	0.6669(0.4931)
γ_{Mar}	0.0276(0.0295)	0.0219(0.0260)	0.0279(0.0250)	0.0027(0.0024)	0.0554(0.0736)
γ_{Apr}	0.0549(0.0588)	0.0424(0.0547)	0.0400(0.0468)	0.0039(0.0046)	0.0821(0.1230)
γ_{May}	0.1436(0.1174)	0.1609*(0.0954)	0.0917(0.1052)	0.0093(0.0150)	0.1991(0.1919)
γ_{Jun}	0.2299**(0.1043)	0.2409***(0.0895)	0.1738*(0.0953)	0.0171(0.0163)	0.3557*(0.1968)
γ_{Jul}	0.3266**(0.1413)	0.3468***(0.1274)	0.2367(0.1691)	0.0234(0.0271)	0.4943*(0.2899)
γ_{Aug}	0.0420+(0.0275)	0.0372*(0.0220)	0.0380(0.0292)	0.0036(0.0027)	0.0735(0.0923)
γ_{Sep}	0.0413(0.0657)	0.0293(0.0642)	0.0427(0.0705)	0.0040(0.0064)	0.0794(0.1656)
γ_{Oct}	0.1857***(0.0301)	0.1837***(0.0302)	0.1680***(0.0313)	0.0161*(0.0087)	0.3293(0.2374)
γ_{Nov}	−0.1167***(0.0236)	−0.1131***(0.0228)	−0.1026***(0.0277)	−0.0098**(0.0047)	−0.2002(0.1630)
γ_{Dec}	−0.0839***(0.0123)	−0.0792***(0.0122)	−0.0770***(0.0143)	−0.0074**(0.0031)	−0.1502(0.1231)
ω	−0.0040(0.0046)	−0.0040(0.0043)	−0.0015(0.0032)	0.0088+(0.0059)	−0.0085(0.0076)
β	0.9858***(0.0085)	0.9868***(0.0083)	0.9881***(0.0069)	0.9884***(0.0068)	0.9888***(0.0065)
α	0.0610***(0.0171)	0.0582***(0.0197)	0.0512***(0.0117)	0.0503***(0.0129)	0.0489***(0.0126)
λ_0	−0.2236(0.4365)	−0.2549(0.4859)	0.0037(0.0452)	0.8879*(0.5254)	−0.6147(0.5876)
ν	2.6425***(0.3900)	2.9339***(0.8613)	1.7225***(0.4380)	2.4287***(0.5400)	0.1859***(0.0666)
η	NA	0.6544***(0.0596)	1.1663***(0.3648)	0.2474***(0.0808)	1.2909**(0.6250)
τ	NA	0.0401(0.0311)	NA	NA	NA
LL	−1.0515	−1.0496	**−1.0400**	−1.0404	−1.0410
AIC	2.1357	2.1355	**2.1145**	2.1153	2.1165
BIC	2.2176	2.2265	**2.2009**	2.2017	2.2029
HQC	2.1667	2.1699	**2.1472**	2.1480	2.1492
C_1	0.9858	0.9868	0.9881	0.9884	0.9888
C_2	0.8060	0.8137	0.7809	0.7885	0.7976
C_3	−0.1272	−0.1232	−0.1544	−0.1524	−0.1459

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn criterion (HQC). Bold numbers indicate superior statistical performance. $C_1 < 1$ indicates that filter λ_t is covariance stationarity. $C_2 < 1$ indicates that those elements of the information matrix that correspond to the parameters of λ_t are finite. $C_3 < 0$ indicates that filter λ_t is invertible. $^+$, *, ** and *** indicate significance at the 15%, 10%, 5% and 1% levels, respectively. Robust standard errors, estimated by using the Huber–White sandwich estimator, are presented in parentheses.

Table 4. Seasonality analysis for crude oil production, natural gas exports and production, and coal exports (monthly data).

Variable	Crude oil production, Russia	Natural gas exports, Russia	Natural gas production, Russia	Coal exports, Russia
Ticker	RUSOTTDY Index	RUCUNGAS Index	RUSGTOMA Index	RUCUCOLE Index
Start date	January 2005	January 2006	August 2010	February 2005
End date	January 2020	November 2019	December 2019	November 2019
c	$9.4182^{***}(0.0446)$	$25.2733^{**}(11.0975)$	$65043.5347^{***}(1289.1595)$	$-24509.4033^{+}(15520.2691)$
δ_{Feb}	$-0.0080(0.0189)$	$8.0477(9.3414)$	$-6329.5951^{***}(765.1319)$	$3186.7250(8929.7279)$
δ_{Mar}	$-0.0099(0.0232)$	$24.3546^{**}(9.5310)$	$-3817.8768^{***}(1171.1445)$	$15437.5148^{+}(9337.8243)$
δ_{Apr}	$-0.0418^{+}(0.0258)$	$38.7039^{***}(9.7059)$	$-8221.8309^{***}(1176.7371)$	$24728.3578^{**}(9716.3523)$
δ_{May}	$-0.0470^{+}(0.0295)$	$53.3742^{***}(9.8403)$	$-10498.4685^{***}(1275.0333)$	$35358.6940^{***}(10002.6871)$
δ_{Jun}	$-0.0323(0.0294)$	$66.6370^{***}(9.9670)$	$-16316.7674^{***}(1123.4340)$	$47073.8433^{***}(10156.6389)$
δ_{Jul}	$-0.0402(0.0338)$	$80.0136^{***}(9.9349)$	$-16332.2609^{***}(992.4660)$	$55699.5792^{***}(10265.2960)$
δ_{Aug}	$-0.0195(0.0334)$	$93.4112^{***}(10.0498)$	$-16562.5187^{***}(1197.4445)$	$66516.0282^{***}(10349.3310)$
δ_{Sep}	$0.0279(0.0317)$	$106.7442^{***}(10.2313)$	$-13186.1337^{***}(1060.7622)$	$78211.5304^{***}(10653.6648)$
δ_{Oct}	$0.0580^{*}(0.0333)$	$122.1338^{***}(10.2753)$	$-4598.0633^{***}(1112.5632)$	$88059.7390^{***}(11016.0403)$
δ_{Nov}	$0.0428^{+}(0.0279)$	$138.4087^{***}(10.3177)$	$-3969.7576^{***}(861.1828)$	$99048.9542^{***}(11614.7613)$
δ_{Dec}	$0.0242(0.0278)$	$156.5331^{***}(10.0873)$	$-67.3165(572.2116)$	$108588.9328^{***}(13733.1356)$
θ_1	$0.0105^{***}(0.0008)$	$-0.4160^{***}(0.1429)$	$-148.2865^{***}(49.3452)$	$392.3526(284.4923)$
θ_2	$0.0000(0.0000)$	$0.0038^{***}(0.0010)$	$1.7973^{***}(0.4230)$	$0.5434(1.4546)$

Notes: OLS-HAC estimates for the following regression model are presented: $y_t = c+\delta_{\text{Jan}}D_{\text{Jan},t}+\ldots+\delta_{\text{Dec}}D_{\text{Dec},t}+\theta_1 t+\theta_2 t^2+v_t$, where $t=1,\ldots,T$ are monthly observations. $^{+}$, *, ** and *** indicate significance at the 15%, 10%, 5% and 1% levels, respectively. Robust standard errors, estimated by using the Newey–West HAC estimator, are presented in parentheses. *Source*: Bloomberg

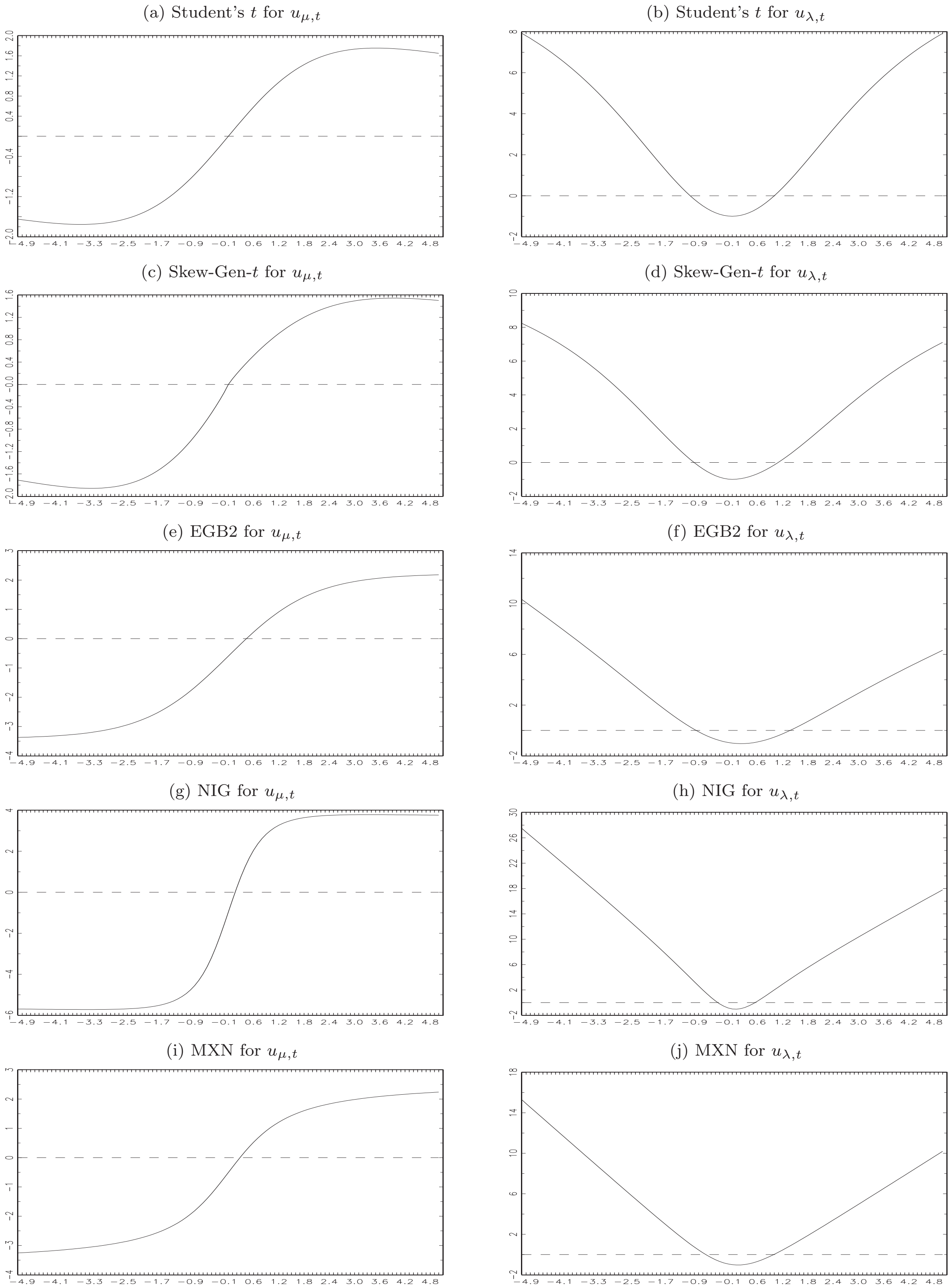


Figure 1. Score functions, as functions of ϵ_t, for the RUB to USD currency exchange rate.
Notes: The ML estimates of the shape parameters are used, and it is assumed that $\lambda_t = 0$.

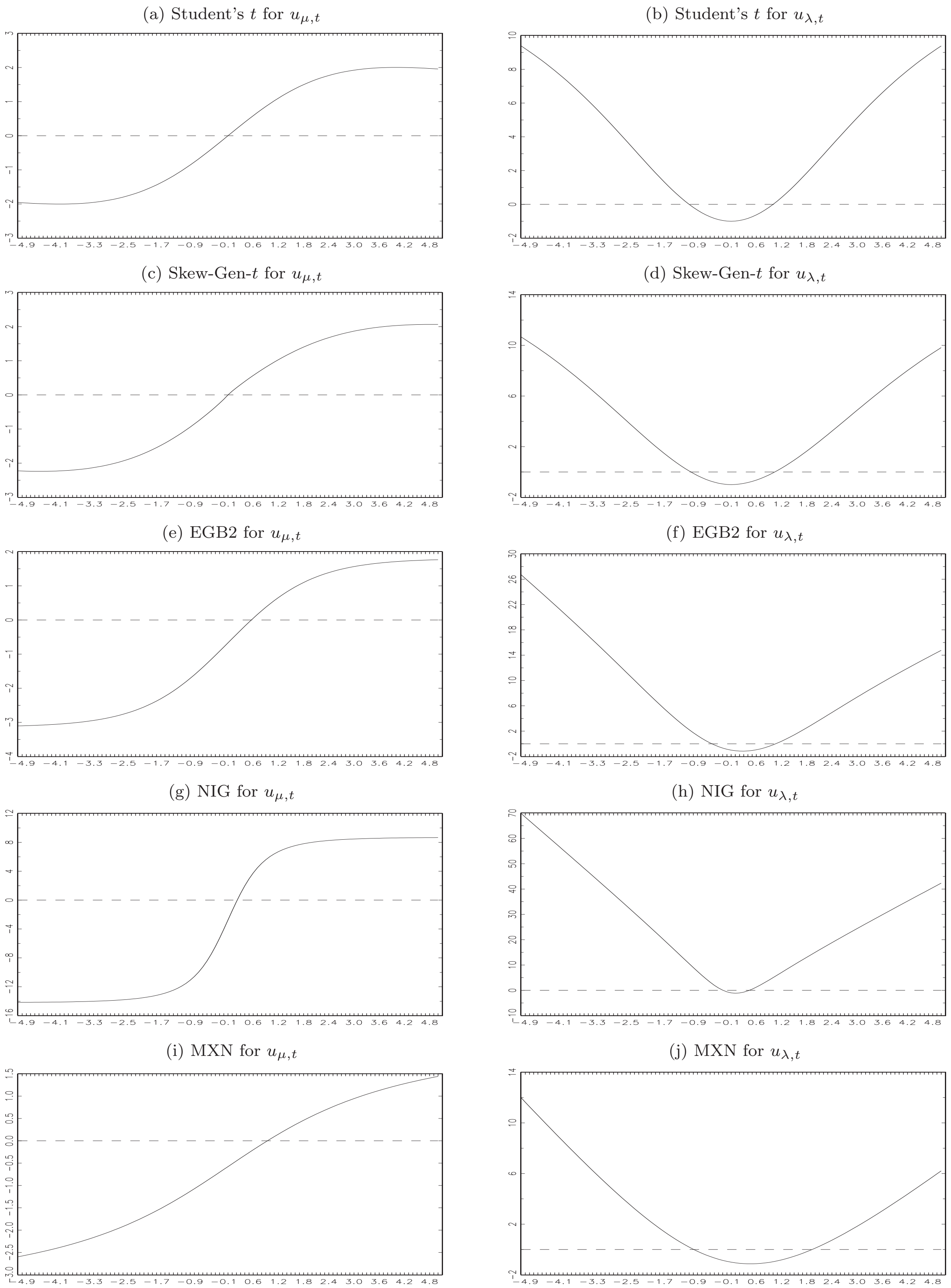


Figure 2. Score functions, as functions of ϵ_t, for the RUB to EUR currency exchange rate.
Notes: The ML estimates of the shape parameters are used, and it is assumed that $\lambda_t = 0$.

(a) RUB to USD for the period of 4 January 1999 to 27 January 2020 (weekly data).

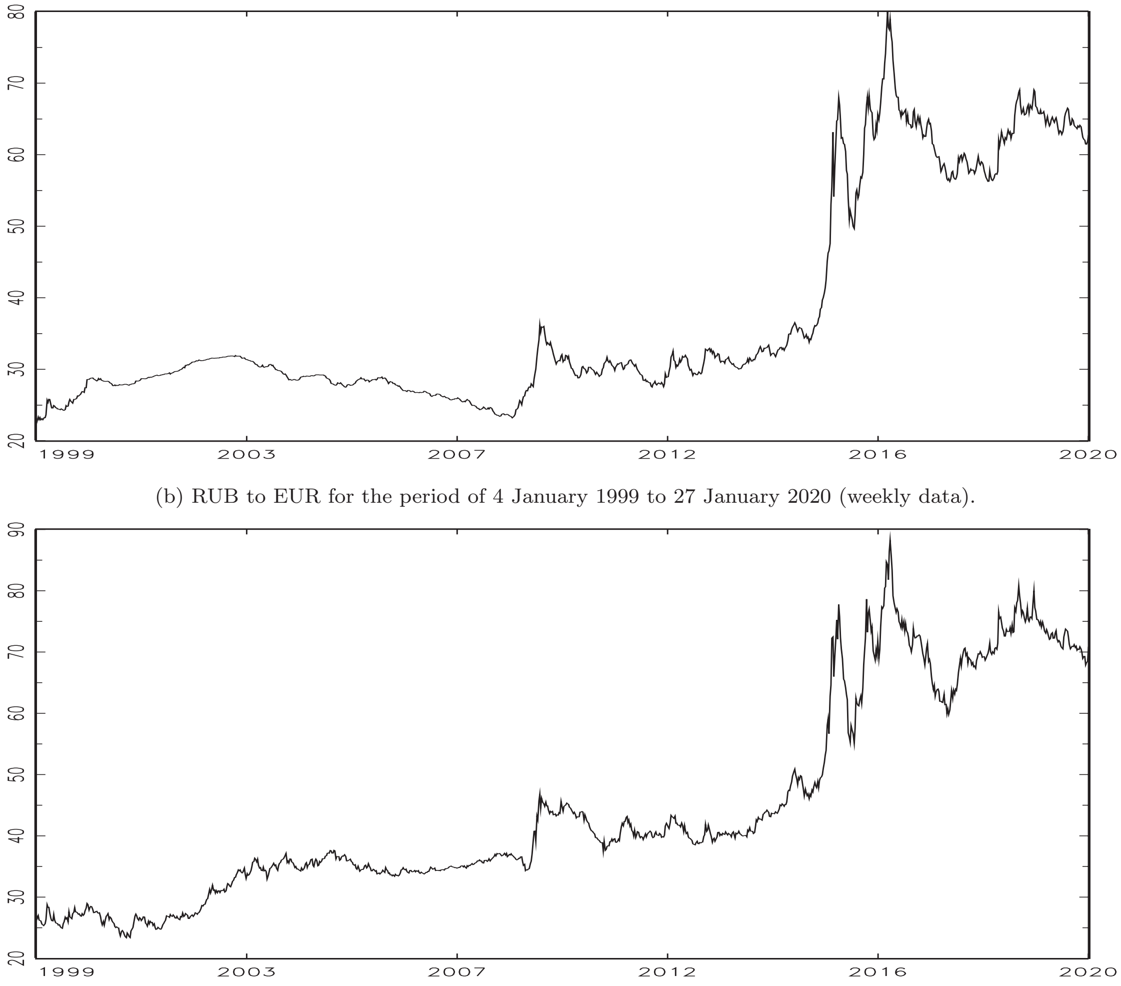
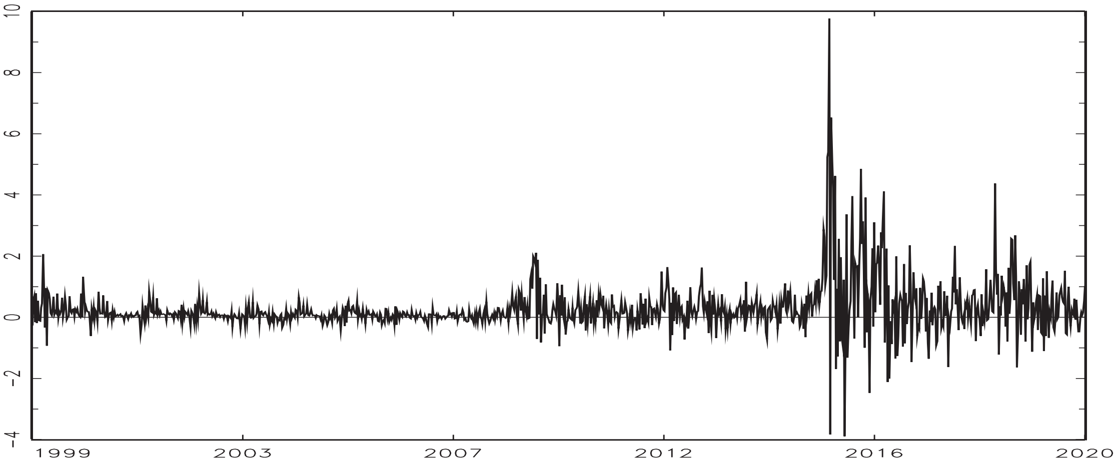
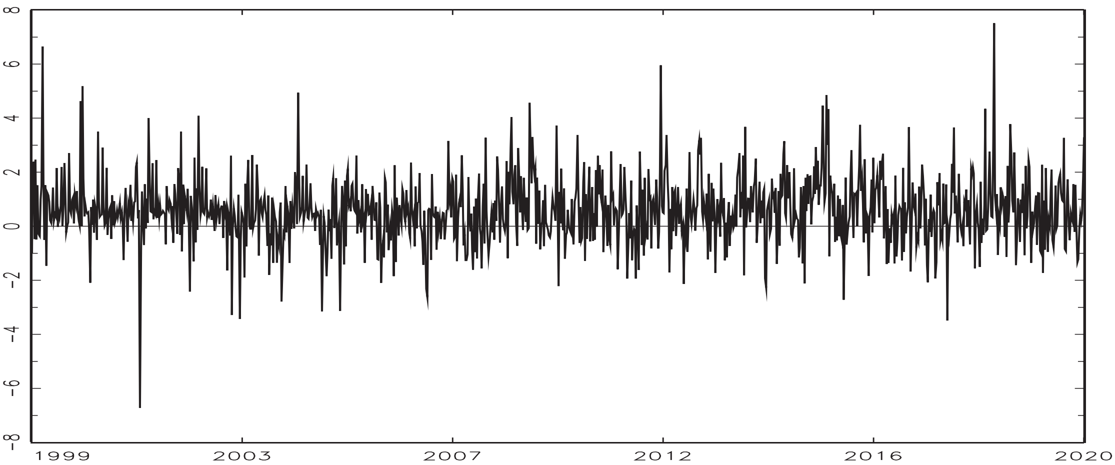


Figure 3. Evolution of the RUB to USD and RUB to EUR exchange rates.

(a) v_t for the period of 4 January 1999 to 27 January 2020 (weekly data).



(b) ϵ_t for the period of 4 January 1999 to 27 January 2020 (weekly data).



(c) s_t/μ_t for the period of 4 January 1999 to 27 January 2020 (weekly data).

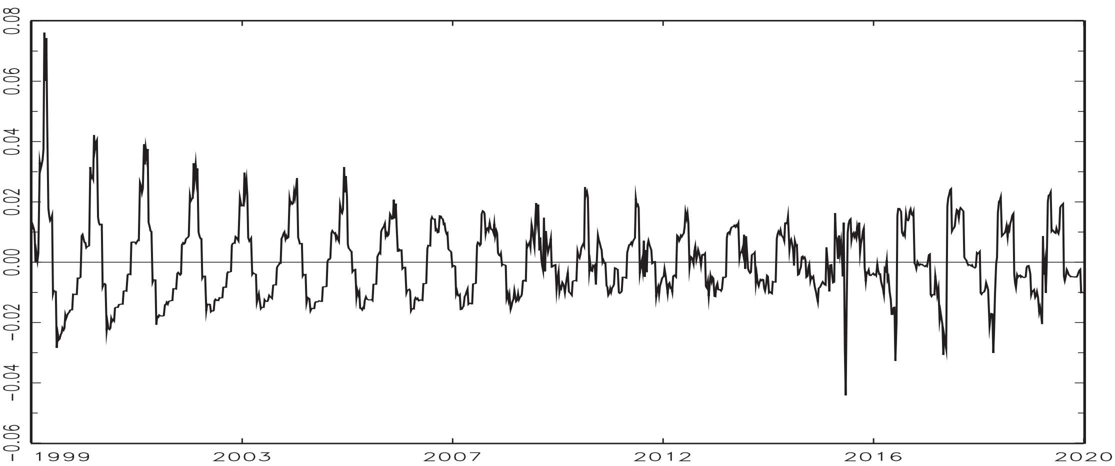
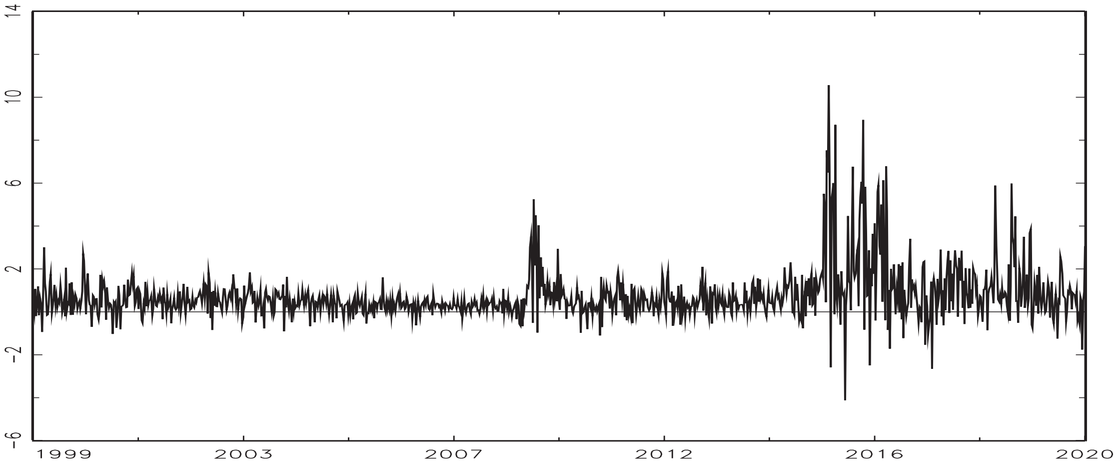
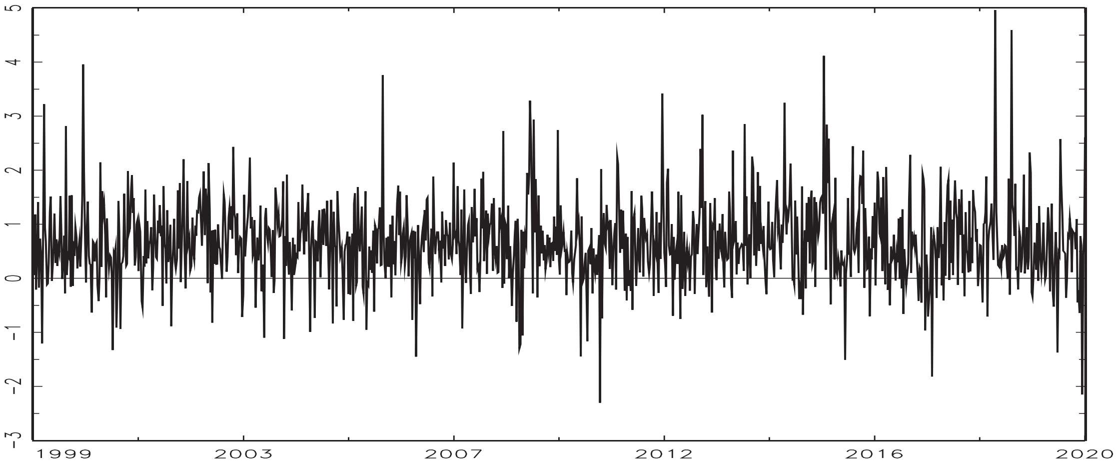


Figure 4. Time series components of the RUB to USD exchange rate for the EGB2 distribution.

(a) v_t for the period of 4 January 1999 to 27 January 2020 (weekly data).



(b) ϵ_t for the period of 4 January 1999 to 27 January 2020 (weekly data).



(c) s_t/μ_t for the period of 4 January 1999 to 27 January 2020 (weekly data).

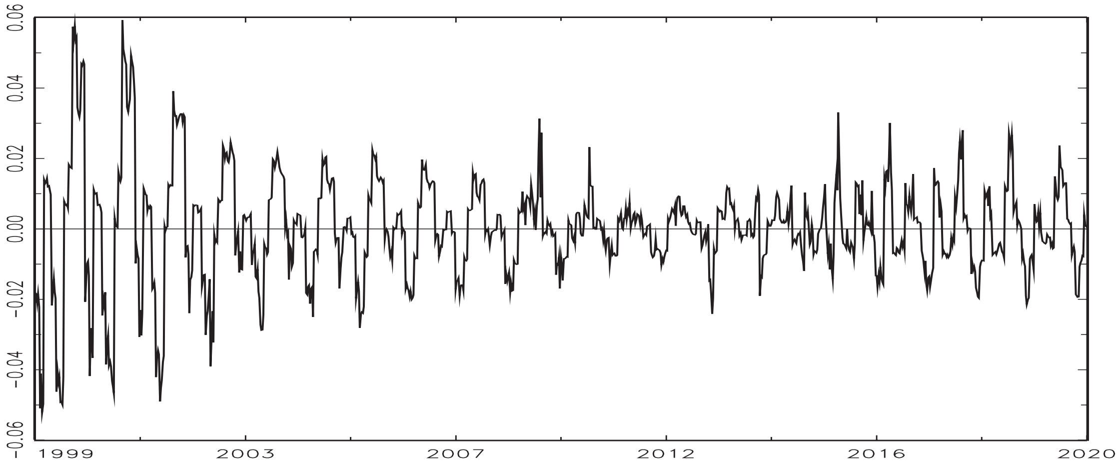


Figure 5. Time series components of the RUB to EUR exchange rate for the EGB2 distribution.

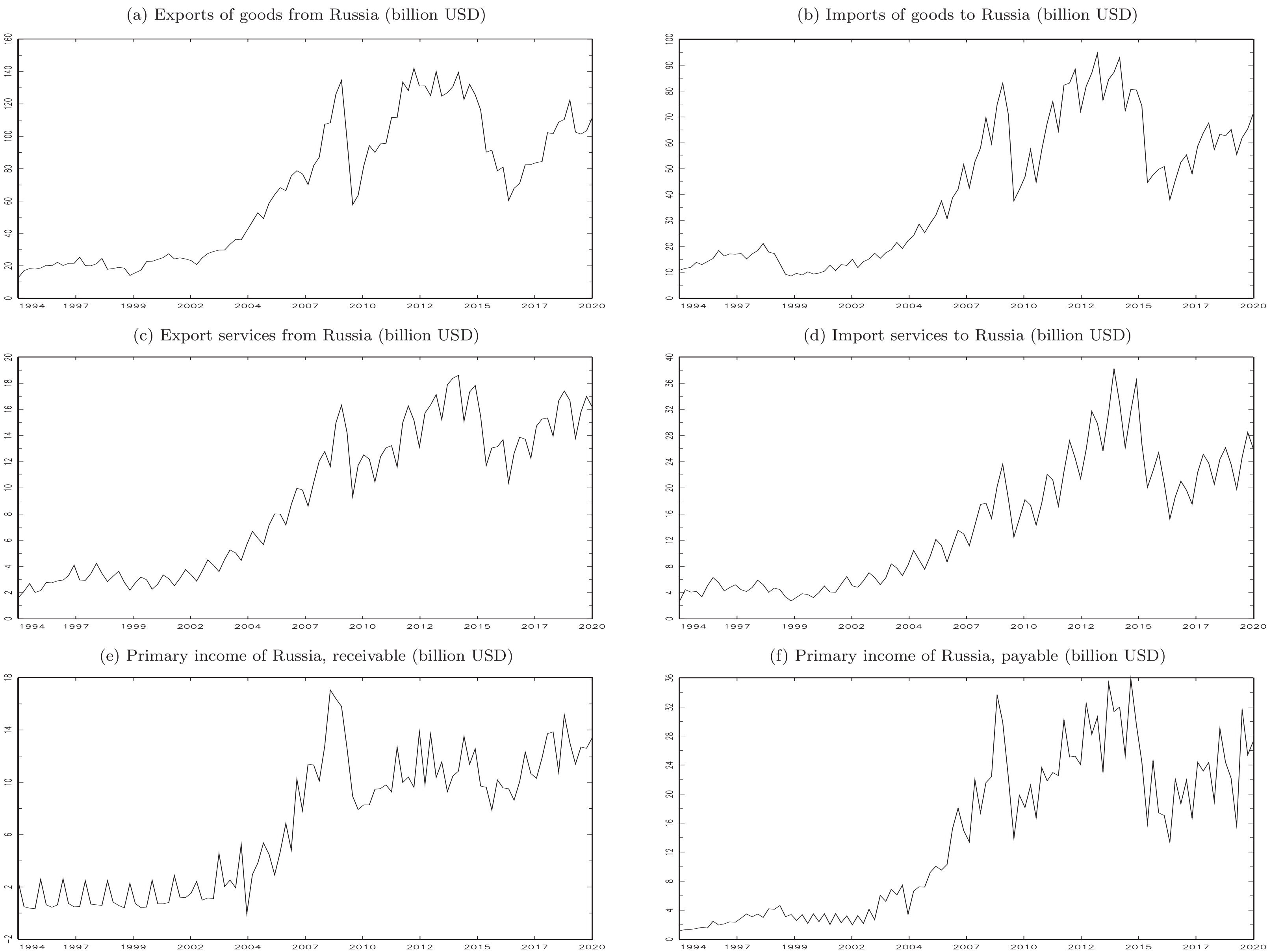


Figure 6. Exports, imports and primary income of the Russian Federation for the period of 1994 Q1 to 2019 Q4 (not seasonally adjusted quarterly data).