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Volatility forecasting for the coronavirus pandemic using quasi-score-driven models

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Abstract:

This is the first empirical study in the literature, in which the statistical and volatility forecasting performances of the recent quasi-score-driven EGARCH (exponential generalized autoregressive conditional heteroscedasticity) models are evaluated. Quasi-score-driven EGARCH models are compared with all relevant score-driven EGARCH models from the literature, and the asymmetric power ARCH (A-PARCH) and GARCH models. The following score-driven distributions are studied: Student's t-distribution; general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian distribution (NIG); Meixner distribution (MXN). All combinations of these distributions are used for (i) the distribution of the dependent variable, and (ii) the distribution which defines the quasi-score function updating term of the quasi-score-driven filters. Daily data are used for the Standard & Poor's 500 (S&P 500) index. We find that quasi-score-driven EGARCH is superior to score-driven EGARCH, A-PARCH, and GARCH. In-sample results are reported for the period of 2000 to 2020, providing evidence in favour of the quasi-score-driven EGARCH model for the last two decades. Out-of-sample forecasting results are reported for the period of the coronavirus pandemic for 2020, providing evidence in favour of the quasi-score-driven EGARCH model for a crisis period.

Keywords: Quasi-score-driven models, coronavirus pandemic, COVID-19, dynamic conditional score, generalized autoregressive score

JEL classification: C22, C51, C52, G17

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1 Introduction

In this paper, empirical results are reported on the statistical performance and volatility forecasting accuracy of the recent quasi-score-driven volatility model (Blasques, Francq, and Laurent 2020). The quasi-score-driven volatility model is an extension of the score-driven volatility model, where the latter model is introduced in the works of Creal, Koopman, and Lucas (2008) and Harvey and Chakravarty (2008). Both models are observation-driven models (Cox 1981): (i) Score-driven models are updated by the partial derivatives of the log conditional density of the dependent variable with respect to a dynamic parameter, and the updating terms are named score functions. (ii) For the quasi-score-driven models of this paper, the updating terms are defined as the partial derivatives of the log conditional density of a distribution, which may be different from the distribution of the dependent variable. The updating terms of the quasi-score-driven models are named quasi-score functions. A score-driven model is obtained as a special case of a corresponding quasi-score-driven model, if the distribution of the dependent variable coincides with the distribution which defines the quasi-score function.

The research question of the present paper is whether the statistical and forecasting performances of the quasi-score-driven volatility models are superior to the statistical and forecasting performances of the score-driven and the classical dynamic volatility models. To the best of our knowledge, this question has not been studied in the body of literature.

We investigate this research question in a general way, to obtain valid conclusions about the performances of quasi-score-driven volatility models. The analysis is general due to the following points:

(i) All relevant probability distributions from the literature of score-driven volatility models are considered, and from those distributions all possible combinations of the probability distribution of the dependent variable, and the probability distribution which defines the quasi-score function, are created. Detailed mathematical formulations for all quasi-score-driven volatility models are presented.

(ii) Three classical volatility models are considered: A-PARCH (asymmetric power autoregressive conditional heteroscedasticity) (Ding et al. 1993), Gaussian-GARCH (generalized ARCH) with leverage effects (Engle 1982; Bollerslev 1986; Glosten, Jagannathan, and Runkle 1993), and t-GARCH with leverage effects (Bollerslev 1987; Glosten, Jagannathan, and Runkle 1993). The consideration of these volatility models is motivated by the work of Hansen and Lunde (2005), in which the volatility forecasting performances of 330 GARCH-type models are compared. The results of Hansen and Lunde (2005) indicate that the volatility forecasting performances of A-PARCH and GARCH with leverage effects (Black 1976) are very difficult to beat by using classical volatility model alternatives.

(iii) Data for the Standard & Poor's 500 (S&P 500) stock index and its realized volatility are used. The former variable is a general representation of the stock market valuation of United States (US) firms, and the latter variable is used as a proxy of true volatility for volatility forecasting performance evaluation. For these two variables, the maximum observation periods which are available from our data sources are crossed, providing the sample period of January 2000 to December 2020.

The practically most relevant empirical application of this paper is an out-of-sample volatility forecasting performance analysis for the period of the coronavirus pandemic. We perform one-step ahead volatility forecasting for the period of January 2020 to December 2020, by using a rolling-window approach of estimation and forecasting. The dynamic volatility models are compared by using the Giacomini–White test of out-of-sample forecasting accuracy (Giacomini and White 2006).

The empirical results indicate that the in-sample statistical and forecasting performances of the quasi-score-models are superior to the in-sample statistical and forecasting performances of the score-driven and the classical dynamic volatility models. The out-of-sample results indicate that, for the highly volatile period of the coronavirus pandemic, the quasi-score-driven models forecast volatility more accurately than the score-driven and the classical dynamic volatility models.

The remainder of this paper is organized as follows: Section 2 reviews the literature. Section 3 presents the quasi-score-driven model. Section 4 presents the statistical inference method. Section 5 presents the empirical results. Section 6 concludes. Technical details are presented in an Appendix.

2 Review of the literature

2.1 Classical dynamic volatility models

In the work of Engle (1982), the ARCH model of dynamic volatility is introduced, which is extended in the works of Bollerslev (1986, 1987) to the Gaussian-GARCH and t-GARCH models, respectively. In the work of Nelson (1991), the EGARCH model is introduced, in which the dynamics of the log conditional variance of returns are formulated, and leverage effects (Black 1976) are included in the log conditional variance equation. In the work of Glosten, Jagannathan, and Runkle (1993), the GARCH model is extended, by including leverage effects in the conditional variance equation. In the work of Ding, Engle, and Granger (1993), the A-PARCH model for a power of the conditional standard derivation is introduced, which generalizes the ARCH and GARCH models, approximates the long memory property of stock returns, and includes leverage effects in the filter driving conditional volatility.

In the work of Hansen and Lunde (2005), empirical evidence is presented for the volatility forecasting performances of the A-PARCH and GARCH models. The subject matter of the present paper is in relation to several classical dynamic volatility models from the body of literature, because the statistical and forecasting performances of quasi-score-driven models are compared with the statistical and forecasting performances of the A-PARCH, Gaussian-GARCH, and t-GARCH models.

2.2 Score-driven volatility models

The first score-driven volatility model in the literature is the Beta-t-EGARCH model (Harvey and Chakravarty 2008), which assumes the Student's t-distribution for financial returns. In relation to Beta-t-EGARCH, we also refer to the works of Creal, Koopman, and Lucas (2013) and Harvey (2013). For the ML estimation of score-driven models, we refer to the theoretical results of the works of Creal, Koopman, and Lucas (2011, 2013), Harvey (2013), Blasques, Koopman, and Lucas (2015, 2017), and Blasques, Gorgi, Lucas, and Wintenberger (2018). In the work of Blasques, Koopman, and Lucas (2015) it is shown for score-driven filters, such as Beta-t-EGARCH, that a score-driven update, asymptotically and in expectation, reduces the Kullback–Leibler divergence in favour of the true data generating process at every step. The authors also show that only score-driven updates have this property. In the work of Blasques, Lucas, and van Vlodrop (2020) simulation-based results are presented, which support the use of the score-driven filters for finite samples.

Alternatives to Beta-t-EGARCH are: GED-EGARCH (general error distribution EGARCH) (Harvey 2013); Beta-Skew-t-EGARCH (skewed t-distribution EGARCH) (Harvey and Sucarrat 2014); EGB2-EGARCH (exponential generalized beta distribution of the second kind EGARCH) (Caivano and Harvey 2014); Beta-Skew-Gen-t-EGARCH (skewed generalized t-distribution EGARCH) (Harvey and Lange 2017); NIG-EGARCH (normal-inverse Gaussian distribution EGARCH) (Blazsek, Ho, and Liu 2018); MXN-EGARCH (Meixner distribution EGARCH) (Blazsek and Haddad 2020).

In the recent work of Blasques, Francq, and Laurent (2020), the class of quasi-score-driven models is introduced, and general theoretical results on the statistical inference of quasi-score-driven models are developed. As an example, the authors present the quasi-score-driven Beta-t-GARCH model (Harvey and Chakravarty), in which the degrees of freedom parameters of the t-distribution of the dependent variable, and the t-distribution which defines the quasi-score function, differ. For the quasi-score-driven Beta-t-GARCH model, theoretical results of statistical inference are presented, Monte Carlo simulation experiments about the small sample properties of the quasi-likelihood (QL) estimator and the ML estimator are shown, and empirical results about statistical performance for 408 stocks from the S&P 500 index are reported. In the present paper, we report extended empirical results on the statistical performances and the volatility forecasting accuracies of a variety of quasi-score-driven EGARCH models for the S&P 500, for which all models are estimated by using the ML method.

3 Quasi-score-driven EGARCH models

The quasi-score-driven EGARCH models are state space models, which are specified by using the measurement equation for the return of a financial asset, and the transition equation for a dynamic variable that drives volatility. The measurement equation for the financial return y_t is:

$$y_t = \exp(\lambda_t)\epsilon_t \tag{1}$$

for $t = 1, \ldots, T$. The constant parameter in equation (1) is set to zero, which has implications on the transformations of the observed return data with respect to risk premium dynamics. Volatility dynamics are specified according to EGARCH for the dynamic scale parameter $\exp(\lambda_t)$, which is conditional on $\mathcal{F}_{t-1} \equiv (y_1, \ldots, y_{t-1}, \Theta)$, where Θ is the vector of time-invariant parameters. The conditional standard deviation of $y_t|\mathcal{F}_{t-1}$ is denoted σ_t, and it is interpreted as the one-step ahead volatility forecast.

The independent and identically distributed (i.i.d.) error term ϵ_t, is specified according to the following alternatives: (i) the Student's t-distribution; (ii) general error distribution (GED) (Harvey 2013); (iii) generalized t-distribution (Gen-t); (iv) skewed generalized t-distribution (Skew-Gen-t) (McDonald and Michelfelder 2017); (v) exponential generalized beta distribution of the second kind (EGB2) (Caivano and Harvey 2014); (vi) normal-inverse Gaussian (NIG) distribution (Barndorff-Nielsen and Halgreen 1977); (vii) the Meixner distribution (MXN) (Schoutens 2002). These distributions are the most relevant ones in the literature of score-driven EGARCH models, which we use in order to reach general conclusions on the in-sample and out-of-sample performances of quasi-score-driven volatility models. The log conditional density of the dependent variable is denoted $\ln f(y_t|\mathcal{F}_{t-1}, \Theta)$.

The transition equation is a quasi-score-driven EGARCH model with leverage effects:

$$\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{t-1} + \alpha^* \mathrm{sgn}(-\epsilon_{t-1})(u_{t-1} + 1) \tag{2}$$

for $t = 2, \ldots, T$, where the time-invariant parameters are ω, β, α, and α^*, and parameter λ_1 is used in order to initialize filter λ_t at $t = 1$. Moreover, $\mathrm{sgn}(\cdot)$ is the signum function that indicates positive or negative unexpected return for the previous period, in order to capture leverage effects.

The updating term in equation (2) is the quasi-score function with respect to λ_t that is defined as $u_t = \partial \ln g(y_t|\mathcal{F}_{t-1}, \Theta)/\partial\lambda_t$, where $g(y_t|y_1, \ldots, y_{t-1}, \Theta) \equiv g(y_t|\mathcal{F}_{t-1}, \Theta)$ is a conditional density function. We show that u_t is i.i.d. with finite variance for all score-driven and quasi-score-driven models of this paper. For $g(y_t|\mathcal{F}_{t-1}, \Theta)$, the same probability distribution alternatives are considered as for $f(y_t|\mathcal{F}_{t-1}, \Theta)$. In the Appendix, for each probability distribution of this paper, the formulas of log-density $\ln f(y_t|\mathcal{F}_{t-1}, \Theta)$, score function u_t, and conditional volatility σ_t are presented.

If the conditional densities $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$ coincide, then a score-driven EGARCH model is obtained. For example, if the Student's t-distribution $t(\nu)$ is selected for both $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$, then the Beta-$t$-EGARCH model with leverage effects (Harvey and Chakravarty 2008; Harvey 2013) is obtained. For the score-driven EGARCH models, Θ includes $(\omega, \beta, \alpha, \alpha^*, \lambda_1)$, and the common shape parameters of $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$.

If $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$ differ, then a quasi-score-driven EGARCH model is obtained. The quasi-score-driven models include the cases when the same distribution is used for $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$, but the shape parameters in those densities differ. For example, $t(\nu)$ is selected for $f(y_t|\mathcal{F}_{t-1}, \Theta)$, and $t(\nu^\dagger)$ is selected for $g(y_t|\mathcal{F}_{t-1})$, where $\nu \neq \nu^\dagger$ (Blasques, Francq, and Laurent 2020). If $\nu = \nu^\dagger$, then the Beta-t-EGARCH model with leverage effects is obtained. Hence, the score-driven models are special cases of the quasi-score-driven models. For the quasi-score-driven EGARCH models, Θ includes $(\omega, \beta, \alpha, \alpha^*, \lambda_1)$, and all shape parameters of $f(y_t|\mathcal{F}_{t-1}, \Theta)$ and $g(y_t|\mathcal{F}_{t-1}, \Theta)$.

In Figure 1, for each probability distribution, we present the score function u_t, as a function of ϵ_t. For each distribution we present the estimate of u_t, by using S&P 500 data for the period of 3 January 2000 to 14 December 2020. By interpreting Figure 1, in the following we show that the variance of u_t is finite: For the Student's t, Gen-t, and Skew-Gen-t distributions, u_t is a continuous and bounded function of ϵ_t. Hence, all moments of u_t are finite for the Student's t, Gen-t, and Skew-Gen-t distributions. For the GED, all moments of ϵ_t are finite, and u_t increases at a lower rate than ϵ_t^2 as $|\epsilon_t| \to \infty$. Hence, the moments of u_t at least up to the second moment are finite. For the EGB2, NIG, and MXN distributions, all moments of ϵ_t are finite, and u_t increases linearly as $|\epsilon_t| \to \infty$. Hence, the moments of u_t at least up to the second moment are finite. An important further result is that u_t is a continuous function of the i.i.d. ϵ_t error term for all distributions. Hence, u_t is also i.i.d. for all score-driven and quasi-score-driven models (White 2001).

In a first step, we estimated equation (2) under the restriction $\alpha^* = 0$, because some distributions of this paper which define u_t are skewed probability distributions. Those distributions make a difference between positive and negative unexpected returns within the error term. Nevertheless, the empirical results for $\alpha^* = 0$ and $\alpha^* \neq 0$ for the S&P 500 indicate that, in addition to the use of a skewed probability distribution for ϵ_t, the use of the leverage effects term greatly improves the volatility forecasting accuracy. Therefore, all results of this paper are estimated according to equation (2).

By using the quasi-score-driven EGARCH model, general conclusions can be obtained about the research question of this paper: Are the in-sample statistical and out-of-sample forecasting performances of quasi-score-driven EGARCH models superior to the in-sample statistical and out-of-sample forecasting performances of score-driven EGARCH and classical volatility models?

[APPROXIMATE LOCATION OF FIGURE 1]

4 Statistical inference

The parameters of the quasi-score-driven models are estimated by using the ML method (Wooldridge 1994; Davidson and MacKinnon 2004):

$$\hat{\Theta} = \arg\max_{\Theta} \mathrm{LL}(y_1, \ldots, y_T|\Theta) = \arg\max_{\Theta} \sum_{t=1}^{T} \ln f(y_t|\mathcal{F}_{t-1}, \Theta) \tag{3}$$

For theoretical results on ML, we refer to the work of Blasques, Francq, and Laurent (2020). The maximization of equation (3) is performed numerically for all quasi-score-driven models. Alternative sets of initial values of parameters are used for the estimation of each model, to find the global maximum of the likelihood function. The computer codes are available from the authors upon request.

5 Empirical results

5.1 Data

Daily S&P 500 index s_t data are used for the period of 3 January 2000 to 14 December 2020 (source of data: Yahoo Finance). Daily S&P 500 log-returns are $\tilde{y}_t = 100 \times \ln(s_t/s_{t-1})$ for $t = 1, \ldots, T$ (in % points), and s_0 is from pre-sample data. Descriptive statistics of $\tilde{y}_t$ are presented in Table 1-A.

For the quasi-score-driven models the constant parameter in equation (1) is set to zero. Therefore, alternative ARMA (AR moving average) specifications are estimated in a first step for $\tilde{y}_t$. For lag-order selection, we use the specific to general approach and the Bayesian information criterion (BIC) (Davidson and MacKinnon 2004) (Table 1-B). According to the results, the AR(2) specification provides the lowest BIC. Hence, the dependent variable of all volatility models of this paper is defined by the residuals of the AR(2) model, and those residuals, named unexpected returns, are denoted y_t. Descriptive statistics of y_t are presented in Table 1-C. From Table 1-A and Table 1-C, we highlight that the serial correlation is changed from -0.1130 for $\tilde{y}_t$ to 0.0001 for y_t. The evolution of S&P 500 log-returns $\tilde{y}_t$ and unexpected returns y_t are presented in Figures 2-A and 2-B, respectively.

The use of y_t, instead of the observed $\tilde{y}_t$, has the following advantages for model performance comparison: (i) Mean dynamics are modelled in the same way for all volatility models. Hence, the volatility forecasting performances of those models are more comparable. As alternatives to y_t, we also used conditional mean models with AR and QAR (quasi-AR) (Harvey 2013) risk premium specifications for $\tilde{y}_t$, but the volatility forecasting performances of those models were inferior to the volatility forecasting performances of the specifications of this paper. (ii) Each econometric model includes only one dynamic equation, which reduces the numerical problems with the numerical ML estimation procedures. The robust estimation procedures are needed for the out-of-sample volatility forecasting procedures of this paper, which involve hundreds of estimations for each model. (iii) The theoretical results on the ML estimator of the work of Blasques, Francq, and Laurent (2020) can be used for the quasi-score-driven volatility models of the present paper, but those conditions are not valid for quasi-score-driven risk premium plus quasi-score-driven volatility models.

[APPROXIMATE LOCATION OF TABLE 1 AND FIGURE 2]

5.2 Statistical performance

The statistical performances of the volatility models are compared, by using the following likelihood-based model selection metrics: LL, Akaike information criterion (AIC), BIC, Hannan–Quinn criterion (HQC) (Davidson and MacKinnon 2004). The use of AIC, BIC, and HQC for the selection of score-driven models is suggested in the work of Harvey (2013, p. 75). The estimates of LL, AIC, BIC, and HQC for the period of 3 January 2000 to 14 December 2020 are presented in Tables 2-A and 2-B.

The main conclusions of the in-sample analysis of statistical performances are the following: (i) For each probability distribution, there are some quasi-score-driven models which have better statistical performances than the score-driven model. This is indicated by bold numbers in Tables 2-A and 2-B. (ii) For the classical volatility models, the statistical performance of the t-GARCH with leverage effects model is superior to the statistical performances of the A-PARCH and Gaussian-GARCH with leverage effects models. (iii) There are quasi-score-driven models with better statistical performances than the t-GARCH with leverage effects model. This is indicated by using ** in Tables 2-A and 2-B for the best-performing model from all specifications, which is always a quasi-score-driven model.

[APPROXIMATE LOCATION OF TABLE 2]

5.3 Prediction accuracy

One-step ahead forecasts of volatility σ_t are compared with a proxy of true volatility. Prediction accuracies are compared by using the following loss functions (Hansen and Lunde 2005; Patton 2011):

$$\begin{aligned} &\mathrm{MSE}_{1,i,t} = (\sigma_t^* - \sigma_{i,t})^2 && \mathrm{MSE}_{2,i,t} = [(\sigma_t^*)^2 - \sigma_{i,t}^2]^2 \\ &\mathrm{QLIKE}_{i,t} = \left\{ \frac{(\sigma_t^*)^2}{\sigma_{i,t}^2} - \ln\left[\frac{(\sigma_t^*)^2}{\sigma_{i,t}^2}\right] - 1 \right\} && \mathrm{R^2LOG}_{i,t} = \left\{ \ln\left[\frac{(\sigma_t^*)^2}{\sigma_{i,t}^2}\right] \right\}^2 \\ &\mathrm{MAE}_{1,i,t} = |\sigma_t^* - \sigma_{i,t}| && \mathrm{MAE}_{2,i,t} = |(\sigma_t^*)^2 - \sigma_{i,t}^2| \end{aligned} \tag{4}$$

for model i and for each period of the forecasting window $t = 1, \dots, T_f$, where σ_t^* is a proxy of true volatility. In this paper, the square root of realized variance of daily S&P 500 returns is used for σ_t^* (source of data: Oxford-Man Institute of Quantitative Finance (OMI), https://realized.oxford-man.ox.ac.uk/data/download). From the realized variance data file of OMI, variable 'rv5' is used, as in the work of Harvey and Lange (2018). The realized variance data for the S&P 500 are available from January 2000, which determines the time period of the full sample of the present paper.

In-sample results—The estimates of the mean loss functions for the period of 1 February 2000 to 14 December 2020 are presented in Tables 3-A and 3-B. In-sample one-step ahead forecasting of volatility is performed, by using the parameter estimates for the period of 3 January 2000 to 14 December 2020. The first month of the observations, i.e. January 2000, is excluded from the forecasting window, to reduce the negative effects of the initialization of the volatility filter on parameter estimation precision.

In Tables 3-A and 3-B, each panel presents the forecasting accuracy for a particular distribution of the dependent variable. In each panel, the mean loss functions of the score-driven model are presented in the first line, and the mean loss functions of the quasi-score-driven models are presented in the remaining lines. In the last panel of Tables 3-A and 3-B, the mean loss functions of the A-PARCH, Gaussian-GARCH with leverage effects, and t-GARCH with leverage effects models are presented.

The main conclusions of in-sample forecasting accuracy are the following: (i) For each distribution, there are quasi-score-driven models which provide more accurate volatility forecasts than the score-driven model. This is indicated by bold numbers in Tables 3-A and 3-B. (ii) There are quasi-score-driven models, which forecast more precisely than the classical A-PARCH, Gaussian-GARCH with leverage effects, and t-GARCH with leverage effects models. This is indicated by using ** in Tables 3-A and 3-B for the best-performing model from all specifications, which is always a quasi-score-driven model.

Out-of-sample results—We use rolling windows for one-step ahead out-of-sample volatility forecasting. The initial data window is for the period of 3 January 2000 to 31 December 2019 (5,031 trading days). The forecasting window is for the period of 2 January 2020 to 14 December 2020 (241 trading days). The selection of this forecasting window is motivated by the period of the coronavirus pandemic, during which the volatility of the stock markets have significantly increased. In this paper, one of the objectives is to study the forecasting precision of the quasi-score-driven models for crisis periods.

All models are estimated for the initial data window, and a one-step ahead volatility forecast is estimated for each model for trading day that follows the data window. Then, the first observation of the data window is excluded from the data window, and a new last observation is added to the data window. All models are estimated for the new data window, and a new one-step ahead volatility forecast is estimated for each model. This rolling-window procedure, for which the data window always includes 5,031 observations, is repeated until the end of the forecasting window. Descriptive statistics of y_t for the out-of-sample forecasting window are presented in Table 1-D.

In Tables 4-A and 4-B, each panel presents the forecasting accuracy for a particular probability distribution of the dependent variable. In each panel, the mean loss functions of the score-driven model are presented in the first line, and the mean loss functions of the quasi-score-driven models are presented in the remaining lines. In the last panel of Tables 4-A and 4-B, the mean loss functions of the A-PARCH, Gaussian-GARCH with leverage effects, and t-GARCH with leverage effects models are presented, which shows that the mean loss function estimate of A-PARCH is lower than the mean loss function estimates of the Gaussian-GARCH with leverage effects, and t-GARCH with leverage effects.

To study the differences between the loss functions of quasi-score-driven and score-driven models, and the differences between the loss functions of quasi-score-driven and classical volatility models, we use the Giacomini–White test. In each panel of score-driven models in Tables 4-A and 4-B, the statistical significance of the Giacomini–White test statistic is added to those mean loss function estimates of quasi-score-driven models, which provide significantly more precise volatility forecasts than the score-driven model. In addition, in the last panel of Tables 4-A and 4-B, the forecasting accuracy of A-PARCH is compared with the forecasting accuracy of the best-performing quasi-score-driven model (that quasi-score-driven model is indicated by bold mean loss function numbers).

The main conclusions of the out-of-sample analysis are the following: (i) For each probability distribution, there are several quasi-score-driven models which provide significantly more accurate volatility forecasts than the score-driven model. (ii) There are quasi-score-driven models, which forecast volatility significantly more precisely than the A-PARCH model. (iii) By comparing the superior forecasting accuracy of the quasi-score-driven EGARCH model with the forecasting accuracies of the score-driven EGARCH and classical volatility models, Tables 3 and 4 indicate that the superior forecasting performance of quasi-score-driven EGARCH model is much clearer for the period of the coronavirus pandemic than for the two-decade period of 2000 to 2020. The latter result supports the use of the quasi-score-driven volatility models for volatility forecasting during crisis periods.

[APPROXIMATE LOCATION OF TABLES 3-4]

6 Conclusions

Score-driven models are among the most important contributions to the literature of time series econometrics in the past decade, with more than 200 publications in academic journals until the date of this paper. Quasi-score-driven models are recent extensions of score-driven models, which may improve the statistical and forecasting performances of score-driven models. This issue has been the subject matter of the present empirical study. In particular, we have compared the statistical and volatility forecasting performances of quasi-score-driven, score-driven, and some classical dynamic volatility models. All relevant probability distributions from the literature of score-driven models have been considered, i.e. the Student's t, GED, Gen-t, Skew-Gen-t, EGB2, NIG, and MXN distributions. We have used daily log-return data for the S&P 500 index, which represents stock market returns in the US.

We have compared in-sample statistical and volatility forecasting performances for the period of January 2000 to December 2020. We have compared the out-of-sample volatility forecasting performances for the period of January 2020 to December 2020, which includes the period of the coronavirus pandemic. The empirical results have supported that the quasi-score-driven volatility models, for all probability distributions, are superior to the score-driven volatility models and the classical dynamic volatility models. Our results motivate the practical use of the quasi-score-driven models for volatility forecasting, especially for crisis periods such as the period of the coronavirus pandemic.

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Appendix

Student's t-distribution—The conditional distribution of return y_t is the non-standardized Student's t-distribution $y_t|(\mathcal{F}_{t-1}, \Theta) \sim t[0, \exp(\lambda_t), \exp(\delta_1)]$, where the degrees of freedom is $\exp(\delta_1)$ (Harvey and Chakravarty; Harvey 2013). The conditional volatility of y_t is $\sigma_t = \exp(\lambda_t)\{\exp(\delta_1)/[\exp(\delta_1) - 2]\}^{1/2}$. The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1}, \Theta) = \ln \Gamma\left[\frac{\exp(\delta_1) + 1}{2}\right] - \ln \Gamma\left[\frac{\exp(\delta_1)}{2}\right] \tag{A.1}$$

$$-\frac{\ln[\exp(\delta_1)\pi]}{2} - \lambda_t - \frac{\exp(\delta_1) + 1}{2} \ln\left\{1 + \frac{\epsilon_t^2}{\exp(\delta_1)}\right\}$$

where $\epsilon_t = y_t \exp(-\lambda_t)$. The score function with respect to λ_t is:

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1}, \Theta)}{\partial \lambda_t} = \frac{[\exp(\delta_1) + 1]\epsilon_t^2}{\exp(\delta_1) + \epsilon_t^2} - 1 \tag{A.2}$$

GED—The conditional distribution of y_t is the non-standardized GED distribution, denoted by $y_t|(\mathcal{F}_{t-1}, \Theta) \sim \text{GED}[0, \exp(\lambda_t), \exp(\delta_1)]$ (Harvey 2013). The conditional volatility of y_t is

$$\sigma_t = \exp(\lambda_t) 2^{\exp(-\delta_1)} \times \left\{\frac{\Gamma[3\exp(-\delta_1)]}{\Gamma[\exp(-\delta_1)]}\right\}^{1/2} \tag{A.3}$$

The log conditional probability density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1}, \Theta) = -[1 + \exp(-\delta_1)]\ln(2) - \lambda_t - \ln\Gamma[1 + \exp(-\delta_1)] - \frac{1}{2}|\epsilon_t|^{\exp(\delta_1)} \tag{A.4}$$

where $\epsilon_t = y_t \exp(-\lambda_t)$. The score function with respect to λ_t is

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1}, \Theta)}{\partial \lambda_t} = \frac{\exp(\delta_1)}{2}|\epsilon_t|^{\exp(\delta_1)} - 1 \tag{A.5}$$

Gen-t-distribution—The conditional distribution of y_t is the non-standardized Get-t distribution that is $y_t|(\mathcal{F}_{t-1}, \Theta) \sim \text{Gen}-t[0, \exp(\lambda_t), \exp(\delta_1), \exp(\delta_2)]$, where $\exp(\lambda_t)$ is the scale parameter, $\exp(\delta_1)$ is the degrees of freedom, and $\exp(\delta_2)$ is the peakedness parameter (Harvey and Sucarrat 2014; Harvey and Lange 2017). The conditional volatility of y_t is:

$$\sigma_t = \exp(\lambda_t)[\exp(\delta_1)]^{\exp(-\delta_2)} \times \left\{\frac{\Gamma\left[\frac{3}{\exp(\delta_2)}\right]\Gamma\left[\frac{\exp(\delta_1)-2}{\exp(\delta_2)}\right]}{\Gamma\left[\frac{1}{\exp(\delta_2)}\right]\Gamma\left[\frac{\exp(\delta_1)}{\exp(\delta_2)}\right]}\right\}^{1/2} \tag{A.6}$$

respectively. The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1}, \Theta) = \delta_2 - \lambda_t - \ln(2) - \frac{\delta_1}{\exp(\delta_2)} - \ln\Gamma\left[\frac{\exp(\delta_1)}{\exp(\delta_2)}\right] \tag{A.7}$$

$$-\ln\Gamma\left[\frac{1}{\exp(\delta_2)}\right]+\ln\Gamma\left[\frac{\exp(\delta_1)+1}{\exp(\delta_2)}\right]-\frac{\exp(\delta_1)+1}{\exp(\delta_2)}\ln\left[1+\frac{|\epsilon_t|^{\exp(\delta_2)}}{\exp(\delta_1)}\right]$$

where $\epsilon_t = y_t\exp(-\lambda_t)$. The score function with respect to λ_t is

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\lambda_t} = \frac{|\epsilon_t|^{\exp(\delta_2)}[\exp(\delta_1)+1]}{|\epsilon_t|^{\exp(\delta_2)}+\exp(\delta_1)} - 1 \tag{A.8}$$

Skew-Gen-t distribution—The conditional distribution of the unexpected return is

$$y_t|(\mathcal{F}_{t-1},\Theta) \sim \text{Skew-Gen-}t[0,\exp(\lambda_t),\tanh(\delta_1),\exp(\delta_2),\exp(\delta_3)] \tag{A.9}$$

where $\exp(\lambda_t)$ is the scale parameter, $\tanh(\delta_1)$ is the skewness parameter, $\exp(\delta_2)$ is the degrees of freedom parameter, and $\exp(\delta_3)$ is the shape parameter that sets the peakedness of the probability distribution (Harvey and Sucarrat 2014; Harvey and Lange 2017). The conditional volatility of y_t is

$$\sigma_t = \exp(\lambda_t)[\exp(\delta_2)]^{\exp(-\delta_3)}\times \tag{A.10}$$

$$\times\left\{\frac{[3\tanh^2(\delta_1)+1]B\left[\frac{3}{\exp(\delta_3)},\frac{\exp(\delta_2)-2}{\exp(\delta_3)}\right]}{B\left[\frac{1}{\exp(\delta_3)},\frac{\exp(\delta_2)}{\exp(\delta_3)}\right]}-\frac{4\tanh^2(\delta_1)B^2\left[\frac{2}{\exp(\delta_3)},\frac{\exp(\delta_2)-1}{\exp(\delta_3)}\right]}{B^2\left[\frac{1}{\exp(\delta_3)},\frac{\exp(\delta_2)}{\exp(\delta_3)}\right]}\right\}^{1/2}$$

where $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function. The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1},\Theta) = \delta_3-\lambda_t-\ln(2)-\frac{\delta_2}{\exp(\delta_3)}-\ln\Gamma\left[\frac{\exp(\delta_2)}{\exp(\delta_3)}\right] \tag{A.11}$$

$$-\ln\Gamma\left[\frac{1}{\exp(\delta_3)}\right]+\ln\Gamma\left[\frac{\exp(\delta_2)+1}{\exp(\delta_3)}\right]$$

$$-\frac{\exp(\delta_2)+1}{\exp(\delta_3)}\ln\left\{1+\frac{|\epsilon_t|^{\exp(\delta_3)}}{[1+\tanh(\delta_1)\mathrm{sgn}(\epsilon_t)]^{\exp(\delta_3)}\times\exp(\delta_2)}\right\}$$

where $\epsilon_t = y_t\exp(-\lambda_t)$. The score function with respect to λ_t is

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\lambda_t} = \frac{|\epsilon_t|^{\exp(\delta_3)}[\exp(\delta_2)+1]}{|\epsilon_t|^{\exp(\delta_3)}+[1+\tanh(\delta_1)\mathrm{sgn}(\epsilon_t)]^{\exp(\delta_3)}\exp(\delta_2)} - 1 \tag{A.12}$$

EGB2 distribution—The conditional distribution of the unexpected return is

$$y_t|(\mathcal{F}_{t-1},\Theta) \sim \text{EGB2}[0,\exp(-\lambda_t),\exp(\delta_1),\exp(\delta_2)] \tag{A.13}$$

where $\exp(\lambda_t)$ is the scale parameter, and $\exp(\delta_1)$ and $\exp(\delta_2)$ are two shape parameters (Caivano and Harvey 2014). The conditional volatility of y_t is $\sigma_t = \exp(\lambda_t)\{\Psi^{(1)}[\exp(\delta_1)]+\Psi^{(1)}[\exp(\delta_2)]\}^{1/2}$, where

$\Psi^{(j)}(\cdot)$ is the polygamma function of order j. The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1},\Theta) = \exp(\delta_1)\epsilon_t - \lambda_t - \ln\Gamma[\exp(\delta_1)] - \ln\Gamma[\exp(\delta_2)] \tag{A.14}$$

$$+\ln\Gamma[\exp(\delta_1)+\exp(\delta_2)] - [\exp(\delta_1)+\exp(\delta_2)]\ln[1+\exp(\epsilon_t)]$$

where $\epsilon_t = y_t\exp(-\lambda_t)$. The score function with respect to λ_t is

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\lambda_t} = [\exp(\delta_1)+\exp(\delta_2)]\frac{\epsilon_t\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\delta_1)\epsilon_t - 1 \tag{A.15}$$

NIG distribution—The conditional distribution of the unexpected return y_t is

$$y_t|(\mathcal{F}_{t-1},\Theta) \sim \text{NIG}[0,\exp(\lambda_t),\exp(\delta_1-\lambda_t),\exp(\delta_1-\lambda_t)\tanh(\delta_2)] \tag{A.16}$$

where $\exp(\lambda_t)$ is the scale parameter, and $\exp(\delta_1)$ and $\exp(\delta_2)$ are two shape parameters (Blazsek, Ho, and Liu 2018). The conditional volatility of y_t is

$$\sigma_t = \frac{\exp[(\lambda_t-\delta_1)/2]}{[1-\tanh^2(\delta_2)]^{3/4}} \tag{A.17}$$

The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1},\Theta) = \delta_1 - \lambda_t - \ln(\pi) + \exp(\delta_1)[1-\tanh^2(\delta_2)]^{1/2} \tag{A.18}$$

$$+\exp(\delta_1)\tanh(\delta_2)\epsilon_t + \ln K^{(1)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right] - \frac{1}{2}\ln(1+\epsilon_t^2)$$

where $K^{(j)}(\cdot)$ is the modified Bessel function of the second kind of order j, and $\epsilon_t = y_t\exp(-\lambda_t)$. The score function with respect to λ_t is

$$u_t = \frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\lambda_t} = -1 - \exp(\delta_1)\tanh(\delta_2)\epsilon_t + \frac{\epsilon_t^2}{1+\epsilon_t^2} \tag{A.19}$$

$$+\frac{\exp(\delta_1)\epsilon_t^2}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\delta_1)\sqrt{1+\epsilon_t^2}\right]}$$

MXN distribution—The conditional distribution of the unexpected return y_t is

$$\epsilon_t \sim \text{MXN}[0,1,\pi\tanh(\delta_1),\exp(\delta_2)] \tag{A.20}$$

where δ_1 and δ_2 are shape parameters (Blazsek and Haddad 2020). The conditional volatility of y_t is

$$\sigma_t = \left\{\frac{\exp(\lambda_t+\delta_2)}{\cos[\pi\tanh(\delta_1)]+1}\right\}^{1/2} \tag{A.21}$$

The log conditional density of y_t is

$$\ln f(y_t|\mathcal{F}_{t-1},\Theta) = -\lambda_t + 2\exp(\delta_2)\ln\{2\cos[\pi\tanh(\delta_1)/2]\} - \ln(2\pi) \tag{A.22}$$
$$-\ln\Gamma\{2\exp(\delta_2)\} + \pi\tanh(\delta_1)\frac{y_t-\mu_t}{\exp(\lambda_t)} + 2\ln\left|\Gamma\left[\exp(\delta_2) + i\frac{y_t-\mu_t}{\exp(\lambda_t)}\right]\right|$$

where $\epsilon_t = (y_t - \mu_t)\exp(-\lambda_t)$, $\cos(\cdot)$ is the cosine function, $\tanh(\cdot)$ is the hyperbolic tangent function, and i is the imaginary unit. We define $g(\lambda_t) = \Gamma[\exp(\delta_2) + i(y_t - \mu_t)\exp(-\lambda_t)]$, for which $\partial \ln|g(\lambda_t)|/\partial\lambda_t = \mathrm{Re}[g'(\lambda_t)/g(\lambda_t)]$, where $\lambda_t \in \mathbb{R}$ and $\mathrm{Re}(\cdot)$ is the real part of a complex number. Since $\Gamma'(\cdot) = \Gamma(\cdot)\Psi^{(0)}(\cdot)$, the score function with respect to λ_t is:

$$\frac{\partial \ln f(y_t|\mathcal{F}_{t-1},\Theta)}{\partial\lambda_t} = u_t = 2\mathrm{Re}\left\{-i\epsilon_t\Psi^{(0)}[\exp(\delta_2) + i\epsilon_t]\right\} - \pi\tanh(\delta_1)\epsilon_t - 1 \tag{A.23}$$

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Table 1: Descriptive statistics of daily S&P 500 returns (% points).

A. Descriptive statistics (full sample window)	Log-return $\tilde{y}_t$	Absolute log-return $\|\tilde{y}_t\|$
Start date	3 January 2000	3 January 2000
End date	14 December 2020	14 December 2020
Sample size T	$5,272$	$5,272$
Minimum	-12.7652	0.0000
Maximum	10.9572	12.7652
Average	0.0172	0.8189
Standard deviation	1.2564	0.9529
Skewness	-0.3922	3.5094
Excess kurtosis	10.9341	21.9943
Corr$(\tilde{y}_t, \tilde{y}_{t-1})$	-0.1130	
Corr$(\|\tilde{y}_t\|, \|\tilde{y}_{t-1}\|)$	0.3079	
Corr$(\tilde{y}_t, \|\tilde{y}_{t-1}\|)$	0.0257	
Corr$(\|\tilde{y}_t\|, \tilde{y}_{t-1})$	-0.1320	
B. BIC for different ARMA(p,q) specifications of $\tilde{y}_t$ (full sample window)		
AR(1)	17313.2521	
AR(2)	**17307.6162**	
AR(3)	17312.9131	
ARMA(1,1)	17328.4848	
ARMA(2,1)	17324.7317	
ARMA(3,1)	17343.5343	
C. Descriptive statistics of AR(2) residuals (full sample window)	Unexpected return y_t	Absolute unexpected return $\|y_t\|$
Start date	3 January 2000	3 January 2000
End date	14 December 2020	14 December 2020
Sample size T	$5,272$	$5,272$
Minimum	-11.9269	0.0000
Maximum	10.6691	11.9269
Average	0.0000	0.8153
Standard deviation	1.2468	0.9433
Skewness	-0.5395	3.4279
Excess kurtosis	10.3337	20.8033
Corr(y_t, y_{t-1})	0.0001	
Corr$(\|y_t\|, \|y_{t-1}\|)$	0.2951	
Corr$(y_t, \|y_{t-1}\|)$	0.0229	
Corr$(\|y_t\|, y_{t-1})$	-0.1311	
D. Descriptive statistics (forecasting window)	Unexpected return y_t	Absolute unexpected return $\|y_t\|$
Start date	2 January 2020	2 January 2020
End date	14 December 2020	14 December 2020
Number of forecasts T_f	241	241
Minimum	-11.9269	0.0152
Maximum	8.5315	11.9269
Average	0.0335	1.3638
Standard deviation	2.1736	1.6906
Skewness	-1.0347	3.0557
Excess kurtosis	7.8706	12.1340

Bayesian information criterion (BIC); correlation coefficient (corr). The daily closing value of the Standard & Poor's 500 (S&P 500) index s_t is observed. Log-return is defined by $\tilde{y}_t = \ln(s_t/s_t)$ for $t = 1, \ldots, T$ where s_0 is given by pre-sample data. The following ARMA(p,q) models for $p = 1, 2, 3$ and $q = 0, 1$ are estimated: $\tilde{y}_t = c + \sum_{i=1}^{p} \phi_i \tilde{y}_{t-j} + \sum_{j=0}^{q} \theta_j v_{t-j} + y_t$. Lag-order selection is done by using the specific to general procedure. The bold BIC number indicates the best-performing ARMA specification. In all volatility modes of this paper, y_t is used as a dependent variable.

Table 2-A: In-sample statistical performance for the period of 3 January 2000 to 14 December 2020.

Model	Distribution	LL	AIC	BIC	HQC
Score	t	−1.3385	2.6792	2.6867	2.6818
Quasi-score	t	**−1.3384**	2.6794	2.6882	2.6825
Quasi-score	GED	**−1.3355**	**2.6737**	**2.6825**	**2.6768**
Quasi-score	Gen-t	**−1.3355**	**2.6741**	**2.6841**	**2.6776**
Quasi-score	Skew-Gen-t	**−1.3355**	**2.6745**	**2.6857**	**2.6784**
Quasi-score	EGB2	**−1.3332***	**2.6694***	**2.6794****	**2.6729****
Quasi-score	NIG	**−1.3365**	**2.6760**	**2.6859**	**2.6795**
Quasi-score	MXN	**−1.3350**	**2.6730**	**2.6830**	**2.6765**
Model	Distribution	LL	AIC	BIC	HQC
Score	GED	−1.3448	2.6920	2.6994	2.6946
Quasi-score	t	**−1.3404**	2.6834	**2.6921**	**2.6864**
Quasi-score	GED	**−1.3446**	**2.6920**	2.7007	2.6950
Quasi-score	Gen-t	**−1.3374***	**2.6779***	**2.6878***	**2.6814***
Quasi-score	Skew-Gen-t	**−1.3374**	**2.6782**	**2.6895**	**2.6822**
Quasi-score	EGB2	**−1.3435**	**2.6900**	2.7000	**2.6935**
Quasi-score	NIG	**−1.3462**	**2.6955**	2.7055	2.6990
Quasi-score	MXN	**−1.3447**	**2.6925**	2.7025	2.6960
Model	Distribution	LL	AIC	BIC	HQC
Score	Gen-t	−1.3376	2.6779	2.6866	2.6809
Quasi-score	t	−1.3381	2.6793	2.6892	2.6827
Quasi-score	GED	**−1.3353**	**2.6736**	**2.6836**	**2.6771**
Quasi-score	Gen-t	**−1.3353**	**2.6740**	**2.6852**	**2.6779**
Quasi-score	Skew-Gen-t	**−1.3353**	**2.6744**	2.6868	**2.6787**
Quasi-score	EGB2	**−1.3329****	**2.6692****	**2.6804***	**2.6731***
Quasi-score	NIG	**−1.3363**	**2.6759**	2.6871	**2.6798**
Quasi-score	MXN	**−1.3347**	**2.6728**	**2.6840**	**2.6767**
Model	Distribution	LL	AIC	BIC	HQC
Score	Skew-Gen-t	−1.3377	2.6785	2.6885	2.6820
Quasi-score	t	−1.3379	2.6793	2.6905	2.6832
Quasi-score	GED	**−1.3367**	**2.6768**	**2.6880**	**2.6807**
Quasi-score	Gen-t	**−1.3351**	**2.6740**	**2.6865**	**2.6784**
Quasi-score	Skew-Gen-t	**−1.3367**	**2.6775**	2.6913	2.6823
Quasi-score	EGB2	**−1.3341***	**2.6721***	**2.6845***	**2.6764***
Quasi-score	NIG	**−1.3374**	2.6786	2.6911	2.6829
Quasi-score	MXN	**−1.3359**	**2.6756**	**2.6881**	**2.6800**
	Classical model	LL	AIC	BIC	HQC
	A-PARCH	−1.3545	2.7114	2.7188	2.7140
	G-GARCH	−1.3598	2.7215	2.7277	2.7236
	t-GARCH	−1.3385*	2.6793*	2.6868*	2.6820*

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH); log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn criterion (HQC). In each panel, bold numbers indicate that the likelihood-based performance of a quasi-score-driven model is superior to the likelihood-based performance of the score-driven model. In each panel, * indicates the best-performing specification. In this table, ** indicates the best-performing specification from Tables 2-A and 2-B. The parameter estimates and the computer codes for all models are available from the authors upon request.

Table 2-B: In-sample statistical performance for the period of 3 January 2000 to 14 December 2020.

Model	Distribution	LL	AIC	BIC	HQC
Score	EGB2	−1.3391	2.6808	2.6895	2.6839
Quasi-score	t	−1.3392	2.6815	2.6915	2.6850
Quasi-score	GED	**−1.3363**	**2.6757**	**2.6856**	**2.6791**
Quasi-score	Gen-t	**−1.3363**	**2.6760**	**2.6872**	**2.6800**
Quasi-score	Skew-Gen-t	**−1.3363**	**2.6764**	**2.6889**	**2.6808**
Quasi-score	EGB2	**−1.3347**	**2.6729**	**2.6841**	**2.6768**
Quasi-score	NIG	**−1.3369**	**2.6772**	**2.6885**	**2.6812**
Quasi-score	MXN	**−1.3336***	**2.6705***	**2.6817***	**2.6744***
Model	Distribution	LL	AIC	BIC	HQC
Score	NIG	−1.3449	2.6924	2.7011	2.6954
Quasi-score	t	**−1.3390**	**2.6811**	**2.6911**	**2.6846**
Quasi-score	GED	**−1.3410**	**2.6850**	**2.6949**	**2.6884**
Quasi-score	Gen-t	**−1.3361***	**2.6757***	**2.6869***	**2.6796***
Quasi-score	Skew-Gen-t	**−1.3410**	**2.6857**	**2.6982**	**2.6901**
Quasi-score	EGB2	**−1.3370**	**2.6774**	**2.6886**	**2.6814**
Quasi-score	NIG	**−1.3393**	**2.6819**	**2.6932**	**2.6859**
Quasi-score	MXN	**−1.3399**	**2.6831**	**2.6943**	**2.6871**
Model	Distribution	LL	AIC	BIC	HQC
Score	MXN	−1.3445	2.6917	2.7004	2.6947
Quasi-score	t	−1.3394	**2.6817**	**2.6917**	**2.6852**
Quasi-score	GED	**−1.3420**	**2.6871**	**2.6971**	**2.6906**
Quasi-score	Gen-t	**−1.3364***	**2.6763***	**2.6875***	**2.6802***
Quasi-score	Skew-Gen-t	**−1.3364**	**2.6767**	**2.6892**	**2.6810**
Quasi-score	EGB2	**−1.3376**	**2.6786**	**2.6898**	**2.6825**
Quasi-score	NIG	**−1.3391**	**2.6816**	**2.6928**	**2.6855**
Quasi-score	MXN	**−1.3372**	**2.6777**	**2.6889**	**2.6816**
	Classical model	LL	AIC	BIC	HQC
	A-PARCH	−1.3545	2.7114	2.7188	2.7140
	G-GARCH	−1.3598	2.7215	2.7277	2.7236
	t-GARCH	−1.3385*	2.6793*	2.6868*	2.6820*

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH); log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn criterion (HQC). In each panel, bold numbers indicate that the likelihood-based performance of a quasi-score-driven model is superior to the likelihood-based performance of the score-driven model. In each panel, * indicates the best-performing specification. The parameter estimates and the computer codes for all models are available from the authors upon request.

Table 3-A: In-sample volatility forecasting performance for the period of 1 February 2000 to 14 December 2020.

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	t	0.1689	3.1554	0.2829	0.7486	0.3079	0.7582
Quasi-score	t	**0.1678**	**3.0929**	**0.2828**	**0.7451**	**0.3075**	**0.7550**
Quasi-score	GED	**0.1664**	**2.9869**	**0.2805**	**0.7342**	**0.3061**	**0.7497**
Quasi-score	Gen-t	**0.1665**	**2.9870**	**0.2805**	**0.7342**	**0.3061**	**0.7497**
Quasi-score	Skew-Gen-t	**0.1665**	**2.9871**	**0.2805**	**0.7342**	**0.3062**	**0.7497**
Quasi-score	EGB2	**0.1670**	**3.0420**	**0.2742***	**0.7191***	**0.3046***	**0.7548**
Quasi-score	NIG	**0.1659**	**2.9842**	**0.2820**	**0.7396**	**0.3062**	**0.7479**
Quasi-score	MXN	**0.1659***	**2.9688***	**0.2785**	**0.7302**	**0.3048**	**0.7467***

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	GED	0.1727	3.1485	0.2864	0.7807	0.3105	0.7589
Quasi-score	t	**0.1589**	**2.9221**	**0.2778**	**0.7363**	**0.3001**	**0.7245**
Quasi-score	GED	**0.1715**	**3.1203**	**0.2848**	**0.7725**	**0.3104**	0.7597
Quasi-score	Gen-t	**0.1589**	**2.8478**	**0.2760***	**0.7255**	**0.3000**	**0.7237**
Quasi-score	Skew-Gen-t	**0.1589****	**2.8476****	**0.2760**	**0.7255***	**0.3000***	**0.7237****
Quasi-score	EGB2	0.1776	**3.1459**	**0.2846**	**0.7646**	0.3179	0.7906
Quasi-score	NIG	**0.1694**	**3.0363**	**0.2821**	**0.7580**	**0.3103**	0.7594
Quasi-score	MXN	0.1770	3.1888	0.2870	**0.7735**	0.3173	0.7851

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	Gen-t	0.1660	3.1151	0.2801	0.7435	0.3048	0.7481
Quasi-score	t	**0.1653**	**3.0493**	0.2810	**0.7403**	0.3052	**0.7468**
Quasi-score	GED	**0.1644**	**2.9518**	**0.2790**	**0.7300**	**0.3043**	**0.7428**
Quasi-score	Gen-t	**0.1644**	**2.9517**	**0.2790**	**0.7300**	**0.3042**	**0.7428**
Quasi-score	Skew-Gen-t	**0.1644**	**2.9518**	**0.2790**	**0.7300**	**0.3043**	**0.7428**
Quasi-score	EGB2	**0.1647**	**2.9942**	**0.2728***	**0.7150***	**0.3026***	**0.7473**
Quasi-score	NIG	**0.1637**	**2.9569**	**0.2800**	**0.7345**	**0.3039**	**0.7404**
Quasi-score	MXN	**0.1636***	**2.9303***	**0.2769**	**0.7256**	**0.3027**	**0.7391***

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	Skew-Gen-t	0.1654	3.1256	0.2796	0.7438	0.3037	0.7447
Quasi-score	t	**0.1641**	**3.0299**	0.2801	**0.7372**	0.3040	**0.7430**
Quasi-score	GED	0.1716	**3.0736**	0.2849	0.7540	0.3120	0.7678
Quasi-score	Gen-t	**0.1633***	**2.9359***	**0.2781***	**0.7269***	**0.3031***	**0.7391***
Quasi-score	Skew-Gen-t	0.1709	**3.0578**	0.2850	0.7537	0.3115	0.7652
Quasi-score	EGB2	0.1726	3.1269	**0.2788**	**0.7368**	0.3107	0.7738
Quasi-score	NIG	0.1715	**3.0663**	0.2864	0.7583	0.3125	0.7673
Quasi-score	MXN	0.1711	**3.0336**	0.2830	**0.7482**	0.3109	0.7651

	Classical model	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
	A-PARCH	0.1649*	3.4271*	0.2730*	0.7216*	0.2982*	0.7425*
	G-GARCH	0.1847	4.4292	0.2780	0.7540	0.3036	0.7878
	t-GARCH	0.2061	5.3026	0.2812	0.7568	0.3147	0.8486

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH). In each panel, bold numbers indicate that the forecasting performance of a quasi-score-driven model is superior to the forecasting performance of the score-driven model. In each panel, * indicates the best-performing specification. Moreover, ** indicates the best-performing specification from Tables 3-A and 3-B. For the in-sample volatility forecasting performance evaluation, the first month of the full sample period (i.e. 20 observations) is excluded, in order to reduce the effects of the initialization on the volatility estimates. The parameter estimates and the computer codes for all models are available from the authors upon request.

Table 3-B: In-sample volatility forecasting performance for the period of 1 February 2000 to 14 December 2020.

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	EGB2	0.1639	3.0140	0.2803	0.7460	0.3035	0.7384
Quasi-score	t	**0.1629**	**3.0042**	**0.2794**	**0.7360**	**0.3029**	0.7385
Quasi-score	GED	**0.1624**	**2.9197**	**0.2774**	**0.7257**	**0.3023**	**0.7359**
Quasi-score	Gen-t	**0.1624**	**2.9198**	**0.2774**	**0.7257**	**0.3023**	**0.7360**
Quasi-score	Skew-Gen-t	**0.1624**	**2.9206**	**0.2775**	**0.7260**	**0.3024**	**0.7359**
Quasi-score	EGB2	**0.1596****	**2.9011**	**0.2709****	**0.7054****	**0.2977****	**0.7287***
Quasi-score	NIG	**0.1627**	**2.8990***	0.2817	**0.7362**	0.3037	**0.7353**
Quasi-score	MXN	**0.1629**	**2.9386**	**0.2717**	**0.7137**	**0.3010**	0.7402

	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	NIG	0.1778	3.2854	0.2889	0.7771	0.3175	0.7873
Quasi-score	t	**0.1641**	**3.0253**	**0.2803**	**0.7389**	**0.3042**	**0.7428**
Quasi-score	GED	**0.1725**	**3.1348**	**0.2850**	**0.7646**	**0.3120**	**0.7681**
Quasi-score	Gen-t	**0.1634***	**2.9355***	**0.2783***	**0.7284***	**0.3034***	**0.7396***
Quasi-score	Skew-Gen-t	**0.1725**	**3.1354**	**0.2850**	**0.7646**	**0.3120**	**0.7682**
Quasi-score	EGB2	**0.1731**	**3.0912**	**0.2788**	**0.7432**	**0.3113**	**0.7741**
Quasi-score	NIG	**0.1767**	**3.1065**	**0.2858**	**0.7608**	**0.3170**	**0.7870**
Quasi-score	MXN	**0.1743**	**3.1886**	**0.2832**	**0.7556**	**0.3134**	**0.7785**

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	MXN	0.1755	3.2079	0.2878	0.7761	0.3153	0.7777
Quasi-score	t	**0.1626**	**2.9962**	**0.2793**	**0.7368**	**0.3028**	**0.7375**
Quasi-score	GED	**0.1714**	**3.1192**	**0.2841**	**0.7636**	**0.3106**	**0.7633**
Quasi-score	Gen-t	**0.1621***	**2.9117**	**0.2774**	**0.7264***	**0.3023***	**0.7351***
Quasi-score	Skew-Gen-t	**0.1621**	**2.9117***	**0.2774**	**0.7265**	**0.3023**	**0.7351**
Quasi-score	EGB2	**0.1721**	**3.0468**	**0.2795**	**0.7479**	**0.3105**	**0.7682**
Quasi-score	NIG	0.1811	3.2181	**0.2854**	**0.7656**	0.3202	0.8019
Quasi-score	MXN	**0.1660**	**2.9647**	**0.2744***	**0.7284**	**0.3040**	**0.7487**

	Classical model	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
	A-PARCH	0.1649*	3.4271*	0.2730*	0.7216*	0.2982*	0.7425*
	G-GARCH	0.1847	4.4292	0.2780	0.7540	0.3036	0.7878
	t-GARCH	0.2061	5.3026	0.2812	0.7568	0.3147	0.8486

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH). In each panel, bold numbers indicate that the forecasting performance of a quasi-score-driven model is superior to the forecasting performance of the score-driven model. In each panel, * indicates the best-performing specification. Moreover, ** indicates the best-performing specification from Tables 3-A and 3-B. For the in-sample volatility forecasting performance evaluation, the first month of the full sample period (i.e. 20 observations) is excluded, in order to reduce the effects of the initialization on the volatility estimates. The parameter estimates and the computer codes for all models are available from the authors upon request.

Table 4-A: Out-of-sample volatility forecasting performance for the period of 2 January 2020 to 14 December 2020.

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	t	0.4791	19.8737	0.4020	0.8723	0.4999	2.0355
Quasi-score	t	0.4524***	18.1662**	0.4019	0.8429***	0.4852***	1.9437***
Quasi-score	GED	0.4628	17.5558	0.3854	0.8374**	0.4849**	1.9411^{+}
Quasi-score	Gen-t	0.4630	17.5635	0.3857	0.8377**	0.4850**	1.9417^{+}
Quasi-score	Skew-Gen-t	0.4561	17.5483	0.3959	0.8183***	0.4763***	1.9144*
Quasi-score	EGB2	0.4462**	17.9275	0.3417**	0.7633***	0.4701***	1.9321*
Quasi-score	NIG	0.4526*	17.1190^{+}	0.3992	0.8413*	0.4823**	1.9167**
Quasi-score	MXN	0.4638	17.1635	0.3781^{+}	0.8414^{+}	0.4876^{+}	1.9401^{+}

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	GED	0.5307	17.7700	0.4101	1.0283	0.5521	2.1443
Quasi-score	t	0.4090***	16.1257**	0.3795	0.8152***	0.4655***	1.8140***
Quasi-score	GED	0.4175***	15.6183***	0.3672**	0.8024***	0.4637***	1.8104***
Quasi-score	Gen-t	0.4175***	15.6187***	0.3673**	0.8019***	0.4636***	1.8103***
Quasi-score	Skew-Gen-t	0.4175***	15.6180***	0.3673**	0.8018***	0.4636***	1.8102***
Quasi-score	EGB2	**0.4045*****	15.9314**	0.3331***	**0.7370*****	**0.4501*****	1.8008***
Quasi-score	NIG	0.4163***	15.4125***	0.3783	0.8247***	0.4669***	1.8057***
Quasi-score	MXN	0.4200***	**15.3590*****	0.3637***	0.8187***	0.4684***	1.8191***

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	Gen-t	0.4776	19.4142	0.3918	0.8795	0.5003	2.0303
Quasi-score	t	0.4367***	17.4327***	0.3949	0.8322***	0.4781***	1.8982***
Quasi-score	GED	0.4459**	16.8155^{+}	0.3795*	0.8242***	0.4771***	1.8931***
Quasi-score	Gen-t	0.4459**	16.8150^{+}	0.3795*	0.8242***	0.4771***	1.8931***
Quasi-score	Skew-Gen-t	0.4393***	17.0183^{+}	0.3913	0.8056***	0.4690***	1.8750***
Quasi-score	EGB2	0.4310***	17.1984^{+}	0.3360**	0.7506***	0.4627***	1.8863***
Quasi-score	NIG	0.4451***	16.7554*	0.3852	0.8371***	0.4808***	1.8983***
Quasi-score	MXN	0.4436**	16.3118*	0.3702***	0.8236***	0.4777***	1.8818**

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	Skew-Gen-t	0.4815	19.2428	0.3906	0.8945	0.5042	2.0356
Quasi-score	t	0.4314***	17.1342***	0.3932	0.8277***	0.4754***	1.8805***
Quasi-score	GED	0.4403***	16.5352*	0.3785^{+}	0.8215***	0.4746***	1.8748***
Quasi-score	Gen-t	0.4401***	16.5339*	0.3777^{+}	0.8190***	0.4741***	1.8742***
Quasi-score	Skew-Gen-t	0.4403***	16.5575*	0.3780^{+}	0.8196***	0.4744***	1.8755***
Quasi-score	EGB2	0.4257***	16.9171*	0.3347***	0.7465***	0.4599***	1.8682***
Quasi-score	NIG	0.4492^{+}	15.3946	0.3944	0.8669*	0.4827*	1.8591^{+}
Quasi-score	MXN	0.4388***	16.0529*	0.3699***	0.8218***	0.4757***	1.8661***

	Classical model	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
	A-PARCH	0.5289***	21.9698**	0.3624^{+}	0.8239***	0.4906***	2.0600***
	G-GARCH	0.8290	42.8140	0.4243	1.0947	0.6302	2.9394
	t-GARCH	0.9598	51.1076	0.4528	1.1684	0.6728	3.2296

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH). ***, **, *, and $^{+}$ indicate that the Giacomini–White test statistic is significant at the 1%, 5%, 10%, and 15% levels, respectively. In each panel of score-driven models, the Giacomini–White test compares the volatility forecasting performances of the score-driven model with all quasi-score-driven models. Bold numbers indicate the lowest loss function value from Tables 4-A and 4-B. In the panel of classical models, the Giacomini-White test compares the volatility forecasting performances of the A-PARCH model with the best-performing quasi-score-driven model (i.e. bold numbers). The computer codes for all models are available from the authors upon request.

Table 4-B: Out-of-sample volatility forecasting performance for the period of 2 January 2020 to 14 December 2020.

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	EGB2	0.4788	17.9402	0.3943	0.9200	0.5114	2.0190
Quasi-score	t	0.4323***	17.2569*	0.3932	0.8275***	0.4757***	1.8854***
Quasi-score	GED	0.4434***	16.7710	0.3789	0.8198***	0.4755***	1.8874***
Quasi-score	Gen-t	0.4434***	16.7692	0.3789	0.8198***	0.4755***	1.8874***
Quasi-score	Skew-Gen-t	0.4434***	16.7691	0.3789	0.8198***	0.4755***	1.8873***
Quasi-score	EGB2	0.4246***	16.7923	0.3508***	0.7566***	0.4596***	1.8569***
Quasi-score	NIG	0.4374***	16.3898	0.3935	0.8232***	0.4730***	1.8630***
Quasi-score	MXN	0.4337***	16.8771	0.3244**	0.7537***	0.4664***	1.8931***

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	NIG	0.4748	18.3059	0.3924	0.9016	0.5048	2.0089
Quasi-score	t	0.4354***	17.3918***	0.3948	0.8304***	0.4773***	1.8945***
Quasi-score	GED	0.4461***	16.8707	0.3801^{+}	0.8236***	0.4772***	1.8951***
Quasi-score	Gen-t	0.4461***	16.8711	0.3801^{+}	0.8237***	0.4772***	1.8952***
Quasi-score	Skew-Gen-t	0.4396***	16.9380	0.3908	0.8051***	0.4689***	1.8724***
Quasi-score	EGB2	0.4408**	17.1194	0.3230*	0.7594***	0.4700***	1.9135**
Quasi-score	NIG	0.4423***	17.0532***	0.3822**	0.8474***	0.4832***	1.9113***
Quasi-score	MXN	0.4445*	16.6519	0.3193*	0.7706***	0.4740***	1.9121*

Model	Distribution	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
Score	MXN	0.4838	17.9445	0.3960	0.9316	0.5161	2.0345
Quasi-score	t	0.4285***	17.0795*	0.3911	0.8257***	0.4741***	1.8743***
Quasi-score	GED	0.4390***	16.5766^{+}	0.3773^{+}	0.8182***	0.4738***	1.8751***
Quasi-score	Gen-t	0.4389***	16.5739^{+}	0.3769*	0.8171***	0.4736***	1.8749***
Quasi-score	Skew-Gen-t	0.4352***	16.3581	0.3912	0.8179***	0.4717***	1.8584***
Quasi-score	EGB2	0.4284***	16.4459	**0.3176****	0.7473***	0.4632***	1.8705***
Quasi-score	NIG	0.4129***	15.4800	0.3493***	0.7574***	0.4521***	**1.7959****
Quasi-score	MXN	0.4282***	16.7733	0.3258***	0.7470***	0.4627***	1.8780***

	Classical model	MSE_1	MSE_2	QLIKE	R^2LOG	MAE_1	MAE_2
	A-PARCH	0.5289***	21.9698**	0.3624^{+}	0.8239***	0.4906***	2.0600***
	G-GARCH	0.8290	42.8140	0.4243	1.0947	0.6302	2.9394
	t-GARCH	0.9598	51.1076	0.4528	1.1684	0.6728	3.2296

Student's t-distribution (t); general error distribution (GED); generalized t-distribution (Gen-t); skewed generalized t-distribution (Skew-Gen-t); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian (NIG) distribution; Meixner distribution (MXN); asymmetric power autoregressive conditional heteroscedasticity (A-PARCH); Gaussian-GARCH (G-GARCH). ***, **, *, and $^{+}$ indicate that the Giacomini–White test statistic is significant at the 1%, 5%, 10%, and 15% levels, respectively. In each panel of score-driven models, the Giacomini–White test compares the volatility forecasting performances of the score-driven model with all quasi-score-driven models. Bold numbers indicate the lowest loss function value from Tables 4-A and 4-B. In the panel of classical models, the Giacomini-White test compares the volatility forecasting performances of the A-PARCH model with the best-performing quasi-score-driven model (i.e. bold numbers). The computer codes for all models are available from the authors upon request.

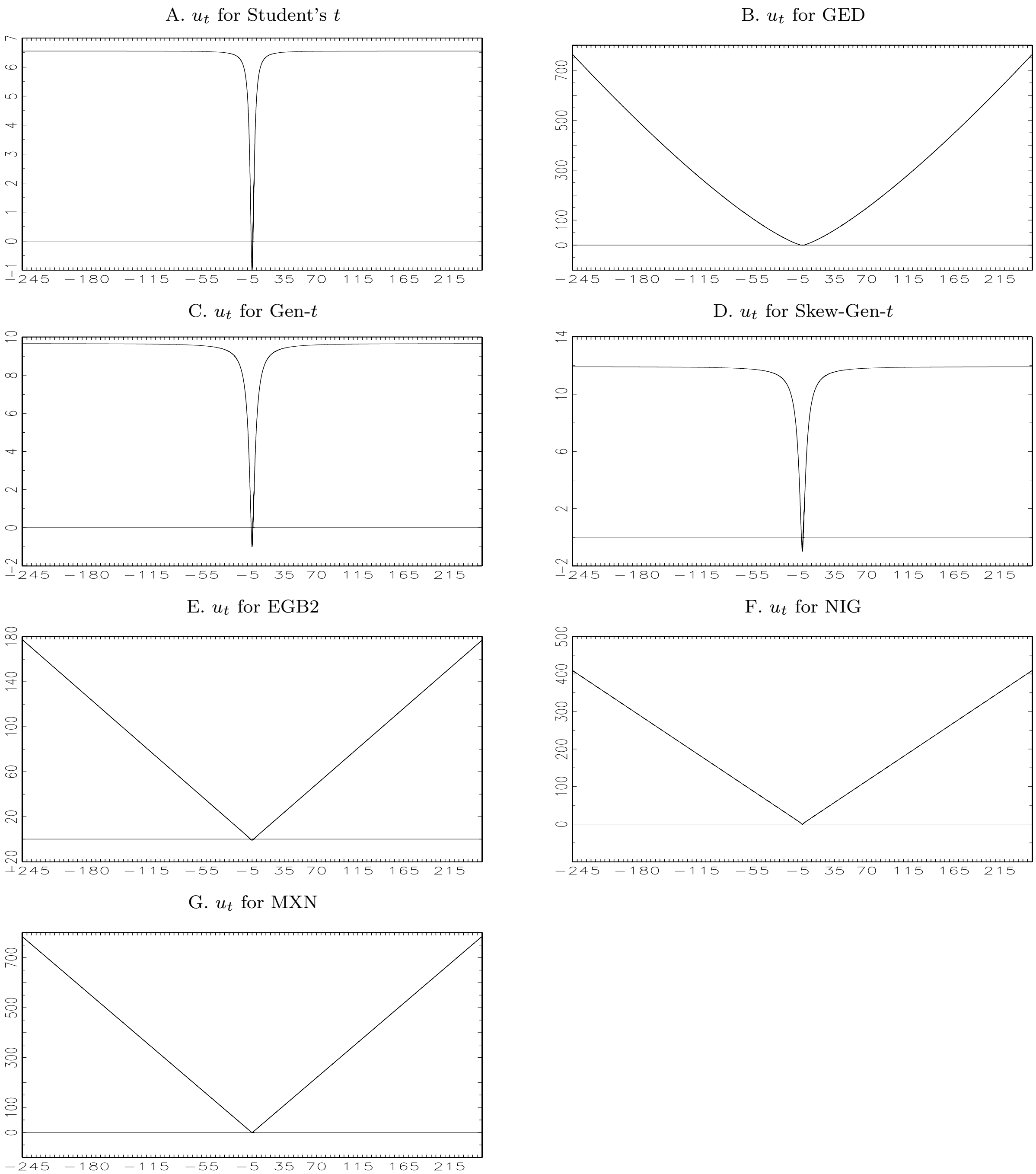

Figure 1: Score function u_t as a function of ϵ_t for the period of 3 January 2000 to 14 December 2020. *Notes:* We use the parameter estimates for the score-driven models. For the x-axis we use $\epsilon_t \in (-250, 250)$, to indicate the asymptotic behaviour of u_t.

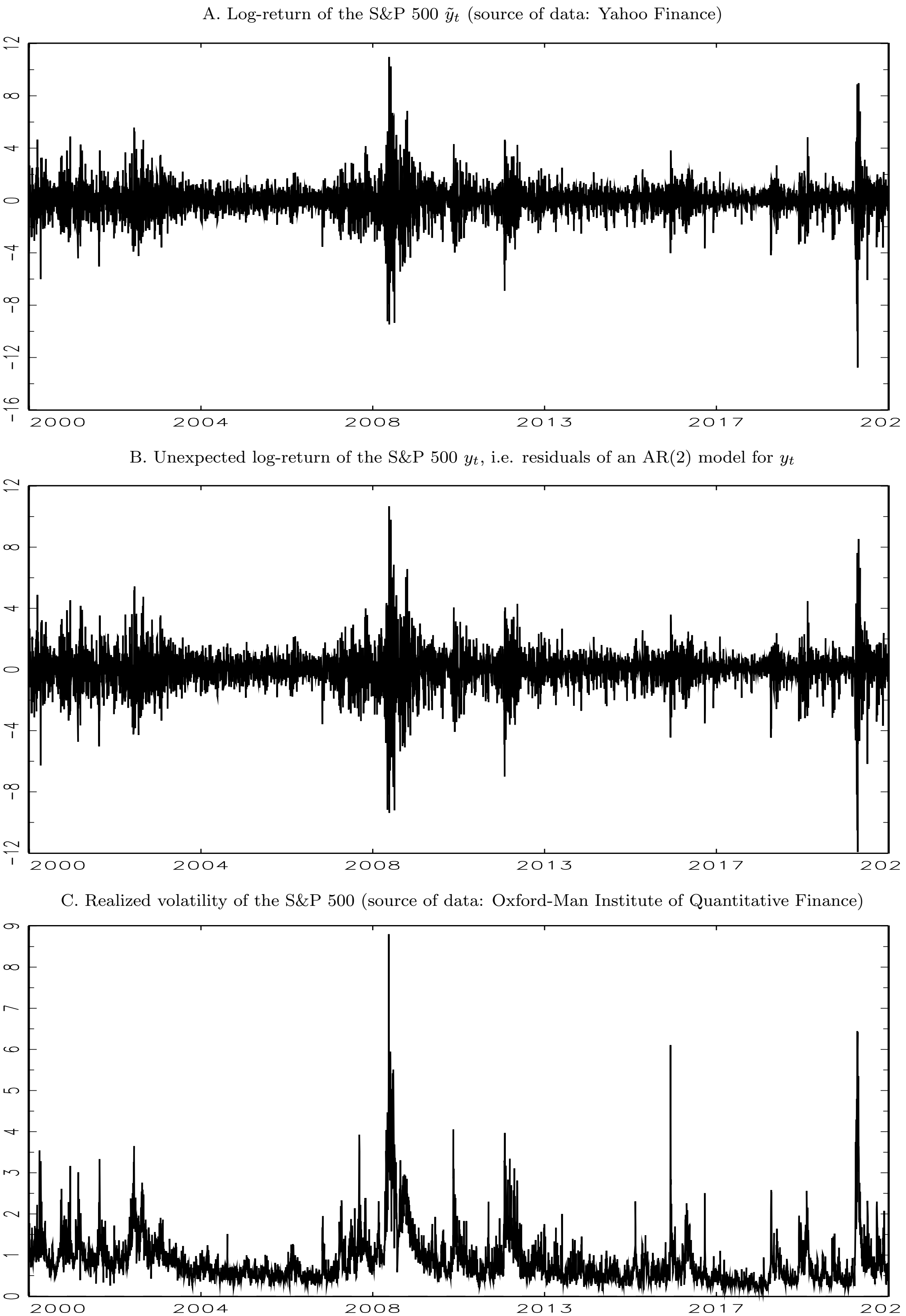

Figure 2: Log-return, unexpected return, and realized volatility for the period of 3 January 2000 to 14 December 2020.