

Highly Competitive Contests: Experimental Evidence

Lucas Rentschler

Universidad Francisco Marroquín

and

Theodore L. Turocy

University of East Anglia

28 August 2015

Idea

The all-pay auction

- ✦ The **all-pay auction** is a model in which two (or more) contestants irrevocably expend resources in pursuit of a prize.
- ✦ One contestant wins the prize.
- ✦ The contestant who expends the most resources wins the prize with certainty.
- ✦ (Contrast with probabilistic contests like the Tullock contest.)
- ✦ In many all-pay auction settings, including the one we study here, equilibrium involves randomised strategies.

Theory literature on all-pay auctions

- ✦ Complete information.
 - ✦ Baye *et al.* (1996).
- ✦ Incomplete information with restrictions on type structures.
 - ✦ Krishna and Morgan (1997).
 - ✦ Siegel (2014).
 - ✦ Both these papers restrict the type structure such that, in equilibrium, a higher type is unambiguously good news (KMS condition).
- ✦ Incomplete information without restrictions on type structures.
 - ✦ Rentschler and Turocy (2014).

Experimental literature on all-pay auctions

- ✦ In the experimental literature on all-pay auctions (for a single, indivisible prize), there are, for the most part, two separate approaches:
 - ✦ Complete information and common value.
 - Gneezy and Smorodinsky (2006).
 - Lugovsky, Puzzello and Tucker (2010).
 - Ernst and Thöni (2013).
 - ✦ Incomplete information with independent types.
 - Aycinena, Baltaduonis and Rentschler (2014).
 - Hyndman, Ozbay and Sujarittanonta (2012).
 - Müller and Schotter (2010).
 - Hoerrisch and Kirchkamp (2007).

Stylised facts from experiments

- ✦ Overdissipation.
 - ✦ The sum of bids often exceeds the value of the prize, especially in early rounds.
- ✦ Bimodal distributions of bids.
 - ✦ With private info: Contestants with low types bid below Nash predictions (often 0) and contestants with high types bid above Nash predictions.
 - ✦ With complete info: Either sit out (spend 0) or spend very aggressively (close to the value).

This project

- ✦ Preceding work by Krishna and Morgan (1997) and Siegel (2014) identify conditions for a monotonic equilibrium.
- ✦ Unlike winner-pay auctions, affiliation of types and values is not a key property for monotonic equilibrium: in fact, the equilibrium is not monotonic when types are “too affiliated.”
- ✦ Highly-affiliated types are a mixed message:
 - ✦ A high type is “good news” in that it means the prize has, or is likely to have, a higher value.
 - ✦ A high type is “bad news” in that it means the other contestant(s) are also likely to bid aggressively.

This project

- ✦ We study a simple environment where there are two possible types and two possible values.
- ✦ We vary how correlated the types are between the two bidders.
- ✦ In a treatment where the prize is common-value, equilibrium is monotonic when types are noisy, but not when they are accurate.
- ✦ \Rightarrow Perhaps a rather counterintuitive prediction?

This project

- ✦ As is well-known from the literature on winner-pay auctions, behaviour in common-value settings may be far from equilibrium.
- ✦ \Rightarrow the winner's curse (and the puzzling failure of participants to learn from experiencing the curse).
- ✦ We therefore also study correlated private values settings.
- ✦ When private values are highly correlated, equilibrium is not monotonic, but it is monotonic when private values are weakly correlated or independent.

Idea
○○○○○○○

Theory
○○○○○○○

Design
○○○○○○○○○○○○

Results
○○○○○○○○○○○○○○○○○○○○

Summary
○

Theory

Basic setup

- ✦ Two contestants, $i = 1, 2$.
- ✦ Two possible values 15 and 30, both equally likely ex ante.
- ✦ Two possible signals 15 and 30.

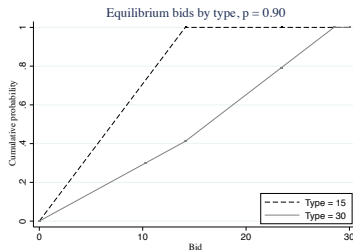
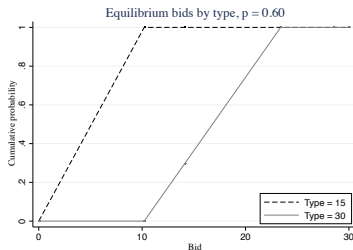
Common-values setting

- ✦ The value of the prize is the same for both participants.
- ✦ Conditional on the realised value of the prize, with probability $p \geq 0.5$ a contestant's type is equal to the value.
- ✦ Signals are conditionally independent (but are correlated due to their dependence on the realised value).

Common-values setting

- ✦ When $p = 0.6$, the equilibrium strategy is (stochastically) monotonic.
- ✦ When $p = 0.9$, the supports of the equilibrium densities overlap.

Common-values setting



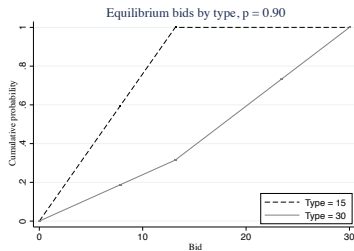
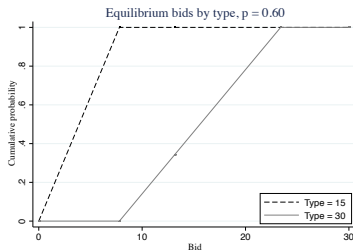
Private-values setting

- ✦ The value of the prize is (potentially) different between participants.
- ✦ There are two states of the world, “high” and “low.”
- ✦ If the state is “high,” each contestant’s value is 30 with probability $p \geq 0.5$.
- ✦ If the state is “low,” each contestant’s value is 15 with probability $p \geq 0.5$.
- ✦ Signals are conditionally independent (but are correlated due to their dependence on the state).

Private-values setting

- ✦ When $p = 0.5$ we have independent private values and the equilibrium strategy is (stochastically) monotonic.
- ✦ When $p = 0.6$, the equilibrium strategy remains (stochastically) monotonic.
- ✦ When $p = 0.9$, the supports of the equilibrium densities overlap.

Private-values setting



Nice properties

- ✦ Simple way to model the potential for “bad news” to be conveyed by a higher signal.
- ✦ Easy environment to explain.
- ✦ The equilibrium is not exactly the same for private and common values with the same p parameter.
 - ✦ But we are interested in the **qualitative** prediction of monotonicity.
 - ✦ We preferred to make the instructions more directly comparable by having the same numbers, rather than have the equilibrium be exactly the same.

Idea
○○○○○○○

Theory
○○○○○○○

Design
○○○○○○○○○○○○

Results
○○○○○○○○○○○○○○○○○○○○

Summary
○

Design

Basic design outline

- ✦ A $2 \times 2 + 1$ design, between-subjects.
- ✦ One dimension: $p = 0.6$ versus $p = 0.9$.
- ✦ Other dimension: Common or private values.
- ✦ Three sessions per treatment cell.

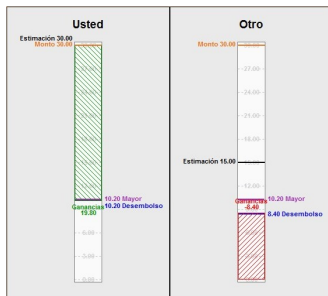
Basic design outline

- ✦ Cohorts of 8 participants.
- ✦ 40 periods (announced in advance).
- ✦ Fixed matching (so 12 independent pairs per treatment)
- ✦ Sessions conducted at Centro Vernon Smith at Universidad Francisco Marroquín, Guatemala, October 2014 through March 2015.
- ✦ Subjects had varying overall experience levels in experiments, but no prior experience with all-pay auctions.

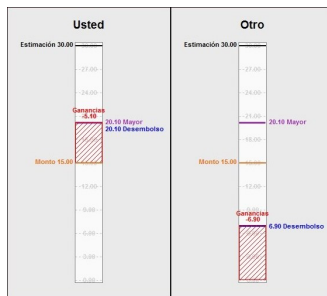
Bonus treatment: Independent private values

- ✦ The case of $p = 0.5$ amounts to independent private values.
- ✦ Basis of comparison with the existing literature.
- ✦ Here, the equilibrium prediction is also monotonic.
- ✦ To explain correlated private values, we tell participants there is a “hidden number,” and their values are determined statistically based on this hidden number.
- ✦ With independent private values, we can dispense with any mention of the hidden number.

Interface and feedback



(a) Positive profit



(b) Negative profit

Interface and feedback

- ✦ Uses a custom slider to display signals and elicit expenditures, based on the software from Turocy, Watson and Battalio (2007), Turocy and Watson (2012) and Turocy and Cason (2014).
- ✦ Decision and feedback presented in a unified way on the same graphical widget.
- ✦ Novelty: Show graphically the feedback from the perspective of the other contestant.
- ✦ Full record sheet on screen at all times.

Timing and payment

- ✦ Pay 10 out of 40 periods, with fixed fee of Q120.
- ✦ Structure of period:
 - ✦ 5 second countdown displaying signal (known to participants)
 - ✦ Choice period lasting $\max(40, \text{time of last choice} + 5)$ seconds (not explicitly stated to participants)
 - ✦ 15 second display of feedback (not explicitly stated)

Hypotheses

Hypothesis 1 (overbidding)

As is generally observed in contests, average bids will exceed the equilibrium prediction, even after experience.

Hypotheses

Hypothesis 2 (learning)

As participants gain experience with the environment, behavior will tend in the direction of equilibrium predictions.

Hypotheses

Hypothesis 3 (monotonicity)

High type contestants will win more often against low type contestants, in the cases where types are not highly correlated.

Hypotheses

Hypothesis 4 (low type bids increase in p)

In both valuation structures average bids of low type contestants will be increasing the degree of correlation between types.

Hypotheses

Hypothesis 5 (earnings of high types is decreasing in p)

In both valuation structures the average earnings of high types will be higher when there is low correlation between types.

Hypotheses

Hypothesis 6 (sum of bids)

In both valuation structures the sum of bids will be increasing in p .

Results

Results: Hypothesis 1 (overbidding)

Values	p	Type = 15		Type = 30	
		Eqm	Data	Eqm	Data
CV	0.60	5.10	11.81** (5.03)	16.80	16.84 (5.57)
CV	0.90	7.06	11.46** (4.63)	15.44	18.74* (4.73)
PV	0.50	3.75	6.21** (2.73)	15.00	19.74** (2.24)
PV	0.60	3.90	8.95** (5.04)	15.60	18.98** (3.55)
PV	0.90	6.58	10.21** (3.35)	16.85	20.73** (2.79)

Summary statistics on average bids conditional on signal.

Results: Hypothesis 1 (overbidding)

Result 1 (overbidding)

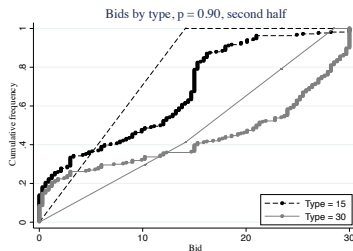
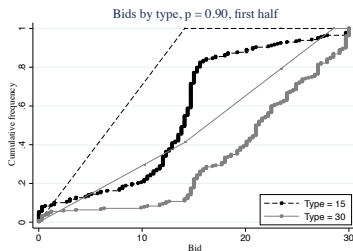
Average bids generally exceeds the equilibrium prediction.

Results: Hypothesis 2 (learning)

Values	p	Type = 15	Type = 30
CV	0.60	-0.63 (5.17)	-0.83 (5.05)
CV	0.90	-3.44* (1.18)	-3.30 (2.01)
PV	0.50	-2.81** (1.69)	-1.81 (1.17)
PV	0.60	-1.79 (4.12)	-2.66* (3.02)
PV	0.90	-2.68** (1.65)	-3.51** (3.23)

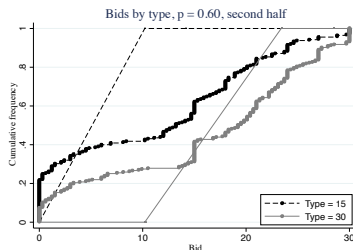
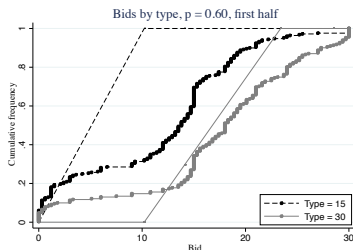
Change in mean bid conditional on type between the first and second half.

Results: Hypothesis 2 (learning)



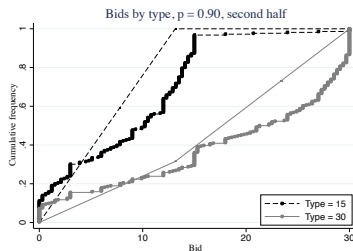
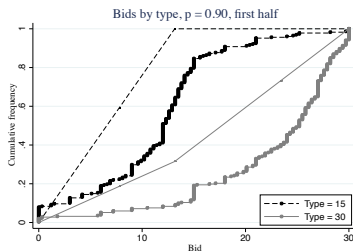
Overall CDFs of bids, common values with highly correlated types ($p = 0.90$).

Results: Hypothesis 2 (learning)



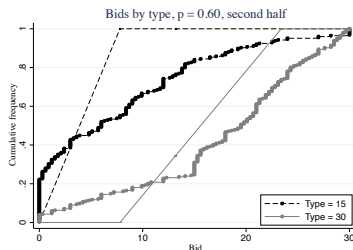
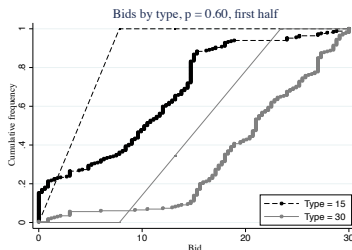
Overall CDFs of bids, common values with low correlation between types ($p = 0.60$).

Results: Hypothesis 2 (learning)



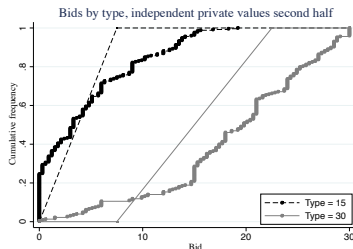
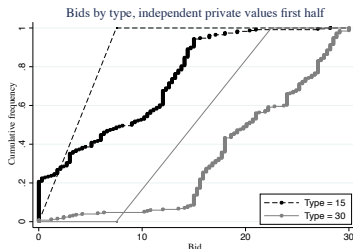
Overall CDFs of bids, private values with high correlation ($p = 0.90$).

Results: Hypothesis 2 (learning)



Overall CDFs of bids, private values with low correlation
($p = 0.60$).

Results: Hypothesis 2 (learning)



Overall CDFs of bids, independent private values ($p = 0.50$).

Results: Hypothesis 2 (learning)

Result 2 (learning)

- ✦ In private values, the distributions of bids tends in the direction of equilibrium in the second half, especially in independent private values.
- ✦ In common values, the distributions of bids tend away from equilibrium, especially with inaccurate signals.
- ✦ In all treatments, bimodal bidding patterns are observed, especially in the second half.
- ✦ Bids in excess of the conditional expected value of the prize are observed, especially in common values.

Results: Hypothesis 2 (learning)

Values	p	Type = 15			Type = 30		
		EV given type	% of bids equal	% of bids above	EV given type	% of bids equal	% of bids above
CV	0.60	21	0.03 (0.04)	0.16 (0.17)	24	0.04 (0.08)	0.22 (0.26)
CV	0.90	16.5	0.01 (0.02)	0.13 (0.22)	28.5	0.01 (0.03)	0.18 (0.24)
PV	0.50	15	0.02 (0.05)	0.02 (0.05)	30	0.05 (0.08)	0.00 (0.00)
PV	0.60	15	0.01 (0.03)	0.16 (0.25)	30	0.01 (0.02)	0.00 (0.00)
PV	0.90	15	0.09 (0.16)	0.03 (0.07)	30	0.14 (0.18)	0.00 (0.00)

Summary statistics on the percentage of bids at, or above, the expected value of the prize conditional on type in the second half of the experiment.

Results: Hypothesis 3 (monotonicity)

- ✦ Equilibrium provides not just a prediction for the difference in average bid with the high type versus low type, but whether the equilibrium is “separating” or (partially) “pooling.”
- ✦ To measure this, we do the following:
 - ✦ For each participant, take their actual set of bids with the low type, and their actual set of bids with the high type.
 - ✦ Pair up each bid from the low type with each bid with the high type.
 - ✦ Compute the proportion of contingencies in which the bid with the high signal would have won.
- ✦ \Rightarrow This is a within-subject measure of separation of bids conditional on type.

Results: Hypothesis 3 (monotonicity)

Values	p	Equilibrium	Data
CV	0.60	1.000	0.696*** (0.026)
CV	0.90	0.794	0.769 (0.038)
PV	0.50	1.000	0.934*** (0.013)
PV	0.60	1.000	0.847*** (0.041)
PV	0.90	0.842	0.830 (0.022)

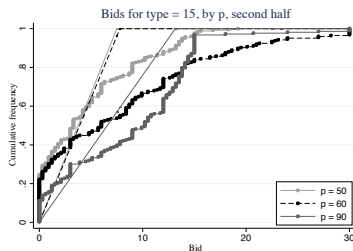
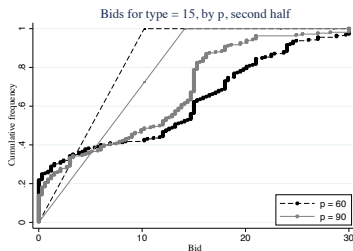
Within-subjects measure of overlap between high-type and low-type bid distributions, by treatment.

Results: Hypothesis 3 (monotonicity)

Result 3 (monotonicity)

- ✦ Behaviour is not monotonic in type for small p .
- ✦ In private values, behaviour is “more” monotonic with small p than large p , as predicted by equilibrium.
- ✦ In common values, behaviour is “less” monotonic with small p than large p , which is the opposite of the equilibrium prediction.

Results: Hypothesis 4 (low type bids increase in p)



Effect of p on CDF of bids for low type contestants in the second half.

Results: Hypothesis 4 (low type bids increase in p)

	All 40 periods		Last 20 periods	
	t_L	t_H	t_L	t_H
Common value	1.327 (1.550)	-2.211 (1.474)	0.712 (1.871)	-2.035 (1.966)
$p = 60$	3.117* (1.550)	-0.999 (1.478)	3.379 ⁺ (1.870)	-1.411 (1.977)
$p = 90$	4.778** (1.550)	1.339 (1.477)	4.879** (1.868)	0.482 (1.977)
CV and $p = 90$	2.080 (2.190)	0.200 (2.088)	2.875 (2.648)	0.905 (2.781)
Constant	5.060*** (1.097)	19.850*** (1.046)	3.559** (1.320)	18.998*** (1.406)
Observations	2316	2484	1134	1266
Left censored	345	92	215	71
Right censored	39	138	22	93
Log-likelihood	-6899.136	-8265.643	-3277.002	-4230.494
p value of Wald test	0.0001	0.0551	0.0027	0.5390

Tobit estimates of the treatment effects on bids by type.

Results: Hypothesis 4 (low type bids increase in p)

Result 4 (low type bids increase in p)

- ✦ In private value auctions, the average bids of low type contestants are bigger when $p > 1/2$.
- ✦ There is no statistical difference between low type bids when $p > 1/2$ in private value auctions.
- ✦ There is no statistical difference between low type bids in common value auctions.

Results: Hypothesis 5 (earnings of high types is decreasing in p)

Values	p	Type = 15		Type = 30	
		Eqm	Data	Eqm	Data
CV	0.60	0.00	-3.75 (6.99)	0.60	-1.43 (6.33)
CV	0.90	0.00	-2.68 (6.10)	0.00	-1.72 (6.85)
PV	0.50	0.00	-0.94 (2.70)	7.50	4.76 (4.63)
PV	0.60	0.00	-2.64 (6.24)	6.50	0.94** (4.08)
PV	0.90	0.00	-1.70 (3.07)	0.00	-3.45* (3.59)

Summary statistics on average earnings conditional on type in the second half.

Results: Hypothesis 5 (earnings of high types is decreasing in p)

Result 5 (learnings of high types is decreasing in p)

- ✦ In private value auctions, the earnings of high type bidders is decreasing in p .
- ✦ When $p = 0.9$ high type bidders in private value auctions have negative earnings on average.
- ✦ The average earnings of high types does not vary with p in common value auctions.

Results: Hypothesis 6 (sum of bids)

Values	p	Equilibrium	Data
CV	0.60	21.90	28.76 (11.32)
CV	0.90	22.50	30.74 (10.51)
PV	0.50	18.75	25.96* (5.94)
PV	0.60	19.50	28.11* (8.00)
PV	0.90	23.43	31.74* (6.21)

Revenue by treatment in the second half.

Results: Hypothesis 6 (sum of bids)

Result 6 (sum of bids)

- ✦ The average sum of bids increases in p in both valuation structures.
- ✦ The sum of bids exceeds equilibrium predictions, on average.
- ✦ The average sum of bids only exceeds predictions when values are private.

Idea
○○○○○○○

Theory
○○○○○○○

Design
○○○○○○○○○○○○

Results
○○○○○○○○○○○○○○○○○○○○

Summary
○

Summary

Summary

- ✦ We contribute to the study of behaviour in laboratory contests in a simple game environment.
- ✦ We focus on a clean comparative static prediction of whether behaviour is monotonic in private signals, varying the correlation between signals.
- ✦ We find that in common-value settings, behaviour is not well-predicted by equilibrium: in fact, the comparative statics predictions are often backward, although this is generally not significant.