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Outlier-robust identification of switching regimes: an application to the S&P 500

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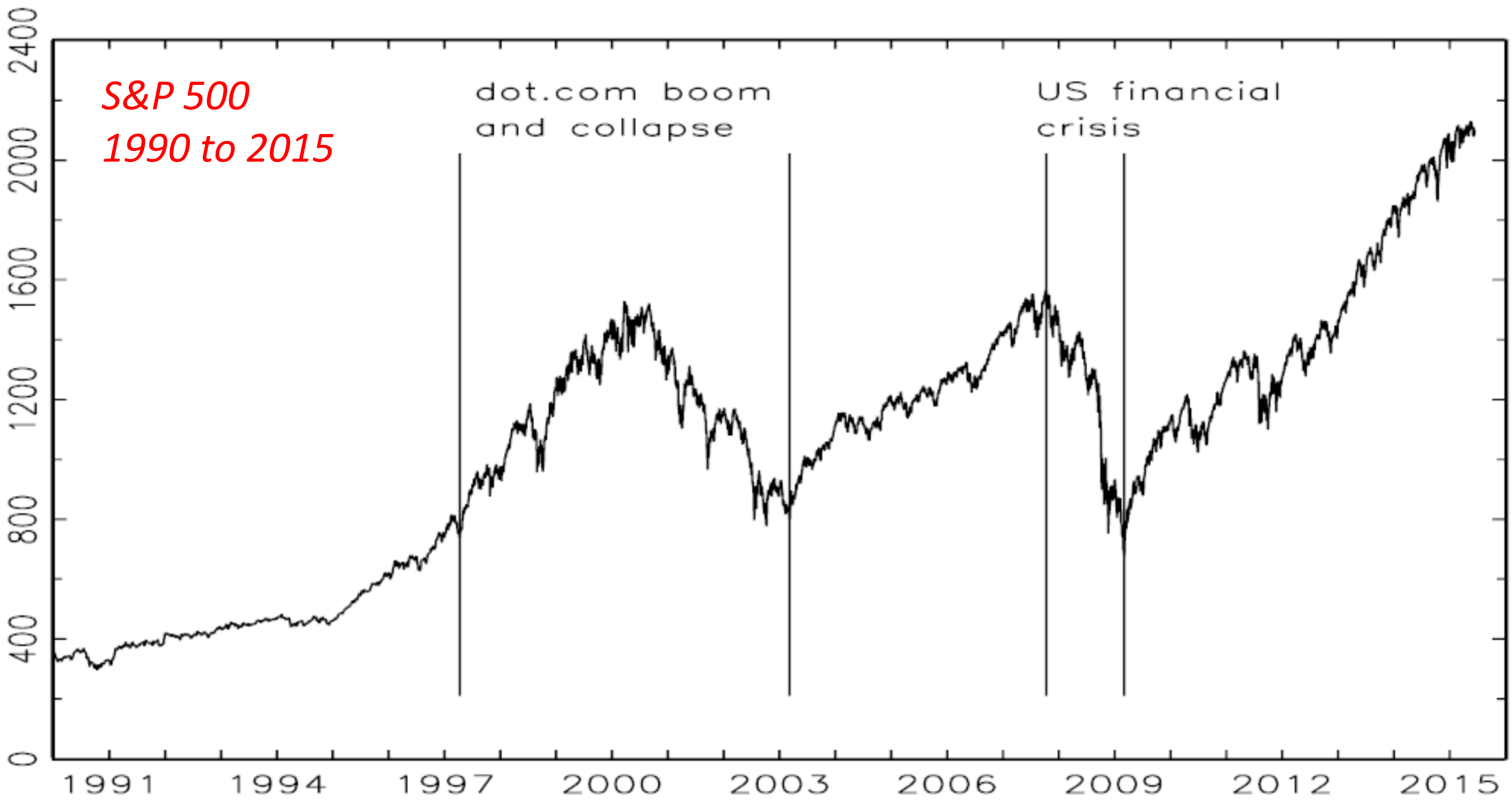
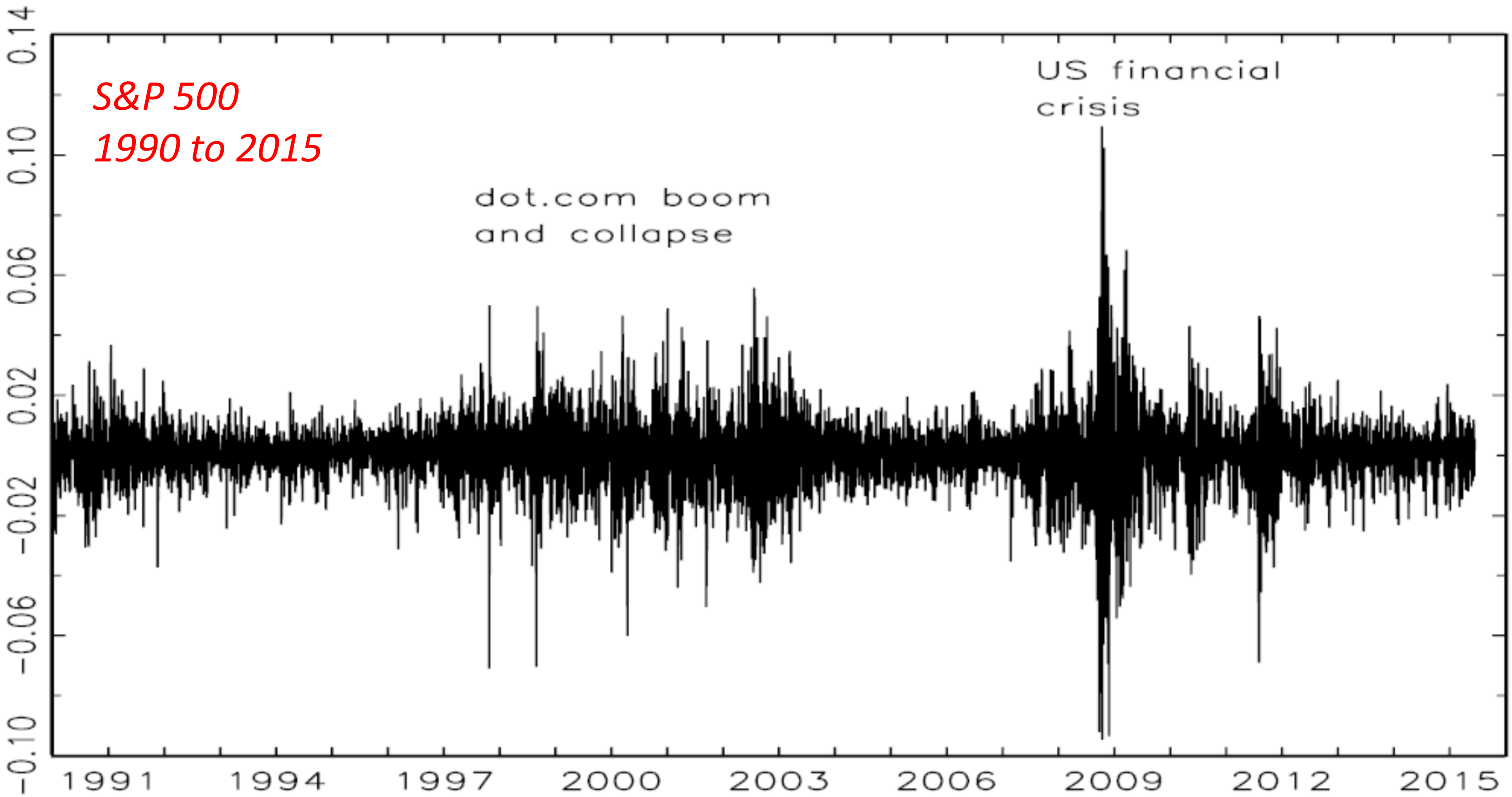

S&P 500
1990 to 2015
dot.com boom
and collapse
US financial
crisis
2400
2000
1600
1200
800
400
0
1991
1994
1997
2000
2003
2006
2009
2012
2015


S&P 500
1990 to 2015
US financial
crisis
dot.com boom
and collapse
0.14
0.10
0.06
0.02
−0.02
−0.06
−0.10
1991
1994
1997
2000
2003
2006
2009
2012
2015

Markov regime-switching models

In financial markets, high and low volatility periods switch each other.

The class of *Markov regime-switching (MS) models* (Hamilton, 1989; Kim and Nelson, 1999) is a possible way of modelling the time-varying probability distribution of asset returns.

MS models assume that data are generated by a mixture of different probability distributions with time-varying parameters which are driven by an underlying Markov chain.

Objective of paper, Outliers

The main objective is to study the performance of different dynamic two-state MS models *for which the treatment of outliers is different*.

“An outlier is an observation which is inconsistent with a model which is thought to be appropriate for the overwhelming majority of the observations” (Harvey, 1989).

For a two-state MS model, outliers are not generated by any of the regimes.

Idea

Suppose that the initial observations of the sample are generated by the low volatility regime.

When the first observation of the high volatility regime arrives, then it will be an outlier for the low volatility regime.

If this observation is followed by subsequent volatile observations, then the MS model will recognize that the process has entered the high volatility regime, and the high volatility observation no longer is an outlier.

Idea

Nevertheless, from time to time, such extreme observations appear that do not belong to any of the regimes.

The question is how effectively a two-regime dynamic regime-switching model can identify the switch between different regimes in these cases.

We show that this crucially depends on how outlier observations are treated by the dynamic model.

Idea

We consider two dynamic models for asset return volatility:

First, the well-known *generalized autoregressive conditional heteroscedasticity (GARCH) model* (Bollerslev, 1986; Taylor, 1986).

Second, the recent and from a statistical point of view very effective *Beta-t-EGARCH model* (Harvey and Chakravarty, 2008) that belongs to class of *dynamic conditional score (DCS) models* (Harvey, 2013).

Idea - GARCH

In GARCH the dynamic equation is updated by the *square of the previous observation* in each time period. Thus, *GARCH accentuates outliers in the conditional variance equation*.

If GARCH is persistent, then outliers will impact future observations through the conditional variance.

Therefore, the regime-switching GARCH model can be rather confused by outlier observations which are not generated by any of the regimes.

Idea - DCS

In DCS models the dynamic equation is updated by lags of the conditional score with respect to a time-dependent parameter.

An important property of these models is that the conditional score discounts extreme observations. Therefore, in DCS models outliers will have little effect on future observations.

Hence, a regime-switching DCS model is not sensitive to outliers and it may identify different regimes effectively.

Data

Daily percentage change in S&P 500 for period

2 January 1990 to 17 June 2015

T=6,416 days are observed

(*source*: Bloomberg)

MS-ARMA(1,1) plus MS-GARCH(1,1) with leverage effects

$$y_t = \mu_t(s_t) + \lambda_t^{1/2}(s_t)\epsilon_t(s_t)$$

$$\epsilon_t(s_t) \sim t[\nu(s_t)] \text{ i.i.d.}$$

Location (MS-ARMA):

$$\begin{aligned}\mu_t(s_t) &= c(s_t) + \phi(s_t)y_{t-1} + \theta(s_t)e_{t-1}(s_t) \\ &= c(s_t) + \phi(s_t)y_{t-1} + \theta(s_t)\lambda_{t-1}^{1/2}(s_t)\epsilon_{t-1}(s_t)\end{aligned}$$

MS-ARMA(1,1) plus MS-GARCH(1,1) with leverage effects

In the location, we avoid path-dependence by

$$e_{t-1}(s_t) = E[e_{t-1}(s_{t-1})|s_t, y_1, \dots, y_{t-1}]$$

Scale (MS-GARCH with leverage effects):

$$\lambda_t(s_t) = \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + \{\alpha(s_t) + \alpha^*(s_t)I[y_{t-1} - \mu_{t-1}(s_t) < 0]\}u_{t-1}(s_t)$$

MS-ARMA(1,1) plus MS-GARCH(1,1) with leverage effects

$$\lambda_{t-1}(s_t) = E[\lambda_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$u_{t-1}(s_t) = E[u_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$u_t(s_t) = \lambda_t(s_t)\epsilon_t^2(s_t)$$

Why to include leverage effects?

"Volatility tends to respond more to falls in stock prices than to rises. One explanation for this phenomenon is that a drop in the share price of a firm will lower the market value and thereby increase the debt-equity ratio. As a result, the risk to investors, as residual claimants, is increased. Hence the term leverage effect" (Harvey, 2013).

MS-QAR(1) plus MS-Beta-t-EGARCH(1,1) with leverage effects

$$y_t = \mu_t(s_t) + \exp[\lambda_t(s_t)]\epsilon_t(s_t)$$

$$\epsilon_t(s_t) \sim t[\nu(s_t)] \text{ i.i.d.}$$

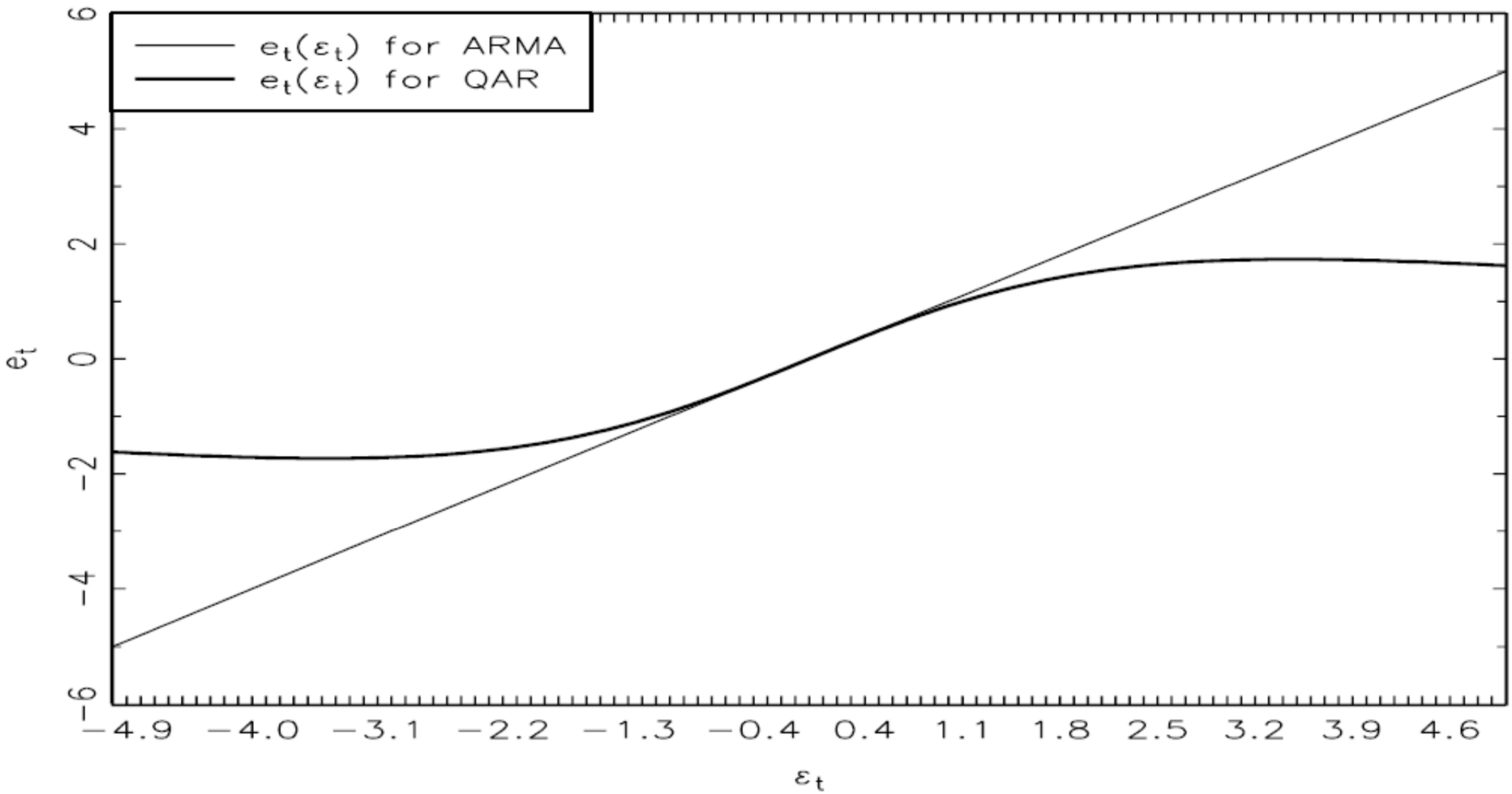
Location (quasi-AR, QAR):

$$\mu_t(s_t) = c(s_t) + \phi(s_t)\mu_{t-1}(s_t) + \theta(s_t)e_{t-1}(s_t)$$

$$\mu_{t-1}(s_t) = E[\mu_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$e_{t-1}(s_t) = E[e_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$e_t(s_t) = \left[1 + \frac{\epsilon_t^2(s_t)}{\nu(s_t)}\right]^{-1} \exp[\lambda_t(s_t)]\epsilon_t(s_t)$$



$e_t(\varepsilon_t)$ for ARMA
$e_t(\varepsilon_t)$ for QAR
6
4
2
0
−2
−4
−6
e_t
−4.9
−4.0
−3.1
−2.2
−1.3
−0.4
0.4
1.1
1.8
2.5
3.2
3.9
4.6
ε_t

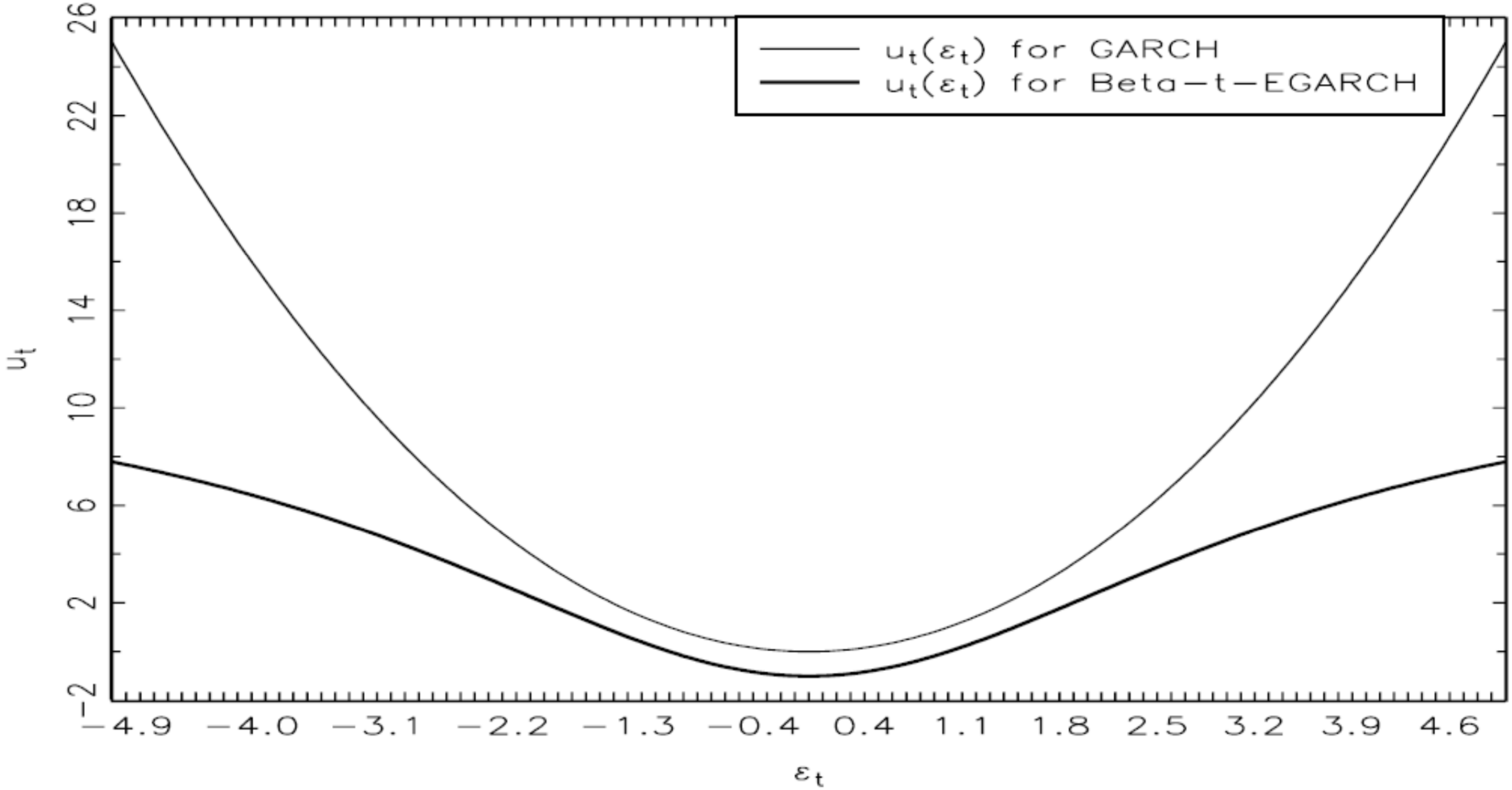
Scale (Beta-t-EGARCH with leverage effects):

$$\lambda_t(s_t) = \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + \alpha(s_t)u_{t-1}(s_t)$$
$$+\alpha^*(s_t)\mathrm{sgn}[\mu_{t-1}(s_t) - y_{t-1}][u_{t-1}(s_t) + 1]$$

$$\lambda_{t-1}(s_t) = E[\lambda_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$u_{t-1}(s_t) = E[u_{t-1}(s_{t-1})|s_t, y_1, \ldots, y_{t-1}]$$

$$u_t(s_t) = \frac{[\nu(s_t) + 1]\epsilon_t^2(s_t)}{\nu(s_t) + \epsilon_t^2(s_t)} - 1$$


$u_t(\varepsilon_t)$ for GARCH
$u_t(\varepsilon_t)$ for Beta-t-EGARCH
u_t
ε_t
26
22
18
14
10
6
2
−2
−4.9
−4.0
−3.1
−2.2
−1.3
−0.4
0.4
1.1
1.8
2.5
3.2
3.9
4.6

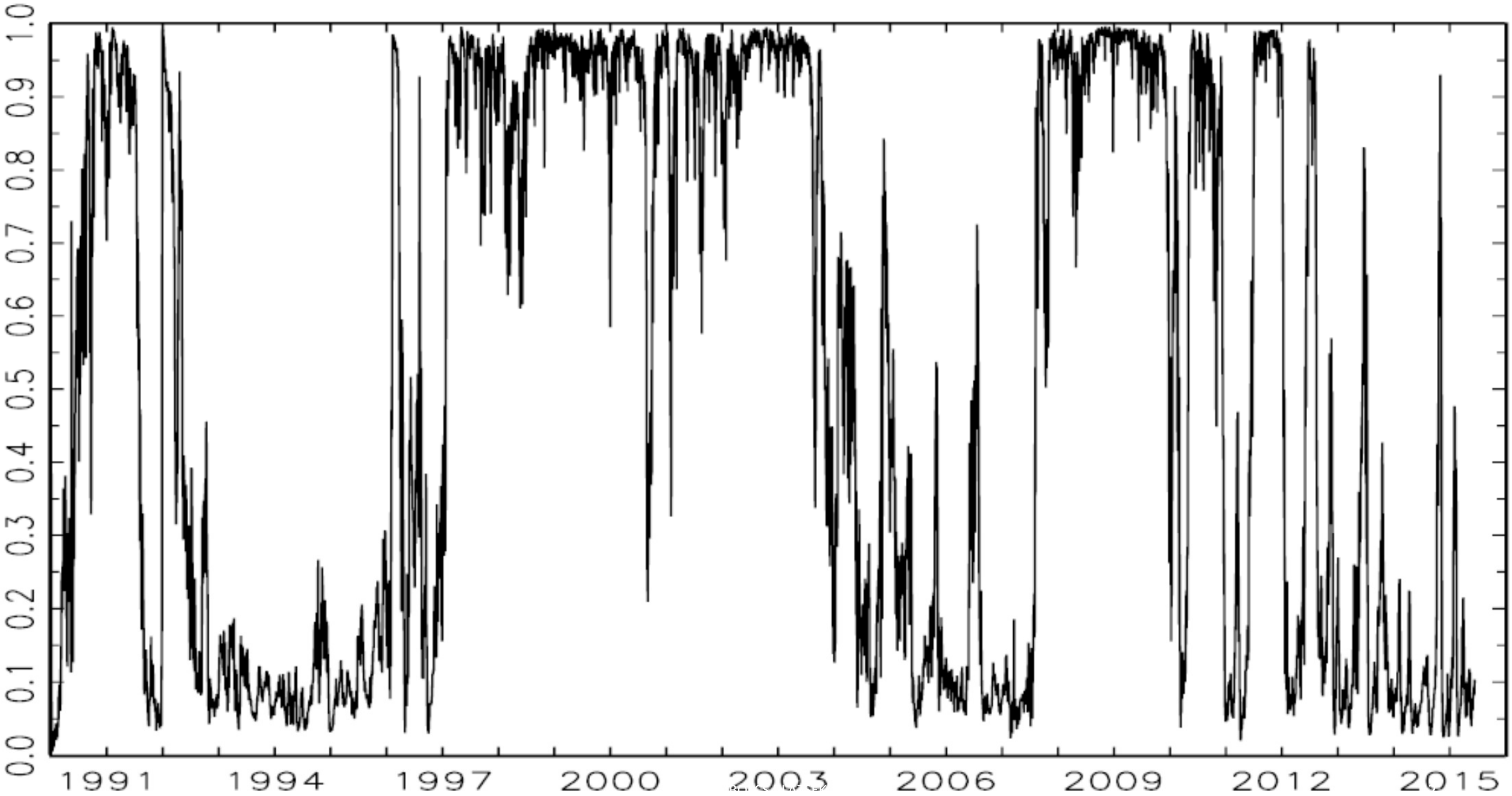
Results

We are not able to estimate MS-ARMA plus MS-GARCH with leverage effects. Some parameters converge to the boundary of the parameter space in the numerical maximization of the log likelihood function.

MS-QAR plus MS-Beta-t-EGARCH with leverage effects can be estimated very effectively. The ML estimation procedure is robust.

For MS-QAR plus MS-Beta-t-EGARCH with leverage effects we have the next filtered probability series:

Filtered probabilities of the high volatility regime



Results

The main difference between the benchmark regime-switching model and the MS-DCS model considered is *the way in which observations are transformed in the innovation term of conditional location and scale equations*.

The failure of MS-ARMA plus MS-t-GARCH with leverage effects and the success of MS-QAR plus MS-Beta-t-EGARCH with leverage effects supports the main point of this work, outlined in the introduction.

Results

We compared the full model, MS-QAR plus MS-Beta-t-EGARCH with leverage effects with:

a) MS-QAR plus MS-Beta-t-EGARCH without leverage effects
b) QAR plus Beta-t-EGARCH with leverage effects
c) QAR plus Beta-t-EGARCH without leverage effects

Results

The MS-QAR plus MS-Beta-t-EGARCH with leverage effects is superior to all alternatives with respect to LL, AIC, BIC and HQC.

The leverage effects coefficient is highly significant in MS-QAR plus MS-Beta-t-EGARCH with leverage effects.

Conditions of covariance stationarity (Abramson and Cohen, 2007) hold for MS-QAR plus MS-Beta-t-EGARCH with leverage effects.

Thank you for your attention!

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