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# Score-driven equity plus gold portfolios before and during the COVID-19 pandemic

Astrid Ayala · Szabolcs Blazsek* · Adrian Licht
School of Business, Universidad Francisco Marroquín, Guatemala City, 01010, Guatemala

**Abstract:** We study the out-of-sample performances of portfolios, for which the mean and covariance matrix of returns are updated using score-driven filters. To the best of our knowledge, the present paper provides the most complete analysis on score-driven portfolios in the literature. We compare the performances of 2,720 portfolios for score-driven copulas, 40 portfolios for DCC (dynamic conditional correlation) models, and the naïve portfolio. For the score-driven portfolios, we use eight score-driven copulas (Clayton; rotated Clayton; Frank; Gaussian; Gumbel; rotated Gumbel; Plackett; Student's $t$), four portfolio optimization strategies (minimum-variance; Sharpe ratio-based mean-variance; two utility function-based mean-variance), five portfolio weight updating frequencies (weekly; monthly; quarterly; semi-annual; annual), and 17 combinations of (i) AR (autoregressive)-$t$-GARCH (generalized autoregressive conditional heteroskedasticity), (ii) QAR (quasi-AR)-Beta-$t$-EGARCH (exponential GARCH), (iii) QAR-Beta-Gen-$t$-EGARCH (generalized $t$-distribution), (iv) QAR-exponential generalized beta distribution of the second kind (EGB2)-EGARCH, and (v) QAR-normal-inverse Gaussian (NIG)-EGARCH. For the DCC-based portfolios we use AR-Gaussian-GARCH and AR-$t$-GARCH marginals. In the empirical application we study the performances of equity plus gold portfolios. The full sample period is from November 2004 to September 2021. The full investment period is from October 2014 to September 2021, which is divided into the pre-COVID-19 (pre-coronavirus disease of 2019) and the COVID-19 periods. Our results provide suggestions on the choice of the econometric model specifications and portfolio strategies for equity-gold portfolio investors. Several score-driven portfolios statistically outperform the naïve and DCC portfolios, and the superiority of score-driven portfolios is more important for the COVID-19 period than for the pre-COVID-19 period.

**Keywords:** Dynamic conditional score (DCS) models; Generalized autoregressive score (GAS) models; Score-driven copulas; Equity-gold portfolios

**JEL Classification:** C32, C52, C58, G11

*Corresponding author. Address: School of Business, Universidad Francisco Marroquín, Call Manuel F. Ayau, Guatemala City, 01010, Guatemala. Telephone: +502-2338-7783; E-mail: sblazsek@ufm.edu

## 1 Introduction

In this paper, we study the performances of score-driven portfolios by using several alternatives of: (i) classical and score-driven models of marginal distributions, (ii) classical and score-driven models of association, (iii) portfolio optimization strategies, and (iv) portfolio weight updating frequencies, for (v) low- and high-market-volatility periods. Our analysis is motivated by the fact that there are very few works in the literature which study portfolio optimization on score-driven models.

Score-driven models are introduced in the works of Creal et al. (2008, 2011, 2013), Harvey and Chakravarty (2008), and Harvey (2013). Those models are observation-driven models (Cox 1981), for which the dynamic parameters are updated by the partial derivatives of the log conditional density of the dependent variables with respect to dynamic parameters (hereinafter, the updating terms are named score functions). Some of the advantages of the score-driven models over the classical observation-driven time series models are the following: (i) Score-driven models are robust to outliers and missing data (Harvey 2013). (ii) Score-driven models are generalizations of classical observation-driven models (Creal et al. 2013; Harvey 2013). (iii) A score-driven update locally reduces the Kullback–Leibler divergence in expectation at every step, and only the score-driven updates have this asymptotic property (Blasques et al. 2015). These advantages may imply better-performing financial portfolios for well-specified score-driven models than for the classical observation-driven time series models.

Our method of portfolio analysis, which compares several score-driven portfolios, DCC (dynamic conditional correlation) (Engle 2002) portfolios, and the naïve portfolio (DeMiguel et al. 2009), extends the methods of portfolio analysis of Atskanov (2016) and Bernardi and Catania (2018) which are the most relevant papers from the literature on score-driven portfolios. We study the performances of 2,720 portfolio strategies for score-driven copulas, and we compare those performances with the performances of 40 DCC-based portfolios and the naïve portfolio (i.e., the equally weighted portfolio).

For the score-driven portfolios we use a general analysis, and we consider the following alternatives: (i) We use eight score-driven copulas (i.e., Clayton, rotated Clayton, Frank, Gaussian, Gumbel, rotated Gumbel, Plackett, and Student's $t$ copulas). Score-driven copulas are used in the following works: Boudt et al. (2012); Avdulaj and Barunik (2013, 2015); Creal et al. (2013); Harvey (2013); De Lira Salvatierra and Patton (2015); Koopman et al. (2015); Atskanov (2016); Bartels and Ziegelmann (2016); Harvey and Thiele (2016); Koopman et al. (2016); Oh and Patton (2016); Cerrato et al. (2017); Ayala and Blazsek (2018a, 2018b); Bernardi and Catania (2018). (ii) We use four portfolio optimization strategies (i.e., minimum-variance portfolio, Sharpe ratio-based mean-variance portfolio, and utility function-based mean-variance portfolios with two alternative risk aversion coefficients) (DeMiguel et al. 2009; DeMiguel and Nogales 2009; Kritzman et al. 2010; Tu and Zhou 2011; Low et al. 2016), by extending the work of Bernardi and Catania (2018). (iii) We use five portfolio updating frequencies (i.e., weekly, monthly, quarterly, semi-annual, and annual), by extending the works of Atskanov (2016) and Bernardi and Catania (2018). (iv) We use 17 alternative pairs of marginal model specifications, for which we use combinations of the AR (Box and Jenkins 1970) plus $t$-GARCH (autoregressive; generalized AR conditional heteroskedasticity) (Engle 1982; Bollerslev 1986, 1987; Glosten et al. 1993), QAR (quasi-AR) (Harvey 2013) plus Beta-$t$-EGARCH (exponential GARCH) (Harvey and Chakravarty

2008), QAR plus Beta-Gen-$t$-EGARCH (generalized $t$-distribution) (Harvey and Lange 2017), QAR plus exponential generalized beta distribution of the second kind (EGB2)-EGARCH (Caivano and Harvey 2014), and QAR plus normal-inverse Gaussian (NIG) distribution-EGARCH (Blazsek et al. 2018). The QAR-Beta-$t$-EGARCH, QAR-Beta-Gen-$t$-EGARCH, QAR-EGB2-EGARCH, and QAR-NIG-EGARCH models use score-driven updates of the expected return and volatility. All volatility models of this paper include leverage effects, to provide asymmetric updates.

As alternatives to the score-driven approach, we use the naïve portfolio strategy, and 40 portfolio strategies for the classical AR-GARCH-DCC model. The 40 portfolio strategies are obtained by using: (i) four portfolio optimization strategies (i.e., minimum-variance portfolio, Sharpe ratio-based mean-variance portfolio, and utility function-based mean-variance portfolios with two alternative risk aversion coefficients), (ii) five portfolio updating frequencies (i.e., weekly, monthly, quarterly, semi-annual, and annual), and (iii) two probability distributions (i.e., Gaussian and Student's $t$ distributions).

In the empirical application, we study the performances of Standard & Poor's 500 (S&P 500) and gold exchange traded fund (ETF) portfolios. We focus on equity and gold investments, because they represent many investors' portfolios due to the diversification and hedging properties of gold (e.g., Hillier et al. 2006; Smirnova 2016; Thongkairat et al. 2019; Akhtaruzzaman et al. 2021). We use daily excess return data on S&P 500 and gold for the period of November 19, 2004 to September 24, 2021, where excess returns are above the 1-month United States (US) Treasury bill (T-bill) rate. We study portfolio performances for (i) the full investment period (October 27, 2014 to September 24, 2021), which is divided into the (ii) the pre-COVID-19 investment period (October 27, 2014 to February 21, 2020), and (iii) the COVID-19 investment period (February 24, 2020 to September 24, 2021).

First, we compare the value of a 1 USD investment for all alternative portfolio strategies. For each of the full, pre-COVID-19, and COVID-19 investment periods, we find that the 10 best-performing score-driven portfolio strategies are superior to the best-performing Gaussian-DCC and $t$-DCC portfolio strategies, which are superior to the naïve portfolio strategy. We also present the specifications of the marginal distributions, score-driven copulas, portfolio weight updating frequencies, and portfolio optimization strategies, which provide the best-performing score-driven models for each investment period. Based on those results, we provide suggestions for equity-gold portfolio investors on model selection and portfolio strategies for (i) investment periods in which both low- and high-volatility periods are present, and (ii) investment periods in which volatility is persistently low or high.

Second, we test the significance of portfolio return differences among score-driven, DCC, and naïve portfolios: (i) We compute the number of score-driven portfolios (N1), which have significantly higher returns than each Gaussian-DCC and naïve portfolio. We compute the number of score-driven portfolios (N2), which have significantly lower returns than each Gaussian-DCC and naïve portfolio. To make (N1) and (N2) comparable, we divide them by the total number of score-driven portfolios: 2,720. The corresponding proportions are denoted (P1) and (P2), respectively. For all investment periods, i.e., for the full, pre-COVID-19, and COVID-19 investment periods, we find that (P1) > (P2). This result supports the use of the score-driven model, instead of the Gaussian-DCC model (Akhtaruzzaman et al. 2021), for equity-gold portfolios for all investment periods.

Moreover, (ii) we compute the number of score-driven portfolios ($\tilde{\text{N}}1$), which have significantly higher returns than each $t$-DCC and naïve portfolio. We compute the number of score-driven portfolios ($\tilde{\text{N}}2$), which have significantly lower returns than each $t$-DCC and naïve portfolio. To make ($\tilde{\text{N}}1$) and ($\tilde{\text{N}}2$) comparable, we divide them by the total number of score-driven portfolios: 2,720. The corresponding proportions are denoted ($\tilde{\text{P}}1$) and ($\tilde{\text{P}}2$), respectively. For the full investment period and the COVID-19 investment period we find that ($\tilde{\text{P}}1$) $>$ ($\tilde{\text{P}}2$). However, for the pre-COVID-19 investment period we find that ($\tilde{\text{P}}2$) $>$ ($\tilde{\text{P}}1$). These results indicate that the aforementioned superiority of the score-driven equity-gold portfolios to the $t$-DCC portfolios is more important for the high-volatility COVID-19 period than for the low-volatility pre-COVID-19 period.

The remainder of this paper is organized as follows: Section 2 reviews the literature on score-driven portfolios. Section 3 presents the methods. Section 4 presents the data and the empirical results. Section 5 concludes. Technical details and further empirical results are presented in the Appendix.

## 2 Literature review on score-driven portfolios

From the literature on score-driven models, the most relevant works for the present paper are the works of Atskanov (2016) and Bernardi and Catania (2018), because in those works the performances of score-driven model-based and classical model-based portfolios are compared.

First, in the work of Atskanov (2016), daily data for 10 liquid stocks from the Russian stock market are used, and the portfolios weights are updated at monthly, quarterly, semi-annual, and annual frequencies. As objective functions of the portfolio optimization, the Sharpe ratio, the Sortino ratio, and the gain-to-pain ratio are used. For the portfolio weight updates, a 365-day rolling data-window is used. For the marginal distributions, the AR model and the GARCH with leverage effects model are used. For the measurement of dynamic correlations a score-driven Gumbel copula is used. As an alternative to the AR plus GARCH plus score-driven Gumbel copula-based portfolio strategy, the naïve portfolio strategy is also considered in the work of Atskanov (2016).

Our contributions are the following: (i) In the work of Atskanov (2016), the AR-GARCH models of the marginal distributions are combined with a single score-driven model of the association. We consider AR-GARCH and score-driven models for the marginal distributions, which are combined with several score-driven copula alternatives of the association. (ii) In addition to the portfolio weight updating frequencies of Atskanov (2016), we also use the weekly updating frequency.

Second, in the work of Bernardi and Catania (2018), weekly data for the S&P 500, Nikkei 225, FTSE 100, DAX 30, and CAC 40 indices are used. Portfolio weights are updated at the quarterly frequency, by using rolling windows of 449 observations. As an objective function of the portfolio optimization, a constant relative risk aversion (CRRA) utility function is used. For the marginal distributions score-driven models for the Skew $t$-distribution are used, to update the mean, volatility, and degrees of freedom parameters of returns. For dynamic correlations a score-driven Markov-switching (MS) Student's $t$-copula including the following exogenous explanatory variables is used: (i) weekly change in the 3-month T-bill rate, (ii) weekly change in the slope of the yield curve (measured by the difference between the 10-year Treasury rate and the 3-month T-bill rate), and (iii) weekly change in the credit spread between the 10-year BAA rated bonds and the 10-year Treasury rate.

We extend the work of Bernardi and Catania (2018) with respect to the following points: (i) In the volatility filters of the present paper we use leverage effects, which are not included in the volatility filters of Bernardi and Catania (2018). (ii) In addition to the maximization of the utility function, we also consider the maximization of the Sharpe ratio and the minimization of the portfolio variance (i.e., minimum-variance portfolio). (iii) In the work of Bernardi and Catania (2018) a zero risk-free rate is assumed. In the present paper, we use the 1-month T-bill rate for the computation of excess returns. (iv) The weekly, monthly, quarterly, semi-annual, and annual portfolio portfolio weight updating frequencies of the present paper are more general than the quarterly portfolio weight updating frequency of Bernardi and Catania (2018). (v) We use a large variety of alternative score-driven copulas, extending the score-driven copulas from the work of Bernardi and Catania (2018).

## 3 Methods

### 3.1 Portfolio returns

The daily returns of individual assets in the portfolio are $\tilde{r}_{k,t} = (p_{k,t} - p_{k,t-1})/p_{k,t-1}$ for days $t = 1, \ldots, T$, where $k = 1, 2$ indicate the S&P 500 and gold ETFs, respectively. The variable $p_{k,t}$ is the closing price, and we use pre-sample data for $p_{k,0}$. The excess returns are $r_{k,t} = \tilde{r}_{k,t} - r_{f,t}$ for $k = 1, 2$, where $r_{f,t}$ is the 1-month US T-bill rate. We note that for daily returns, we use the simple return, $(p_{k,t} - p_{k,t-1})/p_{k,t-1}$, instead of the log-return, $\ln(p_{k,t}/p_{k,t-1})$, where $\ln(\cdot)$ is the natural logarithm function. This is motivated by the fact that the portfolio excess return formula $r_{P,t} = w'r_t = w_1 r_{1,t} + w_2 r_{2,t}$, where $r_t \equiv (r_{1,t}, r_{2,t})'$ is a vector of portfolio excess returns and $w \equiv (w_1, w_2)'$ is a vector of portfolio weights, is correct for simple returns, but it is not correct for log-returns (e.g., Tsay 2010).

### 3.2 Portfolio strategies

The first investment strategy is the naïve portfolio strategy, which uses 50%-50% portfolio weights for the S&P 500 ETF and the gold ETF for all days of the investment periods. We consider the naïve portfolio in this paper, because several works of the literature present that it is difficult to significantly beat the naïve portfolio by using more sophisticated strategies (e.g., DeMiguel et al. 2009). For the alternative strategies, the optimal portfolio weights are chosen as follows: (i) Minimizing the portfolio variance $\sigma_P^2 = w'\Sigma w$, where $\Sigma$ is the variance-covariance matrix of the excess returns of all assets in the portfolio. (ii) Maximizing the Sharpe ratio $\mu_P/\sigma_P$, where $\mu_P = w'\mu$ is the expected excess portfolio return, where $\mu$ is a column vector of expected excess returns of all assets in the portfolio, and $\sigma_P$ is the volatility of the excess portfolio return. (iii) Maximizing the utility function $\mu_P - (\zeta/2)\sigma_P^2$ by using the alternative risk aversion coefficients: $\zeta = 1$ and $\zeta = 4$ (DeMiguel et al. 2009). For the strategies (i)-(iii), we consider weekly, monthly, quarterly, semi-annual and annual alternative updates of optimal portfolio weights. In the portfolio return application case study of the present paper, we assume that the investor takes long positions in both the S&P 500 ETF and the gold ETF. In the following section, we present the econometric models that we use for the estimation of the expected excess returns $\mu$ and the variance-covariance matrix of excess returns $\Sigma$.

### 3.3 Econometric models of expected return, volatility, and association

#### 3.3.1 Classical models

For the marginal distribution of asset returns, we use the AR(1) (Box and Jenkins 1970) plus GARCH(1,1) with leverage effects (Bollerslev 1987; Glosten et al. 1993) model:

$$r_{k,t} = \mu_{k,t} + v_{k,t} = \mu_{k,t} + \lambda_{k,t}^{1/2}\epsilon_{k,t} \tag{1}$$

$$\mu_{k,t} = c_k + \phi_k r_{k,t-1} = c_k + \phi_k(\mu_{k,t-1} + v_{k,t-1}) \tag{2}$$

$$\lambda_{k,t} = \omega_k + \beta_k\lambda_{k,t-1} + [\alpha_k + \alpha_k^* \mathbb{1}(v_{k,t-1} < 0)]v_{k,t-1}^2 \tag{3}$$

for $k = 1, 2$, where the excess return $r_{k,t}$ is the sum of the expected excess return $\mu_{k,t} = E(r_{k,t}|\mathcal{F}_{t-1}; \Theta)$ and the unexpected excess return $v_{k,t}$, where $\mathcal{F}_{t-1} = \sigma(r_{k,1}, \ldots, r_{k,t-1} : k = 1, 2)$, and $\Theta$ is the vector of the time-invariant parameters. The unexpected excess return is the product of the dynamic scale parameter $\lambda_{k,t}^{1/2}$ and the i.i.d. error term. For the standardized error term, we consider the alternatives $\epsilon_{k,t} \sim N(0, 1)$ and $\epsilon_{k,t} \sim t(\nu_k)$ with the Student's $t$-distribution. The conditional standard deviation of the unexpected excess return (i.e., conditional volatility) for the Gaussian distribution is $\sigma_{k,t} = \mathrm{SD}(r_{k,t}|\mathcal{F}_{t-1}; \Theta) = \lambda_{k,t}^{1/2}$, and for the $t$-distribution is $\sigma_{k,t} = \mathrm{SD}(r_{k,t}|\mathcal{F}_{t-1}; \Theta) = \lambda_{k,t}^{1/2}[\nu_k/(\nu_k - 2)]^{1/2}$. For the filter $\lambda_{k,t}$, we consider the possibility of leverage effects $\alpha_k^*$ for $k = 1, 2$, for which negative unexpected excess returns are identified by using the indicator function $\mathbb{1}(x)$. The filter $\mu_{k,t}$ is initialized by $c_k/(1 - \phi_k)$. The filter $\lambda_{k,t}$ for $k = 1, 2$ is initialized by parameters $\lambda_{1,1}$ and $\lambda_{2,1}$, respectively. For the correlation coefficients, we the multivariate normal distribution and the multivariate $t$-distribution for the DCC model. For the estimation of the AR-Gaussian-GARCH plus Gaussian-DCC and the AR-$t$-GARCH plus $t$-DCC models, we use the two-step maximum likelihood (ML) method, as suggested in the work of Engle (2002).

#### 3.3.2 Score-driven models

First, for the marginal distribution of the excess returns we use five alternatives. For the first alternative we use the classical AR plus $t$-GARCH specification from Section 3.3.1. For the remaining alternatives we use the following score-driven models (Harvey and Chakravarty 2008; Harvey 2013):

$$r_{k,t} = \mu_{k,t} + v_{k,t} = \mu_{k,t} + \exp(\lambda_{k,t})\epsilon_{k,t} \tag{4}$$

$$\mu_{k,t} = c_k + \phi_k\mu_{k,t-1} + \theta_k s_{\mu,k,t-1} \tag{5}$$

$$\lambda_{k,t} = \omega_k + \beta_k\lambda_{k,t-1} + \alpha_k s_{\lambda,k,t-1} + \alpha_k^* \mathrm{sgn}(-v_{k,t-1})(s_{\lambda,k,t-1} + 1) \tag{6}$$

for $k = 1, 2$, where $\exp(\cdot)$ is the exponential function, and for the $\epsilon_{k,t}$ i.i.d. error term we use the following five alternative distributions: $\epsilon_{k,t} \sim t[0, 1, \exp(\nu_k) + 2]$, $\epsilon_{k,t} \sim \text{Gen-}t[0, 1, \exp(\nu_k) + 2, \exp(\eta_k)]$, $\epsilon_{k,t} \sim \mathrm{EGB2}[\exp(\nu_k), \exp(\eta_k)]$, and $\epsilon_{k,t} \sim \mathrm{NIG}[\exp(\nu_k), \exp(\nu_k)\tanh(\eta_k)]$, where $\nu_k \in \mathbb{R}$ and $\eta_k \in \mathbb{R}$.

The updating terms of Eqs. (5) and (6) are the scaled score function $s_{\mu,k,t}$, the score function $s_{\lambda,k,t}$, respectively, which we define later. For $\lambda_{k,t}$, we consider the possibility of leverage effects $\alpha_k^*$ for $k = 1, 2$, for which asymmetry is measured using the signum function $\text{sgn}(\cdot)$. We note that we do not use the skewed generalized $t$-distribution, which is more general than the Student's $t$ and Gen-$t$ distributions, because Eq. (6) captures asymmetries by using the leverage effects term.

In the literature, the sigma-algebra $\mathcal{F}_{t-1}$ includes the initial values of all score-driven filters (e.g., Blasques et al. 2021). In the present paper, we use the same sigma-algebra $\mathcal{F}_{t-1}$ for the score-driven models and for the classical models (Section 3.3.1), because the score-driven filters are initialized by using some elements of $\Theta$; i.e., $\mu_{k,t}$ for $k = 1, 2$ are initialized by $c_k/(1 - \phi_k)$ for $k = 1, 2$, respectively, and $\lambda_{k,t}$ for $k = 1, 2$ are initialized by the parameters $\lambda_{k,1}$ for $k = 1, 2$, respectively.

For each score-driven probability distribution, the log conditional density of $r_{k,t}|(\mathcal{F}_{t-1}; \Theta)$, the scaled score function $s_{\mu,k,t}$, the score function $s_{\lambda,k,t}$, the conditional expected return $E(r_{k,t}|\mathcal{F}_{t-1}; \Theta)$, and the conditional volatility $\sigma(r_{k,t}|\mathcal{F}_{t-1}; \Theta)$ are presented in Appendix A. For the results presented in Appendix A, we refer to the works of Harvey and Chakravarty (2008), Harvey (2013), Caivano and Harvey (2014), Blazsek et al. (2018), and Ayala et al. (2019).

We use all possible combinations of the AR-$t$-GARCH, QAR-Beta-$t$-EGARCH, and QAR-Beta-Gen-$t$-EGARCH models which provide nine alternatives of the marginal models. Moreover, we also use all possible combinations of the AR-$t$-GARCH, EGB2-EGARCH, and NIG-EGARCH models which also provide nine alternatives of the marginal models. As the AR-$t$-GARCH plus AR-$t$-GARCH combination appears in both sets of combinations, the total number of marginal model specifications is 17. The consideration of these two separate groups of models for the marginal distributions is motivated by the fact that (i) the score functions of the QAR-Beta-$t$-EGARCH and QAR-Beta-Gen-$t$-EGARCH models, and (ii) the score functions of the EGB2-EGARCH and NIG-EGARCH models transform the error term $\epsilon_{k,t}$ in very different ways. We present more details on this issue in Appendix A.

Second, for the dynamic association of S&P 500 and gold excess returns, we use the following score-driven copulas: Clayton, rotated Clayton, Frank, Gaussian, Gumbel, rotated Gumbel, Plackett, and Student's $t$ copulas. For all copulas, we model the score-driven parameter of association $\rho_t$. (i) We use the following specification of the dynamic parameter $\tilde{\rho}_t$:

$$\tilde{\rho}_t = \delta + \gamma\tilde{\rho}_{t-1} + \kappa s_{\rho,t-1} \tag{7}$$

where the conditional copula score is given by $s_{\rho,t} = \partial \ln c_t[F_1(r_{1,t}|\mathcal{F}_{t-1}; \Theta), F_2(r_{2,t}|\mathcal{F}_{t-1}; \Theta)]/\partial\rho_t$, where $F_1(\cdot|\cdot)$ and $F_2(\cdot|\cdot)$ are the marginal conditional distribution functions of the S&P 500 ETF and gold ETF excess returns, respectively. For each copula, the copula density function $c_t$ and the copula score $s_{\rho,t}$ are presented in Appendix B. We initialize $\tilde{\rho}_t$ by using $\delta/(1 - \gamma)$. (ii) The score-driven parameter of association $\rho_t$ is determined by using the following transformations: For the Clayton and rotated Clayton copulas: $\rho_t = \exp(\tilde{\rho}_t) - 1 \in (-1, \infty)$. For the Frank copula: $\rho_t = \tilde{\rho}_t$. For the Gaussian and Student's $t$ copulas: $\rho_t = [1 - \exp(-\tilde{\rho}_t)]/[1 + \exp(-\tilde{\rho}_t)] \in (-1, 1)$. For the Gumbel and rotated Gumbel copulas: $\rho_t = \exp(\tilde{\rho}_t) + 1 \in (1, \infty)$. For the Plackett copula: $\rho_t = \exp(\tilde{\rho}_t) \in (0, \infty)$.

We estimate all score-driven models in one step, by using the ML method (Harvey 2013; Blasques et al. 2021). In the literature, several works implement two-step estimation procedures for models with copulas (e.g., Bernardi and Catania 2018). In those papers, the parameters of the marginal distributions are estimated in a first step, and the parameters of the copula are estimated in a second step. We use a one-step estimation procedure, which is motivated by the work of Joe (2015).

## 4 Empirical application

### 4.1 Precious metals in financial portfolios

In the finance literature, the results of several works suggest the use of gold-related financial assets in investment portfolios. Examples of some recent works are the work of Hillier et al. (2006), in which the investment role of precious metals is investigated, by using daily data for the London gold bullion price, the London Free Market platinum price, and the Zurich silver price for the period of 1976 to 2004. As proxies of stock market returns, the S&P 500 and the MSCI EAFE (Morgan Stanley Capital International, Europe-Australasia-Far East) indices are used. Hillier et al. (2006) find that precious metals have low correlations with the stock index, which suggests that precious metals provide diversification opportunities for equity investors.

Another example is the work of Smirnova (2016), in which it is shown that gold and gold mining stocks act as diversifiers and safe havens in market downturns for the period of 1970 to 2012. Smirnova (2016) argues that gold and the S&P 500 move in opposite directions, and that 'preliminary evidence suggests that during serious market downturns gold not only holds its value but tends to go up on average, thus serving a safe haven by protecting the portfolio value in times of distress' (Smirnova 2016, p. 79). A further recent study is the work of Thongkairat et al. (2019), in which GARCH plus copula models are applied, to solve the risk-return portfolio optimization problem for portfolios of stock, gold, and crude oil. Daily time series data of the S&P 500 index, gold price, and West Texas Intermediate (WTI) crude oil price, for the period of 1996 to 2016, are used. The authors report that the optimal portfolio weights are 36.06%, 41.63%, and 22.31% for S&P 500, gold, and crude oil, respectively, which indicate the diversification capabilities of gold.

In another recent and relevant work, Akhtaruzzaman et al. (2021) study the role of gold as a hedge asset in different phases of the COVID-19 crisis. The authors analyze the hedging property of gold for the following periods: (i) pre-COVID-19 period (from January 2, 2018, to December 30, 2019), (ii) COVID-19 Phase I period (from December 31, 2019 to March 16, 2020), and (iii) COVID-19 Phase II period (from March 17 to April 24, 2020). They find that: (i) gold served as a safe haven asset for stock markets during Phase I, and (ii) gold lost its safe haven role during Phase II. By using hourly data, the paper evaluates the optimal weights of gold in several portfolios, in which gold is combined with one of the assets from the following list: S&P 500, Euro Stoxx 50, Nikkei 225, China Financial Times Stock Exchange (FTSE) A50 indices, WTI (West Texas Intermediate) crude oil, USD to EUR (euro) exchange rate, USD to JPY (Japanese Yen) exchange rate, and USD to CNY (Chinese Yuan) exchange rate. The optimal portfolio weights are determined by the optimal hedging portfolio, for which the portfolio variance is minimized (i.e., minimum-variance portfolio). For the estimation of the parameters of the minimum-variance portfolio, the authors use the AR-Gaussian-GARCH model for the marginal distributions, and the Gaussian-DCC model for the dynamic correlation coefficients.

For the analysis of S&P 500 and gold portfolios, our contributions to the work of Akhtaruzzaman et al. (2021) are the following: (i) The pre-COVID-19 and the COVID-19 sample periods of our paper significantly extend the observation period of Akhtaruzzaman et al. (2021). (ii) In addition to the minimum-variance portfolio, we also use the naïve and several mean-variance portfolio strategies. (iii) We significantly extend the AR plus Gaussian-GARCH plus Gaussian-DCC model-based portfolio strategy of Akhtaruzzaman et al. (2021) to the naïve strategy, 40 AR plus GARCH plus DCC model-based strategies (for Gaussian and $t$ distributions), and 2,720 score-driven model-based strategies.

### 4.2 Data

We use daily closing price data for 'SPDR S&P 500 ETF Trust' and 'SPDR Gold Shares' ETFs for the full sample period of November 19, 2004 to September 24, 2021 (data source: Bloomberg). The S&P 500 and gold ETF prices are denoted by $p_{1,t}$ and $p_{2,t}$, respectively, which we use to compute daily returns $\tilde{r}_{k,t}$ for $k = 1, 2$. We use the 1-month US T-bill yield $r_{f,t}$ (data source: Kenneth R. French - Data Library), to compute the daily excess returns $r_{k,t}$ for $k = 1, 2$. In Table 1, the descriptive statistics of daily excess returns $r_{1,t}$ and $r_{2,t}$ for the full sample period are presented. In Fig. 1, the evolution of the S&P 500, gold prices, and daily excess returns for the full sample period is presented.

The full sample period is divided into the pre-investment period (from November 19, 2004 to October 24, 2014) and the full investment period (from October 27, 2014 to September 24, 2021). Furthermore, the full investment period is divided into the pre-COVID-19 investment period (from October 27, 2014 to February 21, 2020), and the COVID-19 investment period (from February 24, 2020 to September 24, 2021). For the three investment periods one-step ahead out-of-sample forecasts of expected excess return, volatility, and correlation coefficients are estimated, by using rolling data windows (each with 2,500 observations). In Table 1, the descriptive statistics for $r_{k,t}$ for the pre-investment period, and for the full, pre-COVID-19, and COVID-19 investment periods are presented.

For all data periods, we find significant partial autocorrelation functions (PACFs) for the S&P 500 returns, non-significant PACFs for gold returns, and significant ARCH test statistics (Engle 1982) for both assets (Table 1). In the econometric specifications of this paper, we use first-order AR and QAR dynamics for expected excess returns, to control for serial correlation in the mean, and we use first-order dynamics for the volatility filters, to control for ARCH effects.

We note that the standard deviation estimates for S&P 500 and gold are lower in the pre-COVID-19 investment period than in the COVID-19 investment period (Table 1). Hence, two regimes are identified for the two subperiods. The best-performing portfolios for the pre-COVID-19 and the COVID-19 investment periods may provide suggestions on the choices of the econometric specification, and the portfolio strategy for equity plus gold portfolio investors.

[APPROXIMATE LOCATION OF TABLE 1 AND FIGURE 1]

### 4.3 Best-performing models and portfolio strategies

In Fig. 2, we present the value of 1 USD investments over the full investment period (Fig. 2(a)), the pre-COVID-19 investment period (Fig. 2(b), and the COVID-19 investment period (Fig. 2(c)), providing a graphical illustration of portfolio performances. Those figures focus on the evolution of the 10 best-performing score-driven copula-based strategies, the best-performing Gaussian-DCC and $t$-DCC model-based strategies, and the naïve strategy. According to the results, for all investment periods of Fig. 2, the performance of the score-driven models (black lines) is superior to that of the best-performing Gaussian-DCC model (green lines), the best-performing $t$-DCC model (red lines), and the naïve strategy (blue lines). For the full investment period and the pre-COVID-19 investment period, Gaussian-DCC is superior to $t$-DCC. For the COVID-19 investment period, $t$-DCC is superior to Gaussian-DCC (although the value estimates are very similar). The naïve portfolio is inferior to all other portfolios of Fig. 2. The model specifications and the portfolio strategies of the 10 best-performing portfolios with score-driven copulas of Fig. 2 are presented in Table 2. Some interesting conclusions are obtained from Table 2 on the best-performing score-driven portfolios:

First, for the S&P 500, for the full investment period the best-performing models of the marginal distribution are always score-driven models, and in particular a score-driven model which uses the NIG or EGB2 specification. For NIG and EGB2, an important property of the score functions is that they discount the new information more than GARCH, but they discount the new information less than the score-driven models which use the Student's $t$ and Gen-$t$ specifications. This result indicates for S&P 500 investors that if the investment period includes different regimes of high and low volatility, then the use of the score-driven marginals with NIG and EGB2 specifications may be the best choices.

For the S&P 500, for the pre-COVID-19 and COVID-19 investment periods (i.e., low-volatility and high-volatility periods, respectively), interestingly, we find that almost all the best-performing score-driven models of the marginal distribution use the Student's $t$ and Gen-$t$ specifications. If we think about the pre-COVID-19 and COVID-19 investment periods as two different regimes, then this result indicates that for different regimes in which volatility is persistently low or persistently high, the score-driven marginals with the Student's $t$ and Gen-$t$ specifications may provide better portfolios than the score-driven marginals with the NIG and EGB2 specifications.

Moreover, for the S&P 500, another interesting finding is that the AR-$t$-GARCH model (which does not discount new information) provides the best-performing portfolios for the low-volatility pre-COVID-19 period (i.e., only two of the marginal models are score-driven out of 10), while the performance of the score-driven marginal distributions increases for the high-volatility COVID-19 period (i.e., 5 out of 10 are score-driven marginals). This result indicates for S&P 500 investors that the use of the AR-$t$-GARCH model is the best choice for a persistently low-volatility period, while the use of the score-driven marginals with the Student's $t$ and Gen-$t$ specifications might be a better choice for a persistently high-volatility period.

Second, for the gold ETF, for the full investment period the best-performing model of the marginal distribution for almost all the cases is the AR-$t$-GARCH model (i.e., only one of the marginal models is score-driven which uses the NIG distribution). This result indicates for gold ETF investors that if the investment period includes different regimes of high and low volatility, then no discounting of the new information is needed to create better-performing portfolios (i.e., AR-$t$-GARCH may be preferred to the score-driven models of the marginal distribution).

For the gold ETF, for the pre-COVID-19 and COVID-19 investment periods, we find that most of the best-performing models are score-driven marginals with the Student's $t$ and Gen-$t$ specifications. We also find that AR-$t$-GARCH provides better-performing portfolios for the low-volatility pre-COVID-19 investment period (i.e., 2 out of 10 are AR-$t$-GARCH) than for the high-volatility COVID-19 investment period (i.e., only 1 out of 10 are AR-$t$-GARCH). These results indicate for gold ETF investors that, for a persistently low-volatility period or for a persistently high-volatility period, the use of the score-driven marginals with the Student's $t$ and Gen-$t$ specifications may be a better choice than the use of AR-$t$-GARCH or score-driven marginals with the NIG and EGB2 specifications.

Third, for the performances of the score-driven copulas, the portfolio performance results for all investment periods show that the best-performing score-driven copulas are the asymmetric Gumbel, rotated Gumbel, Clayton, and rotated Clayton copulas (Tables 2). These are followed by the symmetric Frank copula. The Plackett and Student's $t$ copulas are worse performing, and the Gaussian copula is the worst-performing score-driven copula as it does not appear in Table 2. These result indicate for S&P 500 and gold investors that the asymmetric Gumbel, rotated Gumbel, Clayton, and rotated Clayton copulas may be superior to the symmetric Frank, Plackett, Student's $t$, and Gaussian copulas.

Fourth, for the portfolio weight updating frequencies, for the high-volatility COVID-19 investment period the best-performing portfolios use the highest updating frequency (i.e., weekly update), while for the low-volatility pre-COVID-19 investment period the best-performing portfolios use lower updating frequencies (i.e., monthly, semi-annual, and annual updates). This result is intuitive. For the full investment period, all best-performing portfolios use weekly weight updates, which might be caused by the fact that the full investment period includes the volatile COVID-19 investment period.

Fifth, for the portfolio optimization strategies for all investment periods, the best-performing portfolios use either the Sharpe ratio or the utility function. The results show that the minimum-variance portfolio strategy is the worst-performing strategy, as it does not appear in Table 2. The results also show that for the low-volatility pre-COVID-19 period the Sharpe-ratio seems to perform better than the utility function, and the opposite is found for the high-volatility COVID-19 period.

### 4.4 Statistical analysis of portfolio performances

First, in Tables 3 to 8, we report statistical test results on the differences between the returns of alternative portfolio strategies. We compare the performances of the score-driven portfolios, the DCC-based portfolios, and the naïve portfolio. We start with the Gaussian-DCC model in this section, because we would like to compare the score-driven portfolio strategies with the AR-GARCH-Gaussian-DCC model-based portfolios of Akhtaruzzaman et al. (2021), which is the most relevant paper from the literature on portfolios involving precious metals for the COVID-19 period.

For each pair of portfolio strategies, the comparison is done by regressing the portfolio return difference on a constant, and by using the ordinary least squares (OLS) estimator with heteroskedasticity and autocorrelation consistent (HAC) standard errors (Newey and West 1987) for the constant parameter. Tables 3 and 4 are for the full investment period. Tables 5 and 6 are for the pre-COVID-19 investment period. Tables 7 and 8 are for the COVID-19 investment period. We report results on superior portfolio performances which correspond to the 20% level of significance. The first two columns of each table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with those of the score-driven model-based portfolios.

In Tables 3, 5, and 7, for each benchmark model, we compute the number of score-driven portfolios (N1) which have significantly higher returns than each Gaussian-DCC and naïve portfolio. In Tables 4, 6, and 8, for each benchmark model, we compute the number of score-driven portfolios (N2) which have significantly lower returns than each Gaussian-DCC and naïve portfolio. To make (N1) and (N2) comparable, we divide them by the total number of score-driven portfolios: 2,720. The corresponding proportions are denoted (P1) and (P2), respectively. For all investment periods, i.e., for the full investment period, pre-COVID-19 investment period, and COVID-19 investment period, we find that (P1) > (P2). This result provides support for the use of the score-driven portfolios, instead of the use of the Gaussian-DCC portfolios, for all investment periods.

Second, we present the results on portfolio performance comparison for the 2,720 score-driven model-based portfolios, the 20 $t$-DCC model-based portfolios, and the naïve portfolio (Tables C1 to C6 of Appendix C), for which the statistical tests are performed in the same way as for Tables 2 to 7.

In Tables C1, C3, and C5, for each benchmark model, we compute the number of score-driven portfolios ($\tilde{\text{N}}$1) which have significantly higher returns than each $t$-DCC and the naïve portfolio. In Tables C2, C4, and C6, for each benchmark model, we compute the number of score-driven portfolios ($\tilde{\text{N}}$2) which have significantly lower returns than each $t$-DCC and the naïve portfolio. To make ($\tilde{\text{N}}$1) and ($\tilde{\text{N}}$2) comparable, we divide them by the total number of score-driven portfolios: 2,720. The corresponding proportions are denoted ($\tilde{\text{P}}$1) and ($\tilde{\text{P}}$2), respectively. For the full investment and COVID-19 investment periods we find that ($\tilde{\text{P}}$1) > ($\tilde{\text{P}}$2). However, for the pre-COVID-19 investment period we find that ($\tilde{\text{P}}$2) > ($\tilde{\text{P}}$1). These results indicate that the superiority of the score-driven portfolios to the $t$-DCC portfolios is more important for the high-volatility COVID-19 period than for the low-volatility pre-COVID-19 period. The latter result can be interpreted as a suggestion for the consideration of score-driven portfolios for equity plus gold portfolio investors, during periods of high market volatility.

[APPROXIMATE LOCATION OF TABLES 2 TO 7 AND FIGURE 2]

## 5 Conclusions

We have studied the out-of-sample performances of portfolios of S&P 500 and gold ETFs, for which the expected return, volatility, and association dynamics are updated by using score-driven filters. We have contributed to the works of Atskanov (2016) and Bernardi and Catania (2018) from the literature on score-driven models, because we have used alternative econometric models for the estimation of the portfolio mean and covariance matrix, and we have extended their portfolio strategies.

We have compared the performances of classical naïve portfolio strategy and 40 classical AR-GARCH-DCC model-based strategies with the performances of 2,720 score-driven model-based portfolio strategies, for which we use: (i) eight score-driven copulas (i.e., Clayton, rotated Clayton, Frank, Gaussian, Gumbel, rotated Gumbel, Plackett, and Student's $t$ copulas); (ii) four portfolio optimization strategies (i.e., minimum-variance portfolio, Sharpe ratio-based mean-variance portfolio, and two utility function-based mean-variance portfolios, which use two alternative risk aversion coefficients); (iii) five portfolio updating frequencies (i.e., weekly, monthly, quarterly, semi-annual, and annual); (iv) 17 alternatives for the marginal distributions, for which we use several combinations of AR-$t$-GARCH, QAR-Beta-$t$-EGARCH, QAR-Beta-Gen-$t$-EGARCH, QAR-EGB2-EGARCH, and QAR-NIG-EGARCH.

In the empirical application, we have focused on equity-gold portfolios, which represent many investors' preferences due to the diversification and hedging properties of gold. We have contributed to the work of Akhtaruzzaman et al. (2021) from the literature on equity-gold portfolios for the COVID-19 period, because (i) we have extended their econometric models, and (ii) we have also extended their portfolio strategies. We have found that score-driven portfolios are superior to the naïve strategy and the DCC model-based portfolios for the full, pre-COVID-19, and COVID-19 investment periods. The significance testing results have indicated that the superiority of the score-driven portfolios over the naïve and DCC portfolios is more important for the volatile COVID-19 investment period than for the pre-COVID-19 investment period. Our results motivate the consideration of score-driven models for the optimization of equity plus gold portfolios.

## Declarations

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## Appendix A

In this appendix, for each distribution technical details are presented for the: (i) log conditional density of $r_{k,t}$, (ii) scaled score function $s_{\mu,k,t}$, scale factor $K(\lambda_{k,t})$ for $s_{\mu,k,t}$, and score function $s_{\lambda,k,t}$, (iii) conditional mean and conditional standard deviation of $r_{k,t}$. For the conditioning set, we use $\mathcal{F}_{t-1} = \sigma(r_{k,1}, \ldots, r_{k,t-1} : k = 1, 2)$, and the vector of the constant parameters is denoted $\Theta$.

*Student's t-distribution*—For this distribution $\epsilon_{k,t} \sim t[0, 1, \exp(\nu_k) + 2]$ i.i.d., where $\nu_k \in \mathbb{R}$ is a shape parameter, and $\exp(\cdot)$ is the exponential function. The first two moments of $r_{k,t}$ exist.

(i) The log conditional density of $r_{k,t}$ is

$$\ln f(r_{k,t}|\mathcal{F}_{t-1}; \Theta) = \ln \Gamma \left[ \frac{\exp(\nu_k) + 3}{2} \right] - \ln \Gamma \left[ \frac{\exp(\nu_k) + 2}{2} \right] \tag{A.1}$$

$$-\frac{\ln(\pi)+\ln[\exp(\nu_k)+2]}{2}-\lambda_{k,t}-\frac{\exp(\nu_k)+3}{2}\ln\left\{1+\frac{\epsilon_{k,t}^2}{\exp(\nu_k)+2}\right\}$$

where $\ln(\cdot)$ is the natural logarithm function and $\Gamma(\cdot)$ is the gamma function.

(ii) The score function with respect to $\mu_{k,t}$ is (Harvey 2013):

$$\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_{k,t}}=\frac{[\exp(\nu_k)+2]\exp(\lambda_{k,t})\epsilon_{k,t}}{\epsilon_{k,t}^2+\exp(\nu_k)+2}\times\frac{\exp(\nu_k)+3}{[\exp(\nu_k)+2]\exp(2\lambda_{k,t})}= \tag{A.2}$$

$$=s_{\mu,k,t}\times\frac{\exp(\nu_k)+3}{[\exp(\nu_k)+2]\exp(2\lambda_{k,t})}=s_{\mu,k,t}\times K(\lambda_{k,t})$$

where the scaled score function $s_{\mu,k,t}$ is defined in the second equality, and the scale factor $K(\lambda_{k,t})$ is defined in the last equality. The $s_{\mu,k,t}$ term trims outliers, because $s_{\mu,k,t}\rightarrow_p 0$ when $|\epsilon_{k,t}|\rightarrow\infty$ (Fig. A1(a)). The discounting that is undertaken by $s_{\mu,k,t}$ is identical for the positive and negative sides of the distribution. The score function with respect to $\lambda_{k,t}$ is given by (Harvey and Chakravarty 2008):

$$s_{\lambda,k,t}=\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \lambda_{k,t}}=\frac{[\exp(\nu_k)+3]\epsilon_{k,t}^2}{\exp(\nu_k)+2+\epsilon_{k,t}^2}-1 \tag{A.3}$$

The updating term $s_{\lambda,k,t}$ Winsorizes extreme observations, because $s_{\lambda,k,t}\rightarrow_p c$ ($c>0$ is a real number) when $|\epsilon_{k,t}|\rightarrow\infty$ (Fig. A1(b)). The discounting that is undertaken by $s_{\lambda,k,t}$ is identical for the positive and negative sides of the probability distribution.

(iii) The conditional mean and standard deviation of $r_{k,t}$, respectively, are:

$$E(r_{k,t}|\mathcal{F}_{t-1};\Theta)=\mu_{k,t} \tag{A.4}$$

$$\sigma(r_{k,t}|\mathcal{F}_{t-1};\Theta)=\sigma_{k,t}=\exp(\lambda_{k,t})\left[\frac{\exp(\nu_k)+2}{\exp(\nu_k)}\right]^{1/2} \tag{A.5}$$

*Gen-t distribution*—For this distribution $\epsilon_t\sim$ Gen-$t[0,1,\exp(\nu_k)+2,\exp(\eta_k)]$ i.i.d., where $\nu_k\in\mathbb{R}$, and $\eta_k\in\mathbb{R}$ are shape parameters. For $\exp(\eta_k)=2$, the Gen-$t$ distribution is the Student's $t$-distribution. The first two moments of $r_{k,t}$ exist.

(i) The log-density of $r_{k,t}$ is (Ayala et al. 2019):

$$\ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)=\eta_k-\lambda_{k,t}-\ln(2)-\frac{\ln[\exp(\nu_k)+2]}{\exp(\eta_k)}-\ln\Gamma\left[\frac{\exp(\nu_k)+2}{\exp(\eta_k)}\right] \tag{A.6}$$

$$-\ln\Gamma[\exp(-\eta_k)]+\ln\Gamma\left[\frac{\exp(\nu_k)+3}{\exp(\eta_k)}\right]-\frac{\exp(\nu_k)+3}{\exp(\eta_k)}\ln\left\{1+\frac{|\epsilon_{k,t}|^{\exp(\eta_k)}}{[\exp(\nu_k)+2]}\right\}$$

where $\mathrm{sgn}(\cdot)$ is the signum function.

(ii) The score function with respect to $\mu_t$ is given by:

$$\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_t}= \tag{A.7}$$

$$= \frac{[\exp(\nu_k)+2]\exp(\lambda_{k,t})\epsilon_{k,t}|\epsilon_{k,t}|^{\exp(\eta_k)-2}}{|\epsilon_{k,t}|^{\exp(\eta_k)}+[\exp(\nu_k)+2]} \times \frac{\exp(\nu_k)+3}{[\exp(\nu_k)+2]\exp(2\lambda_{k,t})} =$$

$$= s_{\mu,k,t} \times \frac{\exp(\nu_k)+3}{[\exp(\nu_k)+2]\exp(2\lambda_{k,t})} \equiv u_{\mu,k,t} \times K(\lambda_{k,t})$$

where the scaled score function $s_{\mu,k,t}$ is defined in the second equality, and the scale factor $K(\lambda_{k,t})$ is defined in the last equality. The $s_{\mu,k,t}$ term trims extreme observations, because $s_{\mu,k,t} \to_p 0$ when $|\epsilon_{k,t}| \to \infty$ (Fig. A1(c)). The discounting that is undertaken by $s_{\mu,k,t}$ is not identical for the positive and negative sides of the distribution. The score function with respect to $\lambda_{k,t}$ is (Ayala et al. 2019):

$$s_{\lambda,k,t} = \frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \lambda_{k,t}} = \frac{|\epsilon_{k,t}|^{\exp(\eta_k)}[\exp(\nu_k)+3]}{|\epsilon_{k,t}|^{\exp(\eta_k)}+[\exp(\nu_k)+2]} - 1 \tag{A.8}$$

The updating term $s_{\lambda,k,t}$ Winsorizes outliers, because $s_{\lambda,k,t} \to_p c_1$ when $\epsilon_{k,t} \to -\infty$ and $s_{\lambda,k,t} \to_p c_2$ when $\epsilon_{k,t} \to +\infty$ ($c_1 > 0$ and $c_2 > 0$ are real numbers) (Fig. A1(d)). The Winsorizing of $s_{\lambda,k,t}$ is not identical for the positive and negative sides of the distribution.

(iii) The conditional mean and standard deviation of $r_{k,t}$, respectively, are (Ayala et al. 2019):

$$E(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \mu_{k,t} \tag{A.9}$$

$$\sigma(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \sigma_{k,t} = \exp(\lambda_{k,t})[\exp(\nu_k)+2]^{\exp(-\eta_k)} \times \left\{ \frac{B\left[\frac{3}{\exp(\eta_k)}, \frac{\exp(\nu_k)}{\exp(\eta_k)}\right]}{B\left[\frac{1}{\exp(\eta_k)}, \frac{\exp(\nu_k)+2}{\exp(\eta_k)}\right]} \right\}^{1/2} \tag{A.10}$$

where $B(\cdot,\cdot)$ is the beta function.

*EGB2 distribution*—For this distribution $\epsilon_{k,t} \sim \text{EGB2}[0,1,\exp(\nu_k),\exp(\eta_k)]$ i.i.d., where $\nu_k \in \mathbb{R}$ and $\eta_k \in \mathbb{R}$ are shape parameters. For the EGB2 distribution all moments exist.

(i) The log conditional density is (Caivano and Harvey 2014):

$$\ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \exp(\nu_k)\epsilon_{k,t} - \lambda_{k,t} - \ln\Gamma[\exp(\nu_k)] \tag{A.11}$$

$$-\ln\Gamma[\exp(\eta_k)] + \ln\Gamma[\exp(\nu_k)+\exp(\eta_k)] - [\exp(\nu_k)+\exp(\eta_k)]\ln[1+\exp(\epsilon_{k,t})]$$

(ii) The score function with respect to $\mu_{k,t}$ is given by (Caivano and Harvey 2014):

$$\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_{k,t}} = \exp(-\lambda_{k,t})[\exp(\nu_k)+\exp(\eta_k)]\frac{\exp(\epsilon_{k,t})}{\exp(\epsilon_{k,t})+1} - \exp(-\lambda_{k,t})\exp(\nu_k) \tag{A.12}$$

Moreover, the scaled score function $s_{\mu,k,t}$ and the scale factor $K(\lambda_{k,t})$, respectively, are:

$$s_{\mu,k,t} = \frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_{k,t}} \times \{\Psi^{(1)}[\exp(\nu_k)] + \Psi^{(1)}[\exp(\eta_k)]\}\exp(2\lambda_{k,t}) \tag{A.13}$$

$$K(\lambda_{k,t}) \equiv \{\Psi^{(1)}[\exp(\nu_k)] + \Psi^{(1)}[\exp(\eta_k)]\}^{-1}\exp(-2\lambda_{k,t}) \tag{A.14}$$

where $\Psi^{(1)}(\cdot)$ is the trigamma function. The updating term $s_{\mu,k,t}$ Winsorizes outliers, because $s_{\mu,k,t} \rightarrow_p c_1$ when $\epsilon_{k,t} \rightarrow -\infty$ and $s_{\mu,k,t} \rightarrow_p c_2$ when $\epsilon_{k,t} \rightarrow +\infty$ ($c_1 > 0$ and $c_2 > 0$) (Fig. A1(e)). The Winsorizing that is undertaken by $s_{\mu,k,t}$ is not identical for positive and negative values. The score function with respect to $\lambda_{k,t}$ is (Caivano and Harvey 2014):

$$\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \lambda_{k,t}} = s_{\lambda,k,t} = [\exp(\nu_k) + \exp(\eta_k)] \frac{\epsilon_{k,t} \exp(\epsilon_{k,t})}{\exp(\epsilon_{k,t}) + 1} - \exp(\nu_k)\epsilon_{k,t} - 1 \tag{A.15}$$

The updating term $s_{\lambda,k,t}$ performs a linearly increasing and asymmetric transformation of $\epsilon_{k,t}$, as $|\epsilon_{k,t}| \rightarrow \infty$ (Fig. A1(f)).

(iii) The conditional mean and conditional standard deviation of $r_{k,t}$, respectively, are (Caivano and Harvey 2014):

$$E(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \mu_{k,t} + \exp(\lambda_{k,t}) \left\{ \Psi^{(0)}[\exp(\nu_k)] - \Psi^{(0)}[\exp(\eta_k)] \right\} \tag{A.16}$$

$$\sigma(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \sigma_{k,t} = \exp(\lambda_{k,t}) \left\{ \Psi^{(1)}[\exp(\nu_k)] + \Psi^{(1)}[\exp(\eta_k)] \right\}^{1/2} \tag{A.17}$$

where $\Psi^{(0)}(\cdot)$ is the digamma function.

*NIG distribution*—For this distribution $\epsilon_{k,t} \sim \text{NIG}[0, 1, \exp(\nu_k), \exp(\nu_k)\tanh(\eta_k)]$ i.i.d., where $\nu_k \in \mathbb{R}$ and $\eta_k \in \mathbb{R}$ are shape parameters, and $\tanh(\cdot)$ is the hyperbolic tangent function. For the NIG distribution all moments exist.

(i) The log conditional density is (Blazsek et al. 2018):

$$\ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \nu_k - \lambda_{k,t} - \ln(\pi) + \exp(\nu_k)[1 - \tanh^2(\eta_k)]^{1/2} \tag{A.18}$$

$$+ \exp(\nu_k)\tanh(\eta_k)\epsilon_{k,t} + \ln K^{(1)} \left\{ \exp(\nu_k)\sqrt{1 + \epsilon_{k,t}^2} \right\} - \frac{1}{2} \ln \left\{ 1 + \epsilon_{k,t}^2 \right\}$$

where $K^{(j)}(\cdot)$ is the modified Bessel function of the second kind of order $j$.

(ii) The score function with respect to $\mu_{k,t}$ is given by (Blazsek et al. 2018):

$$\frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_{k,t}} = -\exp(\nu_k - \lambda_{k,t})\tanh(\eta_k) + \frac{\epsilon_{k,t}}{\exp(\lambda_{k,t})(1 + \epsilon_{k,t}^2)} \tag{A.19}$$

$$+ \frac{\exp(\nu_k - \lambda_{k,t})\epsilon_{k,t}}{\sqrt{1 + \epsilon_{k,t}^2}} \times \frac{K^{(0)} \left[ \exp(\nu_k)\sqrt{1 + \epsilon_{k,t}^2} \right] + K^{(2)} \left[ \exp(\nu_k)\sqrt{1 + \epsilon_{k,t}^2} \right]}{2K^{(1)} \left[ \exp(\nu_k)\sqrt{1 + \epsilon_{k,t}^2} \right]}$$

Moreover, the scaled score function $s_{\mu,k,t}$ and the scale factor $K(\lambda_{k,t})$, respectively, are:

$$s_{\mu,k,t} = \frac{\partial \ln f(r_{k,t}|\mathcal{F}_{t-1};\Theta)}{\partial \mu_{k,t}} \times \exp(2\lambda_{k,t}) \tag{A.20}$$

$$K(\lambda_{k,t}) \equiv \exp(-2\lambda_{k,t}) \tag{A.21}$$

The updating term $s_{\mu,k,t}$ Winsorizes outliers, because $s_{\mu,k,t} \to_p c_1$ when $\epsilon_{k,t} \to -\infty$ and $s_{\mu,k,t} \to_p c_2$ when $\epsilon_{k,t} \to +\infty$ ($c_1 > 0$ and $c_2 > 0$ are real numbers) (Fig. A1(g)). The Winsorizing that is undertaken by $s_{\mu,k,t}$ is not identical for the positive and negative sides of the probability distribution. The score function with respect to $\lambda_{k,t}$ is given by (Blazsek et al. 2018):

$$s_{\lambda,k,t} = -1 - \exp(\nu_k)\tanh(\eta_k)\epsilon_{k,t} + \frac{\epsilon_{k,t}^2}{1+\epsilon_{k,t}^2} \tag{A.22}$$

$$+\frac{\exp(\nu_k)\epsilon_{k,t}^2}{\sqrt{1+\epsilon_{k,t}^2}} \times \frac{K^{(0)}\left[\exp(\nu_k)\sqrt{1+\epsilon_{k,t}^2}\right] + K^{(2)}\left[\exp(\nu_k)\sqrt{1+\epsilon_{k,t}^2}\right]}{2K^{(1)}\left[\exp(\nu_k)\sqrt{1+\epsilon_{k,t}^2}\right]}$$

The updating term $s_{\lambda,k,t}$ performs a linearly increasing and asymmetric transformation of $\epsilon_{k,t}$, as $|\epsilon_{k,t}| \to \infty$ (Fig. A1(h)).

(iii) The conditional mean and standard deviation of $r_{k,t}$, respectively, are (Blazsek et al. 2018):

$$E(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \mu_{k,t} + \frac{\exp(\lambda_{k,t})\tanh(\eta_k)}{[1-\tanh^2(\eta_k)]^{1/2}} \tag{A.23}$$

$$\sigma(r_{k,t}|\mathcal{F}_{t-1};\Theta) = \sigma_{k,t} = \left\{\frac{\exp(2\lambda_{k,t}-\nu_k)}{[1-\tanh^2(\eta_k)]^{3/2}}\right\}^{1/2} \tag{A.24}$$

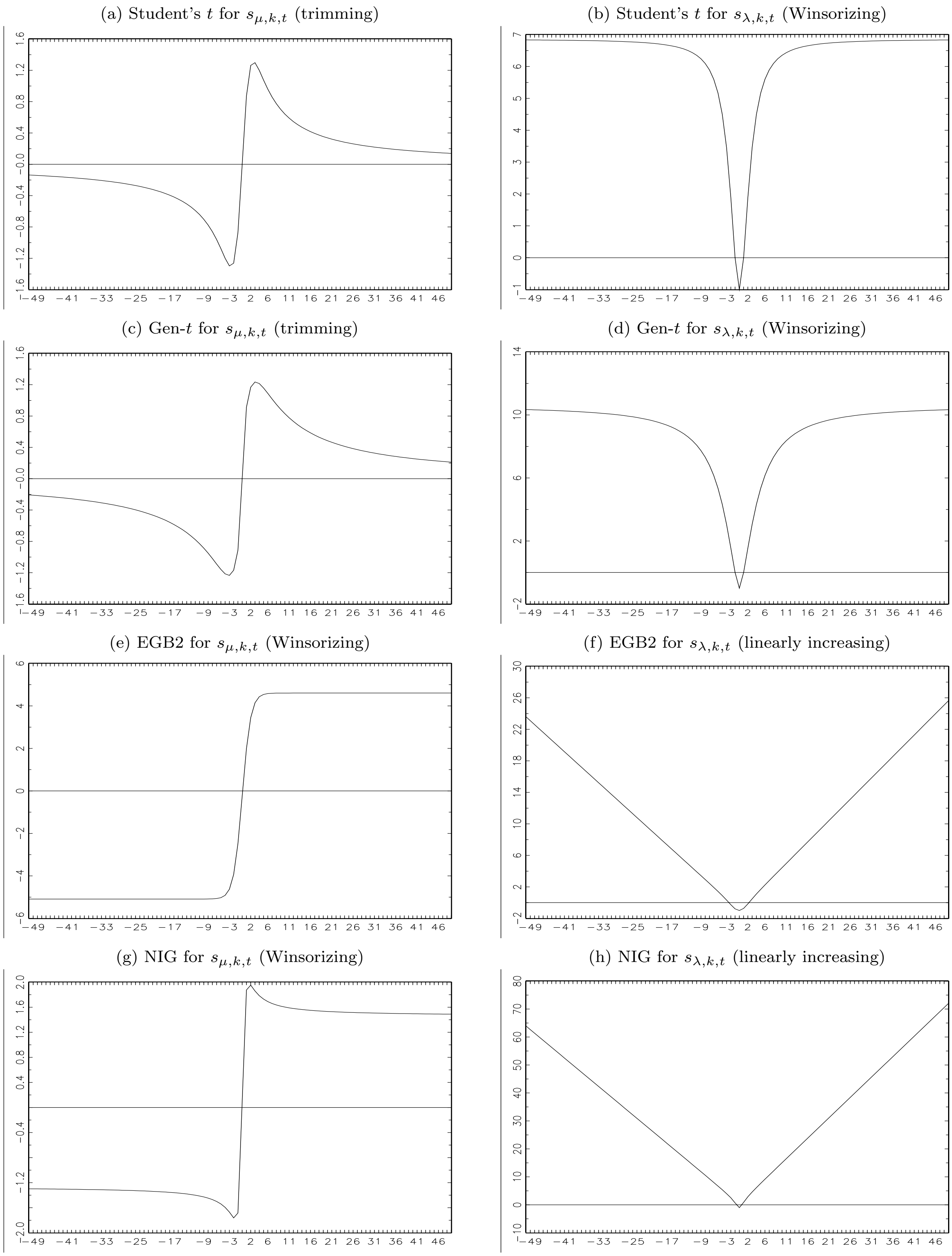


**Fig. A1.** Scaled score function $s_{\mu,k,t}$ and score function $s_{\lambda,k,t}$ estimates, as functions of $\epsilon_t$. *Notes:* ML estimates of the shape parameters with $\lambda_t = 0$ are used. In parentheses, we refer to the asymptotic transformation of outliers, as $|\epsilon_t| \to \infty$.

### Appendix B

*Clayton copula*—The bivariate Clayton copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = (1+\rho_t)(uv)^{-(\rho_t+1)}(u^{-\rho_t}+v^{-\rho_t}-1)^{-(1+2\rho_t)/\rho_t} \tag{B.1}$$

where $u$ and $v$ are realizations of $U[0,1]$ random variables (we use the same notation for the remaining copulas), and $\rho_t \in [-1,\infty)\backslash\{0\}$ (Harvey 2013; Joe 2015). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{1}{1+\rho_t} - \ln(uv) + \frac{1}{\rho_t^2}\ln\left(u^{-\rho_t}+v^{-\rho_t}-1\right) \tag{B.2}$$

$$+\frac{(1+2\rho_t)[u^{-\rho_t}\ln(u)+v^{-\rho_t}\ln(v)]}{\rho_t(u^{-\rho_t}+v^{-\rho_t}-1)}$$

*Rotated Clayton copula*—The bivariate rotated Clayton copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) \tag{B.3}$$

$$= (1+\rho_t)[(1-u)(1-v)]^{-(\rho_t+1)}[(1-u)^{-\rho_t}+(1-v)^{-\rho_t}-1]^{-(1+2\rho_t)/\rho_t}$$

with $\rho_t \in [-1,\infty)\backslash\{0\}$ (Patton 2004). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{1}{1+\rho_t} - \ln[(1-u)(1-v)] \tag{B.4}$$

$$+\frac{1}{\rho_t^2}\ln\left[(1-u)^{-\rho_t}+(1-v)^{-\rho_t}-1\right]+$$

$$+\frac{(1+2\rho_t)[(1-u)^{-\rho_t}\ln(1-u)+(1-v)^{-\rho_t}\ln(1-v)]}{\rho_t[(1-u)^{-\rho_t}+(1-v)^{-\rho_t}-1]}$$

*Frank copula*—The bivariate Frank copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \frac{\rho_t[1-\exp(-\rho_t)]\exp[-\rho_t(u+v)]}{\{1-\exp(-\rho_t)-[1-\exp(-\rho_t u)][1-\exp(-\rho_t v)]\}^2} \tag{B.5}$$

with $\rho_t \in \mathbb{R}\backslash\{0\}$ (Joe 2015). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{1}{\rho_t} + \frac{1}{\exp(\rho_t)-1} - (u+v) \tag{B.6}$$

$$-2\left\{\frac{\exp(-\rho_t)-u\exp(-\rho_t u)-v\exp(-\rho_t v)+(u+v)\exp[-\rho_t(u+v)]}{-\exp(-\rho_t)+\exp(-\rho_t u)+\exp(-\rho_t v)-\exp[-\rho_t(u+v)]}\right\}$$

*Gaussian copula*—The bivariate Gaussian copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) \tag{B.7}$$

$$= \frac{1}{\sqrt{1-\rho_t^2}} \exp\left\{ \frac{2\rho_t \Phi^{-1}(u)\Phi^{-1}(v) - \rho_t^2 \left[ (\Phi^{-1}(u))^2 + (\Phi^{-1}(v))^2 \right]}{2(1-\rho_t^2)} \right\}$$

where $\Phi^{-1}(x)$ is the inverse of the distribution function of $N(0,1)$ and $\rho_t \in [-1,1]$ (Meyer 2013; Joe 2015). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) \tag{B.8}$$

$$= \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{\rho_t}{1-\rho_t^2} + \frac{\Phi^{-1}(u)\Phi^{-1}(v)(1+\rho_t^2) - \rho_t \left[ (\Phi^{-1}(u))^2 + (\Phi^{-1}(v))^2 \right]}{(\rho_t^2-1)^2}$$

*Gumbel copula*—The bivariate Gumbel copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \exp\left\{ - \left[ (-\ln u)^{\rho_t} + (-\ln v)^{\rho_t} \right]^{1/\rho_t} \right\} \times \tag{B.9}$$

$$\times \frac{[\ln(u)\ln(v)]^{\rho_t - 1}}{uv\left[ (-\ln u)^{\rho_t} + (-\ln v)^{\rho_t} \right]^{2-1/\rho_t}} \times \left\{ \left[ (-\ln u)^{\rho_t} + (-\ln v)^{\rho_t} \right]^{1/\rho_t} + \rho_t - 1 \right\}$$

with $\rho_t \in [1,\infty)$ (Joe 2015; Patton 2004). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) \tag{B.10}$$

$$= \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{\xi_1^{1/\rho_t - 1}}{\rho_t^2}\left[ \xi_1 \ln(\xi_1) - \rho_t \xi_2 \right] + \ln(\ln u \ln v) + \frac{(1-2\rho_t)\xi_2}{\rho_t \xi_1}$$

$$- \frac{\ln(\xi_1)}{\rho_t^2} + \frac{\xi_1^{1/\rho_t - 1}\left[ -\xi_1 \ln(\xi_1) + \rho_t \xi_2 \right] + 1}{\rho_t^2 \left( \xi_1^{1/\rho_t} + \rho_t - 1 \right)}$$

where $\xi_1 = (-\ln u)^{\rho_t} + (-\ln v)^{\rho_t}$ and $\xi_2 = (-\ln u)^{\rho_t} \ln(-\ln u) + (-\ln v)^{\rho_t} \ln(-\ln v)$.

*Rotated Gumbel copula*—The bivariate rotated Gumbel copula density function is

$$c_t(u,v) \equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) = \exp\left\{ - \left\{ [-\ln(1-u)]^{\rho_t} + [-\ln(1-v)]^{\rho_t} \right]^{1/\rho_t} \right\} \times \tag{B.11}$$

$$\times \frac{[\ln(1-u)\ln(1-v)]^{\rho_t - 1}}{(1-u)(1-v)\left\{ [-\ln(1-u)]^{\rho_t} + [-\ln(1-v)]^{\rho_t} \right\}^{2-1/\rho_t}} \times$$

$$\times \left\{ \left\{ [-\ln(1-u)]^{\rho_t} + [-\ln(1-v)]^{\rho_t} \right]^{1/\rho_t} + \rho_t - 1 \right\}$$

with $\rho_t \in [1,\infty)$ (Patton 2004). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t} \equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta) \tag{B.12}$$

$$= \frac{\partial \ln c(u,v;\rho_t)}{\partial \rho_t} = \frac{\xi_1^{1/\rho_t - 1}}{\rho_t^2}\left[ \xi_1 \ln(\xi_1) - \rho_t \xi_2 \right] + \ln[\ln(1-u)\ln(1-v)]$$

$$+\frac{(1-2\rho_t)\xi_2}{\rho_t\xi_1}-\frac{\ln(\xi_1)}{\rho_t^2}+\frac{\xi_1^{1/\rho_t-1}\left[-\xi_1\ln(\xi_1)+\rho_t\xi_2\right]+1}{\rho_t^2\left(\xi_1^{1/\rho_t}+\rho_t-1\right)}$$

where $\xi_1=[-\ln(1-u)]^{\rho_t}+[-\ln(1-v)]^{\rho_t}$ and

$$\xi_2=[-\ln(1-u)]^{\rho_t}\ln[-\ln(1-u)]+[-\ln(1-v)]^{\rho_t}\ln[-\ln(1-v)] \tag{B.13}$$

*Plackett copula*—The bivariate Plackett copula density function is

$$c_t(u,v)\equiv c(u,v;\rho_t|\mathcal{F}_{t-1};\Theta)=\frac{\rho_t\left[1+(\rho_t-1)(u+v-2uv)\right]}{\left\{\left[1+(\rho_t-1)(u+v)\right]^2-4\rho_t(\rho_t-1)uv\right\}^{3/2}} \tag{B.14}$$

with $\rho_t\in[0,\infty)\backslash\{1\}$ (Joe 2015). The partial derivative of $\ln c(u,v;\rho_t)$ is

$$s_{\rho,t}\equiv s_\rho(u,v;\rho_t|\mathcal{F}_{t-1};\Theta)= \tag{B.15}$$

$$\frac{\partial\ln c(u,v;\rho_t)}{\partial\rho_t}=-\frac{3\left[(u+v)^2(\rho_t-1)+u+v+2uv(1-2\rho_t)\right]}{\left[(u+v)(\rho_t-1)+1\right]^2-4\rho_t(\rho_t-1)uv}+$$

$$+\frac{u+v-2uv}{(\rho_t-1)(u+v-2uv)+1}+\frac{1}{\rho_t}$$

*Student's t copula*—The bivariate Student's $t$-copula density function is

$$c_t(u,v)\equiv c(u,v;\nu,\rho_t|\mathcal{F}_{t-1};\Theta)=\frac{1}{\sqrt{1-\rho_t^2}}\frac{\Gamma[(\nu+2)/2]\Gamma(\nu/2)}{\Gamma^2[(\nu+1)/2]}\times \tag{B.16}$$

$$\times\frac{\left\{1+\frac{\left[T_\nu^{-1}(u)\right]^2+\left[T_\nu^{-1}(v)\right]^2-2\rho_tT_\nu^{-1}(u)T_\nu^{-1}(v)}{\nu(1-\rho_t^2)}\right\}^{-\frac{\nu+2}{2}}}{\left\{1+\frac{\left[T_\nu^{-1}(u)\right]^2}{\nu}\right\}^{-\frac{\nu+1}{2}}\left\{1+\frac{\left[T_\nu^{-1}(v)\right]^2}{\nu}\right\}^{-\frac{\nu+1}{2}}}$$

where $T_\nu^{-1}(x)$ is the inverse of the distribution function of the Student's $t$-distribution, $\rho_t$ is the correlation coefficient, and the additional parameter $\nu$ denotes degrees of freedom (Joe 2015). The partial derivative of $\ln c(u,v;\nu,\rho_t)$ is

$$s_{\rho,t}\equiv s_\rho(u,v;\nu,\rho_t|\mathcal{F}_{t-1};\Theta)=\frac{\partial\ln c(u,v;\nu,\rho_t)}{\partial\rho_t}=\frac{\rho_t}{1-\rho_t^2}+ \tag{B.17}$$

$$+\frac{\nu+2}{\rho_t^2-1}\times\frac{\rho_t\left\{\left[T_\nu^{-1}(u)\right]^2+\left[T_\nu^{-1}(v)\right]^2\right\}-(\rho_t^2+1)T_\nu^{-1}(u)T_\nu^{-1}(v)}{\left[T_\nu^{-1}(u)\right]^2+\left[T_\nu^{-1}(v)\right]^2-2\rho_tT_\nu^{-1}(u)T_\nu^{-1}(v)-\nu(\rho_t^2-1)}$$

# Appendix C: Portfolio performance for score-driven, $t$-DCC, and naïve portfolios

**Table C1** Full investment period: Number (Ñ1) and proportion ($\tilde{\text{P}}$1) of significantly better-performing score-driven portfolios than $t$-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 87(3.20%) | 8(0.29%) | 7(0.26%) | 9(0.33%) | 7(0.26%) | 20(0.74%) | 20(0.00%) | 8(0.29%) | 8(0.29%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 68(2.50%) | 3(0.11%) | 6(0.22%) | 10(0.37%) | 5(0.18%) | 11(0.40%) | 20(0.74%) | 8(0.29%) | 5(0.18%) |
| Weekly | Mean-Var, Sharpe ratio | 89(3.27%) | 6(0.22%) | 6(0.22%) | 11(0.40%) | 7(0.26%) | 13(0.48%) | 25(0.92%) | 9(0.33%) | 12(0.44%) |
| Weekly | Mean-Var, RA = 1 | 215(7.90%) | 19(0.70%) | 24(0.88%) | 21(0.77%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Weekly | Mean-Var, RA = 4 | 215(7.90%) | 19(0.70%) | 24(0.88%) | 21(0.77%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Monthly | Min-Var | 305(11.21%) | 30(1.10%) | 30(1.10%) | 37(1.36%) | 30(1.10%) | 49(1.80%) | 61(2.24%) | 31(1.14%) | 37(1.36%) |
| Monthly | Mean-Var, Sharpe ratio | 115(4.23%) | 10(0.37%) | 8(0.29%) | 11(0.40%) | 9(0.33%) | 20(0.74%) | 28(1.03%) | 16(0.59%) | 13(0.48%) |
| Monthly | Mean-Var, RA = 1 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Monthly | Mean-Var, RA = 4 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Quarterly | Min-Var | 167(6.14%) | 18(0.66%) | 19(0.70%) | 16(0.59%) | 16(0.59%) | 26(0.96%) | 33(1.21%) | 22(0.81%) | 17(0.63%) |
| Quarterly | Mean-Var, Sharpe ratio | 208(7.65%) | 19(0.70%) | 23(0.85%) | 22(0.81%) | 19(0.70%) | 36(1.32%) | 43(1.58%) | 24(0.88%) | 22(0.81%) |
| Quarterly | Mean-Var, RA = 1 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Quarterly | Mean-Var, RA = 4 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Semi-annual | Min-Var | 230(8.46%) | 20(0.74%) | 26(0.96%) | 27(0.99%) | 25(0.92%) | 35(1.29%) | 47(1.73%) | 24(0.88%) | 26(0.96%) |
| Semi-annual | Mean-Var, Sharpe ratio | 259(9.52%) | 26(0.96%) | 28(1.03%) | 29(1.07%) | 26(0.96%) | 43(1.58%) | 49(1.80%) | 28(1.03%) | 30(1.10%) |
| Semi-annual | Mean-Var, RA = 1 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Semi-annual | Mean-Var, RA = 4 | 216(7.94%) | 19(0.70%) | 24(0.88%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Annual | Min-Var | 238(8.75%) | 22(0.81%) | 27(0.99%) | 27(0.99%) | 25(0.92%) | 38(1.40%) | 48(1.76%) | 24(0.88%) | 27(0.99%) |
| Annual | Mean-Var, Sharpe ratio | 254(9.34%) | 24(0.88%) | 27(0.99%) | 29(1.07%) | 27(0.99%) | 42(1.54%) | 49(1.80%) | 27(0.99%) | 29(1.07%) |
| Annual | Mean-Var, RA = 1 | 217(7.98%) | 19(0.70%) | 25(0.92%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |
| Annual | Mean-Var, RA = 4 | 217(7.98%) | 19(0.70%) | 25(0.92%) | 22(0.81%) | 22(0.81%) | 36(1.32%) | 45(1.65%) | 24(0.88%) | 24(0.88%) |

*Notes:* Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (Ñ1), which perform significantly better than the benchmark portfolio. In parenthesis, we present the proportion of the significantly better-performing portfolios ($\tilde{\text{P}}$1) in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table C2** Full investment period: Number ($\tilde{N}2$) and proportion ($\tilde{P}2$) of significantly worse-performing score-driven portfolios than $t$-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 4(0.15%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 88(3.24%) | 12(0.44%) | 17(0.63%) | 17(0.63%) | 15(0.55%) | 4(0.15%) | 3(0.11%) | 8(0.29%) | 12(0.44%) |
| Weekly | Mean-Var, Sharpe ratio | 12(0.44%) | 1(0.04%) | 6(0.22%) | 2(0.07%) | 1(0.04%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 1(0.04%) |
| Weekly | Mean-Var, RA = 1 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Weekly | Mean-Var, RA = 4 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Monthly | Min-Var | 19(0.70%) | 2(0.07%) | 8(0.29%) | 4(0.15%) | 3(0.11%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 1(0.04%) |
| Monthly | Mean-Var, Sharpe ratio | 19(0.70%) | 2(0.07%) | 8(0.29%) | 4(0.15%) | 3(0.11%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 1(0.04%) |
| Monthly | Mean-Var, RA = 1 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Monthly | Mean-Var, RA = 4 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Quarterly | Min-Var | 87(3.20%) | 15(0.55%) | 17(0.63%) | 14(0.51%) | 18(0.66%) | 5(0.18%) | 3(0.11%) | 5(0.18%) | 10(0.37%) |
| Quarterly | Mean-Var, Sharpe ratio | 58(2.13%) | 8(0.29%) | 15(0.55%) | 8(0.29%) | 11(0.40%) | 4(0.15%) | 3(0.11%) | 4(0.15%) | 5(0.18%) |
| Quarterly | Mean-Var, RA = 1 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Quarterly | Mean-Var, RA = 4 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Semi-annual | Min-Var | 64(2.35%) | 9(0.33%) | 15(0.55%) | 12(0.44%) | 12(0.44%) | 5(0.18%) | 3(0.11%) | 3(0.11%) | 5(0.18%) |
| Semi-annual | Mean-Var, Sharpe ratio | 59(2.17%) | 8(0.29%) | 15(0.55%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 3(0.11%) | 5(0.18%) |
| Semi-annual | Mean-Var, RA = 1 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Semi-annual | Mean-Var, RA = 4 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Annual | Min-Var | 61(2.24%) | 9(0.33%) | 15(0.55%) | 8(0.29%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Annual | Mean-Var, Sharpe ratio | 57(2.10%) | 8(0.29%) | 15(0.55%) | 8(0.29%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 2(0.07%) | 5(0.18%) |
| Annual | Mean-Var, RA = 1 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |
| Annual | Mean-Var, RA = 4 | 64(2.35%) | 10(0.37%) | 16(0.59%) | 9(0.33%) | 11(0.40%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 6(0.22%) |

*Notes:* Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios ($\tilde{N}2$), which perform significantly worse than the benchmark portfolio. In parenthesis, we present the proportion of the significantly worse-performing portfolios ($\tilde{P}2$) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

**Table C3** Pre-COVID-19 investment period: Number ($\tilde{N}1$) and proportion ($\tilde{P}1$) of significantly better-performing score-driven portfolios than $t$-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 18(0.66%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 1(0.04%) | 2(0.07%) | 6(0.00%) | 2(0.07%) | 0(0.00%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 42(1.54%) | 3(0.11%) | 4(0.15%) | 9(0.33%) | 4(0.15%) | 4(0.15%) | 11(0.40%) | 7(0.26%) | 0(0.00%) |
| Weekly | Mean-Var, Sharpe ratio | 33(1.21%) | 2(0.07%) | 3(0.11%) | 8(0.29%) | 2(0.07%) | 4(0.15%) | 9(0.33%) | 5(0.18%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 4 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Monthly | Min-Var | 22(0.81%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 1(0.04%) | 4(0.15%) | 7(0.26%) | 4(0.15%) | 0(0.00%) |
| Monthly | Mean-Var, Sharpe ratio | 17(0.63%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 2(0.07%) | 6(0.22%) | 3(0.11%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 4 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Quarterly | Min-Var | 19(0.70%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 3(0.11%) | 7(0.26%) | 3(0.11%) | 0(0.00%) |
| Quarterly | Mean-Var, Sharpe ratio | 19(0.70%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 3(0.11%) | 7(0.26%) | 3(0.11%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 4 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Semi-annual | Min-Var | 31(1.14%) | 2(0.07%) | 1(0.04%) | 8(0.29%) | 4(0.15%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Semi-annual | Mean-Var, Sharpe ratio | 33(1.21%) | 2(0.07%) | 2(0.07%) | 8(0.29%) | 4(0.15%) | 4(0.15%) | 9(0.33%) | 4(0.15%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 4 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Annual | Min-Var | 23(0.85%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 2(0.07%) | 3(0.11%) | 8(0.29%) | 3(0.11%) | 0(0.00%) |
| Annual | Mean-Var, Sharpe ratio | 25(0.92%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 2(0.07%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Annual | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Annual | Mean-Var, RA = 4 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios ($\tilde{N}1$), which perform significantly better than the benchmark portfolio. In parenthesis, we present the proportion of the significantly better-performing portfolios ($\tilde{P}1$) in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven model portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table C4** Pre-COVID-19 investment period: Number ($\tilde{N}2$) and proportion ($\tilde{P}2$) of significantly worse-performing $t$-DCC and naïve portfolios than score-driven portfolios

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 34(1.25%) | 3(0.11%) | 1(0.04%) | 14(0.51%) | 3(0.11%) | 3(0.11%) | 2(0.00%) | 5(0.18%) | 3(0.11%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 40(1.47%) | 2(0.07%) | 5(0.18%) | 14(0.51%) | 3(0.11%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 4(0.15%) |
| Weekly | Mean-Var, Sharpe ratio | 27(0.99%) | 1(0.04%) | 4(0.15%) | 12(0.44%) | 1(0.04%) | 3(0.11%) | 2(0.07%) | 2(0.07%) | 2(0.07%) |
| Weekly | Mean-Var, RA = 1 | 57(2.10%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 6(0.22%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |
| Weekly | Mean-Var, RA = 4 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |
| Monthly | Min-Var | 87(3.20%) | 10(0.37%) | 11(0.40%) | 21(0.77%) | 11(0.40%) | 9(0.33%) | 8(0.29%) | 7(0.26%) | 10(0.37%) |
| Monthly | Mean-Var, Sharpe ratio | 87(3.20%) | 10(0.37%) | 11(0.40%) | 21(0.77%) | 11(0.40%) | 9(0.33%) | 8(0.29%) | 7(0.26%) | 10(0.37%) |
| Monthly | Mean-Var, RA = 1 | 61(2.24%) | 5(0.18%) | 7(0.26%) | 19(0.70%) | 8(0.29%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Monthly | Mean-Var, RA = 4 | 61(2.24%) | 5(0.18%) | 7(0.26%) | 19(0.70%) | 8(0.29%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Quarterly | Min-Var | 93(3.42%) | 11(0.40%) | 11(0.40%) | 21(0.77%) | 13(0.48%) | 9(0.33%) | 8(0.29%) | 8(0.29%) | 12(0.44%) |
| Quarterly | Mean-Var, Sharpe ratio | 97(3.57%) | 12(0.44%) | 11(0.40%) | 21(0.77%) | 14(0.51%) | 9(0.33%) | 8(0.29%) | 9(0.33%) | 13(0.48%) |
| Quarterly | Mean-Var, RA = 1 | 61(2.24%) | 5(0.18%) | 7(0.26%) | 19(0.70%) | 8(0.29%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Quarterly | Mean-Var, RA = 4 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 6(0.22%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Semi-annual | Min-Var | 69(2.54%) | 8(0.29%) | 9(0.33%) | 20(0.74%) | 6(0.22%) | 6(0.22%) | 7(0.26%) | 7(0.26%) | 6(0.22%) |
| Semi-annual | Mean-Var, Sharpe ratio | 54(1.99%) | 3(0.11%) | 7(0.26%) | 17(0.63%) | 6(0.22%) | 5(0.18%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Semi-annual | Mean-Var, RA = 1 | 59(2.17%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 6(0.22%) |
| Semi-annual | Mean-Var, RA = 4 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |
| Annual | Min-Var | 80(2.94%) | 9(0.33%) | 9(0.33%) | 20(0.74%) | 11(0.40%) | 8(0.29%) | 7(0.26%) | 7(0.26%) | 9(0.33%) |
| Annual | Mean-Var, Sharpe ratio | 68(2.50%) | 5(0.18%) | 8(0.29%) | 19(0.70%) | 9(0.33%) | 7(0.26%) | 5(0.18%) | 7(0.26%) | 8(0.29%) |
| Annual | Mean-Var, RA = 1 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |
| Annual | Mean-Var, RA = 4 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios ($\tilde{N}2$), which perform significantly worse than the benchmark portfolio. In parenthesis, we present the proportion of the significantly worse-performing portfolios ($\tilde{N}2$) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

**Table C5** COVID-19 investment period: Number ($\tilde{N}1$) and proportion ($\tilde{P}1$) of significantly better-performing score-driven portfolios than $t$-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 137(5.04%) | 16(0.59%) | 13(0.48%) | 22(0.81%) | 13(0.48%) | 19(0.70%) | 17(0.00%) | 19(0.70%) | 18(0.66%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 107(3.93%) | 10(0.37%) | 13(0.48%) | 13(0.48%) | 9(0.33%) | 13(0.48%) | 14(0.51%) | 15(0.55%) | 20(0.74%) |
| Weekly | Mean-Var, Sharpe ratio | 43(1.58%) | 3(0.11%) | 4(0.15%) | 7(0.26%) | 4(0.15%) | 8(0.29%) | 10(0.37%) | 4(0.15%) | 3(0.11%) |
| Weekly | Mean-Var, RA = 1 | 299(10.99%) | 36(1.32%) | 28(1.03%) | 46(1.69%) | 25(0.92%) | 36(1.32%) | 43(1.58%) | 41(1.51%) | 44(1.62%) |
| Weekly | Mean-Var, RA = 4 | 301(11.07%) | 37(1.36%) | 28(1.03%) | 46(1.69%) | 26(0.96%) | 36(1.32%) | 43(1.58%) | 41(1.51%) | 44(1.62%) |
| Monthly | Min-Var | 361(13.27%) | 43(1.58%) | 38(1.40%) | 53(1.95%) | 28(1.03%) | 44(1.62%) | 50(1.84%) | 49(1.80%) | 56(2.06%) |
| Monthly | Mean-Var, Sharpe ratio | 211(7.76%) | 22(0.81%) | 18(0.66%) | 30(1.10%) | 17(0.63%) | 29(1.07%) | 33(1.21%) | 33(1.21%) | 29(1.07%) |
| Monthly | Mean-Var, RA = 1 | 301(11.07%) | 36(1.32%) | 28(1.03%) | 47(1.73%) | 25(0.92%) | 35(1.29%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |
| Monthly | Mean-Var, RA = 4 | 301(11.07%) | 36(1.32%) | 28(1.03%) | 47(1.73%) | 25(0.92%) | 35(1.29%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |
| Quarterly | Min-Var | 291(10.70%) | 35(1.29%) | 28(1.03%) | 43(1.58%) | 24(0.88%) | 34(1.25%) | 43(1.58%) | 40(1.47%) | 44(1.62%) |
| Quarterly | Mean-Var, Sharpe ratio | 303(11.14%) | 35(1.29%) | 29(1.07%) | 46(1.69%) | 25(0.92%) | 37(1.36%) | 44(1.62%) | 42(1.54%) | 45(1.65%) |
| Quarterly | Mean-Var, RA = 1 | 300(11.03%) | 36(1.32%) | 28(1.03%) | 46(1.69%) | 25(0.92%) | 35(1.29%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |
| Quarterly | Mean-Var, RA = 4 | 300(11.03%) | 36(1.32%) | 28(1.03%) | 46(1.69%) | 25(0.92%) | 35(1.29%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |
| Semi-annual | Min-Var | 285(10.48%) | 35(1.29%) | 27(0.99%) | 42(1.54%) | 24(0.88%) | 33(1.21%) | 42(1.54%) | 39(1.43%) | 43(1.58%) |
| Semi-annual | Mean-Var, Sharpe ratio | 285(10.48%) | 35(1.29%) | 27(0.99%) | 42(1.54%) | 24(0.88%) | 33(1.21%) | 42(1.54%) | 39(1.43%) | 43(1.58%) |
| Semi-annual | Mean-Var, RA = 1 | 456(16.76%) | 39(1.43%) | 48(1.76%) | 50(1.84%) | 73(2.68%) | 86(3.16%) | 67(2.46%) | 46(1.69%) | 47(1.73%) |
| Semi-annual | Mean-Var, RA = 4 | 456(16.76%) | 39(1.43%) | 48(1.76%) | 50(1.84%) | 73(2.68%) | 86(3.16%) | 67(2.46%) | 46(1.69%) | 47(1.73%) |
| Annual | Min-Var | 324(11.91%) | 37(1.36%) | 32(1.18%) | 50(1.84%) | 27(0.99%) | 40(1.47%) | 46(1.69%) | 44(1.62%) | 48(1.76%) |
| Annual | Mean-Var, Sharpe ratio | 323(11.88%) | 37(1.36%) | 32(1.18%) | 50(1.84%) | 27(0.99%) | 40(1.47%) | 46(1.69%) | 43(1.58%) | 48(1.76%) |
| Annual | Mean-Var, RA = 1 | 302(11.10%) | 36(1.32%) | 28(1.03%) | 47(1.73%) | 25(0.92%) | 36(1.32%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |
| Annual | Mean-Var, RA = 4 | 302(11.10%) | 36(1.32%) | 28(1.03%) | 47(1.73%) | 25(0.92%) | 36(1.32%) | 43(1.58%) | 43(1.58%) | 44(1.62%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios ($\tilde{N}1$), which perform significantly better than the benchmark portfolio. In parenthesis, we present the proportion of the significantly better-performing portfolios ($\tilde{P}1$) in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table C6** COVID-19 investment period: Number ($\tilde{N}2$) and proportion ($\tilde{P}2$) of significantly worse-performing score-driven portfolios than $t$-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 36(1.32%) | 2(0.07%) | 2(0.07%) | 8(0.29%) | 2(0.07%) | 5(0.18%) | 2(0.00%) | 13(0.48%) | 2(0.07%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 175(6.43%) | 24(0.88%) | 23(0.85%) | 20(0.74%) | 20(0.74%) | 12(0.44%) | 16(0.59%) | 35(1.29%) | 25(0.92%) |
| Weekly | Mean-Var, Sharpe ratio | 7(0.26%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 1 | 85(3.13%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 6(0.22%) | 24(0.88%) | 7(0.26%) |
| Weekly | Mean-Var, RA = 4 | 84(3.09%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 5(0.18%) | 24(0.88%) | 7(0.26%) |
| Monthly | Min-Var | 41(1.51%) | 5(0.18%) | 7(0.26%) | 10(0.37%) | 7(0.26%) | 0(0.00%) | 0(0.00%) | 7(0.26%) | 5(0.18%) |
| Monthly | Mean-Var, Sharpe ratio | 10(0.37%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 1(0.04%) | 0(0.00%) | 0(0.00%) | 6(0.22%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Monthly | Mean-Var, RA = 4 | 80(2.94%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 6(0.22%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Quarterly | Min-Var | 91(3.35%) | 10(0.37%) | 10(0.37%) | 13(0.48%) | 11(0.40%) | 10(0.37%) | 9(0.33%) | 23(0.85%) | 5(0.18%) |
| Quarterly | Mean-Var, Sharpe ratio | 49(1.80%) | 5(0.18%) | 4(0.15%) | 6(0.22%) | 5(0.18%) | 6(0.22%) | 2(0.07%) | 18(0.66%) | 3(0.11%) |
| Quarterly | Mean-Var, RA = 1 | 83(3.05%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 6(0.22%) | 23(0.85%) | 6(0.22%) |
| Quarterly | Mean-Var, RA = 4 | 82(3.01%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 5(0.18%) | 23(0.85%) | 6(0.22%) |
| Semi-annual | Min-Var | 122(4.49%) | 15(0.55%) | 13(0.48%) | 18(0.66%) | 16(0.59%) | 11(0.40%) | 11(0.40%) | 30(1.10%) | 8(0.29%) |
| Semi-annual | Mean-Var, Sharpe ratio | 124(4.56%) | 15(0.55%) | 13(0.48%) | 19(0.70%) | 17(0.63%) | 11(0.40%) | 11(0.40%) | 30(1.10%) | 8(0.29%) |
| Semi-annual | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Semi-annual | Mean-Var, RA = 4 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Annual | Min-Var | 69(2.54%) | 8(0.29%) | 6(0.22%) | 13(0.48%) | 7(0.26%) | 7(0.26%) | 4(0.15%) | 21(0.77%) | 3(0.11%) |
| Annual | Mean-Var, Sharpe ratio | 69(2.54%) | 8(0.29%) | 6(0.22%) | 13(0.48%) | 7(0.26%) | 7(0.26%) | 4(0.15%) | 21(0.77%) | 3(0.11%) |
| Annual | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Annual | Mean-Var, RA = 4 | 80(2.94%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 9(0.33%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the $t$-distribution ($t$-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 $t$-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios ($\tilde{N}2$), which perform significantly worse than the benchmark portfolio. In parenthesis, we present the proportion of the significantly worse-performing portfolios ($\tilde{P}2$) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

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**Table 1** Descriptive statistics of daily excess returns

| | Full sample period | | Pre-investment period | | | |
|---|---|---|---|---|---|---|
| Start date | November 19, 2004 | | November 19, 2004 | | | |
| End date | September 24, 2021 | | October 24, 2014 | | | |
| Sample size $T$ | 4,241 | | $2,500$ | | | |
| | S&P 500 ETF | Gold ETF | S&P 500 ETF | Gold ETF | | |
| Minimum | $-0.1095$ | $-0.0878$ | $-0.0985$ | $-0.0878$ | | |
| Maximum | 0.1452 | 0.1128 | 0.1452 | 0.1128 | | |
| Mean | 0.0004 | 0.0003 | 0.0003 | 0.0004 | | |
| Standard deviation | 0.0122 | 0.0114 | 0.0128 | 0.0128 | | |
| Skewness | $-0.0654$ | $-0.1925$ | 0.2387 | $-0.2213$ | | |
| Excess kurtosis | 16.5392 | 6.3661 | 15.6894 | 5.8828 | | |
| PACF(1) | $-0.1202$*** | $-0.0116$ | $-0.092$*** | $-0.0177$ | | |
| PACF(2) | $-0.0307$** | $-0.0055$ | $-0.0908$*** | $-0.0039$ | | |
| PACF(3) | 0.0144 | 0.0016 | 0.0200 | $-0.0043$ | | |
| ARCH(5) statistic | 1168.6600*** | 177.2150*** | 593.105*** | 87.4468*** | | |
| | Full investment period | | Pre-COVID-19 investment period | | COVID-19 investment period | |
| Start date | October 27, 2014 | | October 27, 2014 | | February 24, 2020 | |
| End date | September 24, 2021 | | February 21, 2020 | | September 24, 2021 | |
| Sample size $T$ | $1,741$ | | $1,339$ | | 402 | |
| | S&P 500 ETF | Gold ETF | S&P 500 ETF | Gold ETF | S&P 500 ETF | Gold ETF |
| Minimum | $-0.1095$ | $-0.0537$ | $-0.0421$ | $-0.0347$ | $-0.1095$ | $-0.0537$ |
| Maximum | 0.0905 | 0.0491 | 0.0504 | 0.0491 | 0.0905 | 0.0485 |
| Mean | 0.0006 | 0.0002 | 0.0005 | 0.0002 | 0.0009 | 0.0002 |
| Standard deviation | 0.0111 | 0.0090 | 0.0084 | 0.0082 | 0.0174 | 0.0114 |
| Skewness | $-0.7212$ | $-0.0832$ | $-0.4591$ | 0.2528 | $-0.6983$ | $-0.4966$ |
| Excess kurtosis | 17.5773 | 3.5526 | 3.9355 | 2.5539 | 10.9292 | 3.3289 |
| PACF(1) | $-0.1744$*** | 0.0056 | $-0.0168$ | $-0.0445$ | $-0.3008$*** | 0.0898* |
| PACF(2) | 0.0813*** | $-0.0111$ | $-0.0502$* | $-0.0093$ | 0.1515*** | $-0.0212$ |
| PACF(3) | 0.0201 | 0.0185 | 0.0085 | 0.0560** | 0.0820 | $-0.0458$ |
| ARCH(5) statistic | 621.429*** | 77.2829*** | 160.2750*** | 27.3540*** | 166.9030*** | 22.0163*** |

*Notes:* Standard & Poor's 500 (S&P 500); exchange-traded fund (ETF); Partial autocorrelation function (PACF); autoregressive conditional heteroskedasticity (ARCH); coronavirus disease of 2019 (COVID-19). Daily closing price of each ETF is denoted $p_{k,t}$ for $k \in \{1,2\} \equiv \{\text{S\&P 500}, \text{Gold}\}$. Daily returns are defined as $\tilde{r}_{k,t} = (p_{k,t} - p_{k,t-1})/p_{k,t-1}$ for $k \in \{1,2\}$. Daily excess returns are defined as $r_{k,t} = \tilde{r}_{k,t} - r_{f,t}$ for $k \in \{1,2\}$, where $r_{f,t}$ is the 1-month US Treasury bill (T-bill) yield. The PACF and ARCH lag-orders are reported in parentheses. *, **, and *** show significance at the 10%, 5%, and 1% levels, respectively.

**Table 2** Ranking of score-driven portfolio performances by using portfolio values at the end of the investment periods

| Ranking | S&P 500 marginal | Gold marginal | Copula | Update | Portfolio strategy | Value |
|---|---|---|---|---|---|---|
| (a) Full investment period | | | | | | |
| 1 | QAR-NIG-EGARCH | AR-$t$-GARCH | R Gumbel | Weekly | Mean-Var, RA = 1 | 2.9882 |
| 2 | QAR-EGB2-EGARCH | AR-$t$-GARCH | Plackett | Weekly | Mean-Var, Sharpe ratio | 2.9464 |
| 3 | QAR-EGB2-EGARCH | AR-$t$-GARCH | Frank | Weekly | Mean-Var, RA = 4 | 2.9307 |
| 4 | QAR-EGB2-EGARCH | AR-$t$-GARCH | Gumbel | Weekly | Mean-Var, Sharpe ratio | 2.9296 |
| 5 | QAR-EGB2-EGARCH | QAR-NIG-EGARCH | R Gumbel | Weekly | Mean-Var, Sharpe ratio | 2.9220 |
| 6 | QAR-NIG-EGARCH | AR-$t$-GARCH | R Gumbel | Weekly | Mean-Var, RA = 4 | 2.9197 |
| 7 | QAR-EGB2-EGARCH | AR-$t$-GARCH | Student's $t$ | Weekly | Mean-Var, RA = 4 | 2.9120 |
| 8 | QAR-EGB2-EGARCH | AR-$t$-GARCH | R Gumbel | Weekly | Mean-Var, Sharpe ratio | 2.9086 |
| 9 | QAR-EGB2-EGARCH | AR-$t$-GARCH | Frank | Weekly | Mean-Var, Sharpe ratio | 2.8919 |
| 10 | QAR-NIG-EGARCH | AR-$t$-GARCH | Clayton | Weekly | Mean-Var, RA = 1 | 2.8808 |
| (b) Pre-COVID-19 investment period | | | | | | |
| Ranking | S&P 500 marginal | Gold marginal | Copula | Update | Portfolio strategy | Value |
| 1 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | R Gumbel | Monthly | Mean-Var, Sharpe ratio | 2.1656 |
| 2 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | Gumbel | Monthly | Mean-Var, Sharpe ratio | 2.1503 |
| 3 | QAR-Gen-$t$-EGARCH | AR-$t$-GARCH | Clayton | Annual | Mean-Var, RA = 1 | 2.1441 |
| 4 | QAR-Gen-$t$-EGARCH | AR-$t$-GARCH | Clayton | Semi-annual | Mean-Var, RA = 1 | 2.1165 |
| 5 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | R Gumbel | Monthly | Mean-Var, Sharpe ratio | 2.0946 |
| 6 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | R Gumbel | Quarterly | Mean-Var, Sharpe ratio | 2.0764 |
| 7 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | Gumbel | Quarterly | Mean-Var, Sharpe ratio | 2.0764 |
| 8 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | R Gumbel | Quarterly | Mean-Var, Sharpe ratio | 2.0764 |
| 9 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | Frank | Quarterly | Mean-Var, RA = 1 | 2.0661 |
| 10 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | Frank | Quarterly | Mean-Var, RA = 1 | 2.0577 |
| (c) COVID-19 investment period | | | | | | |
| Ranking | S&P 500 marginal | Gold marginal | Copula | Update | Portfolio strategy | Value |
| 1 | QAR-Beta-$t$-EGARCH | QAR-Gen-$t$-EGARCH | Gumbel | Weekly | Mean-Var, RA = 4 | 1.7652 |
| 2 | AR-$t$-GARCH | QAR-Beta-$t$-EGARCH | R Clayton | Weekly | Mean-Var, RA = 1 | 1.6906 |
| 3 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | R Clayton | Weekly | Mean-Var, RA = 1 | 1.6813 |
| 4 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | Frank | Weekly | Mean-Var, RA = 1 | 1.6807 |
| 5 | QAR-Beta-$t$-EGARCH | QAR-Gen-$t$-EGARCH | R Gumbel | Weekly | Mean-Var, RA = 4 | 1.6781 |
| 6 | AR-$t$-GARCH | AR-$t$-GARCH | Plackett | Weekly | Mean-Var, Sharpe ratio | 1.6766 |
| 7 | AR-$t$-GARCH | QAR-Gen-$t$-EGARCH | R Gumbel | Weekly | Mean-Var, RA = 1 | 1.6741 |
| 8 | QAR-Beta-$t$-EGARCH | QAR-Beta-$t$-EGARCH | Plackett | Weekly | Mean-Var, RA = 4 | 1.6653 |
| 9 | QAR-Beta-$t$-EGARCH | QAR-Beta-$t$-EGARCH | R Gumbel | Weekly | Mean-Var, RA = 4 | 1.6611 |
| 10 | QAR-EGB2-EGARCH | QAR-EGB2-EGARCH | Student's $t$ | Weekly | Mean-Var, RA = 4 | 1.6516 |

*Notes:* Quasi-autoregressive (QAR); normal-inverse Gaussian (NIG); exponential generalized AR conditional heteroskedasticity (EGARCH); exponential generalized beta distribution of the second kind (EGB2); risk aversion (RA); coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for a utility function (Mean-Var, RA).

**Table 3** Full investment period: Number (N1) and proportion (P1) of significantly better-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 87(3.20%) | 8(0.29%) | 7(0.26%) | 9(0.33%) | 7(0.26%) | 20(0.74%) | 20(0.00%) | 8(0.29%) | 8(0.29%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 51(1.88%) | 3(0.11%) | 3(0.11%) | 5(0.18%) | 5(0.18%) | 16(0.59%) | 14(0.51%) | 4(0.15%) | 1(0.04%) |
| Weekly | Mean-Var, Sharpe ratio | 82(3.01%) | 6(0.22%) | 6(0.22%) | 11(0.40%) | 8(0.29%) | 17(0.63%) | 18(0.66%) | 9(0.33%) | 7(0.26%) |
| Weekly | Mean-Var, RA = 1 | 21(0.77%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 4(0.15%) | 6(0.22%) | 6(0.22%) | 1(0.04%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 4 | 12(0.44%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 1(0.04%) | 4(0.15%) | 5(0.18%) | 0(0.00%) | 0(0.00%) |
| Monthly | Min-Var | 83(3.05%) | 7(0.26%) | 8(0.29%) | 10(0.37%) | 8(0.29%) | 15(0.55%) | 19(0.70%) | 9(0.33%) | 7(0.26%) |
| Monthly | Mean-Var, Sharpe ratio | 13(0.48%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 1(0.04%) | 3(0.11%) | 4(0.15%) | 1(0.04%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 112(4.12%) | 10(0.37%) | 9(0.33%) | 16(0.59%) | 8(0.29%) | 21(0.77%) | 26(0.96%) | 11(0.40%) | 11(0.40%) |
| Monthly | Mean-Var, RA = 4 | 70(2.57%) | 7(0.26%) | 7(0.26%) | 6(0.22%) | 8(0.29%) | 14(0.51%) | 16(0.59%) | 7(0.26%) | 5(0.18%) |
| Quarterly | Min-Var | 94(3.46%) | 10(0.37%) | 9(0.33%) | 12(0.44%) | 8(0.29%) | 17(0.63%) | 19(0.70%) | 7(0.26%) | 12(0.44%) |
| Quarterly | Mean-Var, Sharpe ratio | 122(4.49%) | 11(0.40%) | 8(0.29%) | 15(0.55%) | 19(0.70%) | 23(0.85%) | 23(0.85%) | 13(0.48%) | 10(0.37%) |
| Quarterly | Mean-Var, RA = 1 | 66(2.43%) | 7(0.26%) | 7(0.26%) | 5(0.18%) | 7(0.26%) | 15(0.55%) | 14(0.51%) | 6(0.22%) | 5(0.18%) |
| Quarterly | Mean-Var, RA = 4 | 92(3.38%) | 9(0.33%) | 8(0.29%) | 13(0.48%) | 6(0.22%) | 19(0.70%) | 19(0.70%) | 11(0.40%) | 7(0.26%) |
| Semi-annual | Min-Var | 411(15.11%) | 51(1.88%) | 53(1.95%) | 49(1.80%) | 50(1.84%) | 54(1.99%) | 56(2.06%) | 38(1.40%) | 60(2.21%) |
| Semi-annual | Mean-Var, Sharpe ratio | 14(0.51%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 4(0.15%) | 5(0.18%) | 1(0.04%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 1 | 146(5.37%) | 10(0.37%) | 10(0.37%) | 19(0.70%) | 21(0.77%) | 26(0.96%) | 28(1.03%) | 19(0.70%) | 13(0.48%) |
| Semi-annual | Mean-Var, RA = 4 | 306(11.25%) | 35(1.29%) | 32(1.18%) | 37(1.36%) | 36(1.32%) | 45(1.65%) | 48(1.76%) | 32(1.18%) | 41(1.51%) |
| Annual | Min-Var | 119(4.38%) | 12(0.44%) | 12(0.44%) | 18(0.66%) | 9(0.33%) | 23(0.85%) | 24(0.88%) | 9(0.33%) | 12(0.44%) |
| Annual | Mean-Var, Sharpe ratio | 17(0.63%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 2(0.07%) | 6(0.22%) | 5(0.18%) | 0(0.00%) | 0(0.00%) |
| Annual | Mean-Var, RA = 1 | 87(3.20%) | 8(0.29%) | 7(0.26%) | 9(0.33%) | 7(0.26%) | 20(0.74%) | 20(0.74%) | 8(0.29%) | 8(0.29%) |
| Annual | Mean-Var, RA = 4 | 93(3.42%) | 10(0.37%) | 7(0.26%) | 9(0.33%) | 8(0.29%) | 20(0.74%) | 23(0.85%) | 8(0.29%) | 8(0.29%) |

*Notes:* Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N1), which perform significantly better than the benchmark portfolio. In parenthesis, we present the proportion of the significantly better-performing portfolios (P1) in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table 4** Full investment period: Number (N2) and proportion (P2) of significantly worse-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 4(0.15%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 2(0.07%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Weekly | Mean-Var, Sharpe ratio | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 1 | 2(0.07%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 4 | 4(0.15%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) |
| Monthly | Min-Var | 5(0.18%) | 0(0.00%) | 0(0.00%) | 5(0.18%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Monthly | Mean-Var, Sharpe ratio | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 4 | 5(0.18%) | 0(0.00%) | 0(0.00%) | 5(0.18%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Quarterly | Min-Var | 10(0.37%) | 0(0.00%) | 0(0.00%) | 6(0.22%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) |
| Quarterly | Mean-Var, Sharpe ratio | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 1 | 5(0.18%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 4 | 9(0.33%) | 0(0.00%) | 0(0.00%) | 5(0.18%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) |
| Semi-annual | Min-Var | 3(0.11%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Semi-annual | Mean-Var, Sharpe ratio | 14(0.51%) | 0(0.00%) | 0(0.00%) | 6(0.22%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 6(0.22%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 1 | 3(0.11%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 4 | 4(0.15%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) |
| Annual | Min-Var | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Annual | Mean-Var, Sharpe ratio | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Annual | Mean-Var, RA = 1 | 4(0.15%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) |
| Annual | Mean-Var, RA = 4 | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |

*Notes:* Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N2), which perform significantly worse than the benchmark portfolio. In parenthesis, we present the proportion of the significantly worse-performing portfolios (P2) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

**Table 5** Pre-COVID-19 investment period: Number (N1) and proportion (P1) of significantly better-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 18(0.66%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 1(0.04%) | 2(0.07%) | 6(0.00%) | 2(0.07%) | 0(0.00%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 137(5.04%) | 15(0.55%) | 12(0.44%) | 19(0.70%) | 13(0.48%) | 22(0.81%) | 31(1.14%) | 18(0.66%) | 7(0.26%) |
| Weekly | Mean-Var, Sharpe ratio | 63(2.32%) | 4(0.15%) | 5(0.18%) | 11(0.40%) | 4(0.15%) | 9(0.33%) | 19(0.70%) | 9(0.33%) | 2(0.07%) |
| Weekly | Mean-Var, RA = 1 | 10(0.37%) | 2(0.07%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 2(0.07%) | 4(0.15%) | 1(0.04%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 4 | 16(0.59%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 2(0.07%) | 5(0.18%) | 3(0.11%) | 0(0.00%) |
| Monthly | Min-Var | 7(0.26%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 2(0.07%) | 2(0.07%) | 0(0.00%) |
| Monthly | Mean-Var, Sharpe ratio | 18(0.66%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 2(0.07%) | 3(0.11%) | 4(0.15%) | 3(0.11%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 6(0.22%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 2(0.07%) | 1(0.04%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 4 | 6(0.22%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 2(0.07%) | 1(0.04%) | 0(0.00%) |
| Quarterly | Min-Var | 22(0.81%) | 3(0.11%) | 0(0.00%) | 4(0.15%) | 1(0.04%) | 4(0.15%) | 8(0.29%) | 2(0.07%) | 0(0.00%) |
| Quarterly | Mean-Var, Sharpe ratio | 15(0.55%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 1(0.04%) | 3(0.11%) | 4(0.15%) | 1(0.04%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 1 | 16(0.59%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 2(0.07%) | 6(0.22%) | 2(0.07%) | 0(0.00%) |
| Quarterly | Mean-Var, RA = 4 | 14(0.51%) | 2(0.07%) | 0(0.00%) | 4(0.15%) | 0(0.00%) | 2(0.07%) | 5(0.18%) | 1(0.04%) | 0(0.00%) |
| Semi-annual | Min-Var | 578(21.25%) | 85(3.13%) | 90(3.31%) | 56(2.06%) | 82(3.01%) | 72(2.65%) | 76(2.79%) | 46(1.69%) | 71(2.61%) |
| Semi-annual | Mean-Var, Sharpe ratio | 1(0.04%) | 1(0.04%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Semi-annual | Mean-Var, RA = 1 | 128(4.71%) | 12(0.44%) | 13(0.48%) | 21(0.77%) | 19(0.70%) | 16(0.59%) | 25(0.92%) | 11(0.40%) | 11(0.40%) |
| Semi-annual | Mean-Var, RA = 4 | 275(10.11%) | 36(1.32%) | 42(1.54%) | 33(1.21%) | 32(1.18%) | 34(1.25%) | 45(1.65%) | 23(0.85%) | 30(1.10%) |
| Annual | Min-Var | 57(2.10%) | 7(0.26%) | 4(0.15%) | 12(0.44%) | 4(0.15%) | 7(0.26%) | 13(0.48%) | 7(0.26%) | 3(0.11%) |
| Annual | Mean-Var, Sharpe ratio | 3(0.11%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 0(0.00%) |
| Annual | Mean-Var, RA = 1 | 26(0.96%) | 2(0.07%) | 0(0.00%) | 5(0.18%) | 3(0.11%) | 4(0.15%) | 8(0.29%) | 4(0.15%) | 0(0.00%) |
| Annual | Mean-Var, RA = 4 | 28(1.03%) | 2(0.07%) | 0(0.00%) | 7(0.26%) | 1(0.04%) | 5(0.18%) | 9(0.33%) | 4(0.15%) | 0(0.00%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N1), which perform significantly better than the benchmark portfolio. In parenthesis, we present the proportion of the significantly better-performing portfolios (P1) in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table 6** Pre-COVID-19 investment period: Number (N2) and proportion (P2) of significantly worse-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 34(1.25%) | 3(0.11%) | 1(0.04%) | 14(0.51%) | 3(0.11%) | 3(0.11%) | 2(0.00%) | 5(0.18%) | 3(0.11%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 12(0.44%) | 0(0.00%) | 1(0.04%) | 1(0.04%) | 1(0.04%) | 3(0.11%) | 2(0.07%) | 3(0.11%) | 1(0.04%) |
| Weekly | Mean-Var, Sharpe ratio | 5(0.18%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 2(0.07%) | 1(0.04%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 1 | 31(1.14%) | 1(0.04%) | 4(0.15%) | 14(0.51%) | 2(0.07%) | 3(0.11%) | 2(0.07%) | 3(0.11%) | 2(0.07%) |
| Weekly | Mean-Var, RA = 4 | 30(1.10%) | 2(0.07%) | 3(0.11%) | 13(0.48%) | 2(0.07%) | 3(0.11%) | 2(0.07%) | 3(0.11%) | 2(0.07%) |
| Monthly | Min-Var | 118(4.34%) | 17(0.63%) | 14(0.51%) | 28(1.03%) | 16(0.59%) | 10(0.37%) | 8(0.29%) | 9(0.33%) | 16(0.59%) |
| Monthly | Mean-Var, Sharpe ratio | 5(0.18%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 2(0.07%) | 2(0.07%) | 1(0.04%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 130(4.78%) | 19(0.70%) | 20(0.74%) | 27(0.99%) | 21(0.77%) | 10(0.37%) | 8(0.29%) | 9(0.33%) | 16(0.59%) |
| Monthly | Mean-Var, RA = 4 | 125(4.60%) | 17(0.63%) | 16(0.59%) | 28(1.03%) | 17(0.63%) | 10(0.37%) | 10(0.37%) | 9(0.33%) | 18(0.66%) |
| Quarterly | Min-Var | 88(3.24%) | 10(0.37%) | 11(0.40%) | 20(0.74%) | 10(0.37%) | 9(0.33%) | 9(0.33%) | 11(0.40%) | 8(0.29%) |
| Quarterly | Mean-Var, Sharpe ratio | 10(0.37%) | 0(0.00%) | 1(0.04%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 2(0.07%) | 2(0.07%) | 2(0.07%) |
| Quarterly | Mean-Var, RA = 1 | 75(2.76%) | 9(0.33%) | 8(0.29%) | 20(0.74%) | 7(0.26%) | 9(0.33%) | 8(0.29%) | 7(0.26%) | 7(0.26%) |
| Quarterly | Mean-Var, RA = 4 | 85(3.13%) | 10(0.37%) | 10(0.37%) | 21(0.77%) | 10(0.37%) | 9(0.33%) | 8(0.29%) | 11(0.40%) | 6(0.22%) |
| Semi-annual | Min-Var | 11(0.40%) | 0(0.00%) | 0(0.00%) | 6(0.22%) | 0(0.00%) | 2(0.07%) | 2(0.07%) | 1(0.04%) | 0(0.00%) |
| Semi-annual | Mean-Var, Sharpe ratio | 95(3.49%) | 9(0.33%) | 10(0.37%) | 21(0.77%) | 10(0.37%) | 9(0.33%) | 12(0.44%) | 9(0.33%) | 15(0.55%) |
| Semi-annual | Mean-Var, RA = 1 | 28(1.03%) | 0(0.00%) | 3(0.11%) | 13(0.48%) | 2(0.07%) | 3(0.11%) | 2(0.07%) | 3(0.11%) | 2(0.07%) |
| Semi-annual | Mean-Var, RA = 4 | 18(0.66%) | 0(0.00%) | 2(0.07%) | 7(0.26%) | 0(0.00%) | 3(0.11%) | 2(0.07%) | 3(0.11%) | 1(0.04%) |
| Annual | Min-Var | 41(1.51%) | 3(0.11%) | 7(0.26%) | 12(0.44%) | 5(0.18%) | 5(0.18%) | 3(0.11%) | 3(0.11%) | 3(0.11%) |
| Annual | Mean-Var, Sharpe ratio | 44(1.62%) | 2(0.07%) | 4(0.15%) | 17(0.63%) | 4(0.15%) | 6(0.22%) | 4(0.15%) | 4(0.15%) | 3(0.11%) |
| Annual | Mean-Var, RA = 1 | 58(2.13%) | 5(0.18%) | 6(0.22%) | 19(0.70%) | 7(0.26%) | 6(0.22%) | 3(0.11%) | 7(0.26%) | 5(0.18%) |
| Annual | Mean-Var, RA = 4 | 49(1.80%) | 3(0.11%) | 7(0.26%) | 15(0.55%) | 5(0.18%) | 5(0.18%) | 3(0.11%) | 7(0.26%) | 4(0.15%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N2), which perform significantly worse than the benchmark portfolio. In parenthesis, we also present the proportion of the significantly worse-performing portfolios (P2) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

**Table 7** COVID-19 investment period: Number (N1) and proportion (P1) of significantly better-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 137(5.04%) | 16(0.59%) | 13(0.48%) | 22(0.81%) | 13(0.48%) | 19(0.70%) | 17(0.00%) | 19(0.70%) | 18(0.66%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 123(4.52%) | 15(0.55%) | 13(0.48%) | 19(0.70%) | 8(0.29%) | 15(0.55%) | 15(0.55%) | 17(0.63%) | 21(0.77%) |
| Weekly | Mean-Var, Sharpe ratio | 79(2.90%) | 6(0.22%) | 9(0.33%) | 9(0.33%) | 5(0.18%) | 13(0.48%) | 16(0.59%) | 9(0.33%) | 12(0.44%) |
| Weekly | Mean-Var, RA = 1 | 213(7.83%) | 27(0.99%) | 21(0.77%) | 31(1.14%) | 16(0.59%) | 26(0.96%) | 29(1.07%) | 35(1.29%) | 28(1.03%) |
| Weekly | Mean-Var, RA = 4 | 119(4.38%) | 14(0.51%) | 13(0.48%) | 16(0.59%) | 8(0.29%) | 15(0.55%) | 15(0.55%) | 17(0.63%) | 21(0.77%) |
| Monthly | Min-Var | 424(15.59%) | 51(1.88%) | 46(1.69%) | 60(2.21%) | 38(1.40%) | 46(1.69%) | 56(2.06%) | 63(2.32%) | 64(2.35%) |
| Monthly | Mean-Var, Sharpe ratio | 26(0.96%) | 1(0.04%) | 3(0.11%) | 5(0.18%) | 2(0.07%) | 5(0.18%) | 7(0.26%) | 3(0.11%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 1 | 716(26.32%) | 70(2.57%) | 83(3.05%) | 79(2.90%) | 107(3.93%) | 118(4.34%) | 98(3.60%) | 76(2.79%) | 85(3.13%) |
| Monthly | Mean-Var, RA = 4 | 415(15.26%) | 50(1.84%) | 44(1.62%) | 60(2.21%) | 36(1.32%) | 45(1.65%) | 56(2.06%) | 62(2.28%) | 62(2.28%) |
| Quarterly | Min-Var | 340(12.50%) | 36(1.32%) | 32(1.18%) | 58(2.13%) | 29(1.07%) | 39(1.43%) | 46(1.69%) | 44(1.62%) | 56(2.06%) |
| Quarterly | Mean-Var, Sharpe ratio | 402(14.78%) | 42(1.54%) | 42(1.54%) | 62(2.28%) | 44(1.62%) | 53(1.95%) | 53(1.95%) | 46(1.69%) | 60(2.21%) |
| Quarterly | Mean-Var, RA = 1 | 456(16.76%) | 39(1.43%) | 48(1.76%) | 50(1.84%) | 73(2.68%) | 86(3.16%) | 67(2.46%) | 46(1.69%) | 47(1.73%) |
| Quarterly | Mean-Var, RA = 4 | 380(13.97%) | 42(1.54%) | 37(1.36%) | 67(2.46%) | 32(1.18%) | 40(1.47%) | 48(1.76%) | 48(1.76%) | 66(2.43%) |
| Semi-annual | Min-Var | 305(11.21%) | 35(1.29%) | 29(1.07%) | 50(1.84%) | 23(0.85%) | 35(1.29%) | 43(1.58%) | 38(1.40%) | 52(1.91%) |
| Semi-annual | Mean-Var, Sharpe ratio | 305(11.21%) | 35(1.29%) | 29(1.07%) | 50(1.84%) | 23(0.85%) | 35(1.29%) | 43(1.58%) | 38(1.40%) | 52(1.91%) |
| Semi-annual | Mean-Var, RA = 1 | 456(16.76%) | 39(1.43%) | 48(1.76%) | 50(1.84%) | 73(2.68%) | 86(3.16%) | 67(2.46%) | 46(1.69%) | 47(1.73%) |
| Semi-annual | Mean-Var, RA = 4 | 327(12.02%) | 37(1.36%) | 36(1.32%) | 55(2.02%) | 30(1.10%) | 35(1.29%) | 43(1.58%) | 37(1.36%) | 54(1.99%) |
| Annual | Min-Var | 334(12.28%) | 41(1.51%) | 32(1.18%) | 53(1.95%) | 29(1.07%) | 38(1.40%) | 46(1.69%) | 44(1.62%) | 51(1.88%) |
| Annual | Mean-Var, Sharpe ratio | 334(12.28%) | 41(1.51%) | 32(1.18%) | 53(1.95%) | 29(1.07%) | 38(1.40%) | 46(1.69%) | 44(1.62%) | 51(1.88%) |
| Annual | Mean-Var, RA = 1 | 456(16.76%) | 39(1.43%) | 48(1.76%) | 50(1.84%) | 73(2.68%) | 86(3.16%) | 67(2.46%) | 46(1.69%) | 47(1.73%) |
| Annual | Mean-Var, RA = 4 | 302(11.10%) | 38(1.40%) | 28(1.03%) | 47(1.73%) | 25(0.92%) | 35(1.29%) | 44(1.62%) | 42(1.54%) | 43(1.58%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N1), which perform significantly better than the benchmark portfolios. In parenthesis, we present the proportion (P1) of the significantly better-performing portfolios in percentage terms (i.e., the number of better-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on superior portfolio performances which correspond to the 20% level of significance.

**Table 8** COVID-19 investment period: Number (N2) and proportion (P2) of significantly worse-performing score-driven portfolios than Gaussian-DCC and naïve portfolios.

| | | Total | Clayton | R Clayton | Frank | Gaussian | Gumbel | R Gumbel | Plackett | Student's $t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | Naïve strategy | 36(1.32%) | 2(0.07%) | 2(0.07%) | 8(0.29%) | 2(0.07%) | 5(0.18%) | 2(0.00%) | 13(0.48%) | 2(0.07%) |
| Update | Gaussian-DCC-based: | | | | | | | | | |
| Weekly | Min-Var | 62(2.28%) | 11(0.40%) | 11(0.40%) | 12(0.44%) | 7(0.26%) | 0(0.00%) | 1(0.04%) | 12(0.44%) | 8(0.29%) |
| Weekly | Mean-Var, Sharpe ratio | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 0(0.00%) |
| Weekly | Mean-Var, RA = 1 | 15(0.55%) | 1(0.04%) | 1(0.04%) | 7(0.26%) | 1(0.04%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 1(0.04%) |
| Weekly | Mean-Var, RA = 4 | 73(2.68%) | 11(0.40%) | 14(0.51%) | 13(0.48%) | 8(0.29%) | 0(0.00%) | 4(0.15%) | 13(0.48%) | 10(0.37%) |
| Monthly | Min-Var | 22(0.81%) | 2(0.07%) | 2(0.07%) | 9(0.33%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 2(0.07%) |
| Monthly | Mean-Var, Sharpe ratio | 131(4.82%) | 17(0.63%) | 18(0.66%) | 16(0.59%) | 16(0.59%) | 11(0.40%) | 11(0.40%) | 27(0.99%) | 15(0.55%) |
| Monthly | Mean-Var, RA = 1 | 7(0.26%) | 0(0.00%) | 0(0.00%) | 3(0.11%) | 0(0.00%) | 0(0.00%) | 0(0.00%) | 4(0.15%) | 0(0.00%) |
| Monthly | Mean-Var, RA = 4 | 17(0.63%) | 0(0.00%) | 1(0.04%) | 9(0.33%) | 2(0.07%) | 0(0.00%) | 0(0.00%) | 5(0.18%) | 0(0.00%) |
| Quarterly | Min-Var | 50(1.84%) | 5(0.18%) | 5(0.18%) | 6(0.22%) | 5(0.18%) | 6(0.22%) | 2(0.07%) | 18(0.66%) | 3(0.11%) |
| Quarterly | Mean-Var, Sharpe ratio | 25(0.92%) | 2(0.07%) | 1(0.04%) | 1(0.04%) | 2(0.07%) | 4(0.15%) | 1(0.04%) | 13(0.48%) | 1(0.04%) |
| Quarterly | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Quarterly | Mean-Var, RA = 4 | 38(1.40%) | 3(0.11%) | 4(0.15%) | 4(0.15%) | 5(0.18%) | 5(0.18%) | 2(0.07%) | 13(0.48%) | 2(0.07%) |
| Semi-annual | Min-Var | 86(3.16%) | 5(0.18%) | 10(0.37%) | 9(0.33%) | 11(0.40%) | 12(0.44%) | 10(0.37%) | 26(0.96%) | 3(0.11%) |
| Semi-annual | Mean-Var, Sharpe ratio | 86(3.16%) | 5(0.18%) | 10(0.37%) | 9(0.33%) | 11(0.40%) | 12(0.44%) | 10(0.37%) | 26(0.96%) | 3(0.11%) |
| Semi-annual | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Semi-annual | Mean-Var, RA = 4 | 121(4.45%) | 11(0.40%) | 6(0.22%) | 14(0.51%) | 19(0.70%) | 14(0.51%) | 17(0.63%) | 35(1.29%) | 5(0.18%) |
| Annual | Min-Var | 63(2.32%) | 5(0.18%) | 5(0.18%) | 7(0.26%) | 4(0.15%) | 9(0.33%) | 10(0.37%) | 20(0.74%) | 3(0.11%) |
| Annual | Mean-Var, Sharpe ratio | 63(2.32%) | 5(0.18%) | 5(0.18%) | 7(0.26%) | 4(0.15%) | 9(0.33%) | 10(0.37%) | 20(0.74%) | 3(0.11%) |
| Annual | Mean-Var, RA = 1 | 81(2.98%) | 10(0.37%) | 8(0.29%) | 13(0.48%) | 10(0.37%) | 7(0.26%) | 4(0.15%) | 23(0.85%) | 6(0.22%) |
| Annual | Mean-Var, RA = 4 | 80(2.94%) | 5(0.18%) | 9(0.33%) | 13(0.48%) | 5(0.18%) | 9(0.33%) | 10(0.37%) | 23(0.85%) | 6(0.22%) |

*Notes:* Coronavirus disease of 2019 (COVID-19); Clayton copula (Clayton); rotated Clayton copula (R Clayton); Frank copula (Frank); Gaussian copula (Gaussian); Gumbel copula (Gumbel); rotated Gumbel copula (R Gumbel), Plackett copula (Plackett), Student's $t$-copula (Student's $t$); dynamic conditional correlation model for the Gaussian distribution (Gaussian-DCC); minimum-variance strategy (Min-Var); mean-variance strategy for the Sharpe ratio (Mean-Var, Sharpe ratio); mean-variance strategy for the utility function with risk aversion coefficient RA = 1 (Mean-Var, RA = 1); mean-variance strategy for the utility function with risk aversion coefficient RA = 4 (Mean-Var, RA = 4). The first two columns of the table indicate the benchmark portfolio strategies (i.e., 1 naïve and 20 Gaussian-DCC portfolios), whose performances are compared with the performances of the score-driven portfolios. In each row, the table shows the number of score-driven portfolios (N2), which perform significantly worse than the benchmark portfolio. In parenthesis, we present the proportion of the significantly worse-performing portfolios (P2) in percentage terms (i.e., the number of worse-performing score-driven portfolios divided by the total number of score-driven portfolios). For testing the differences between the performances of the alternative portfolio strategies, we use an OLS-HAC (ordinary least squares, heteroskedasticity and autocorrelation consistent) estimator. In this table, we report results on inferior portfolio performances which correspond to the 20% level of significance.

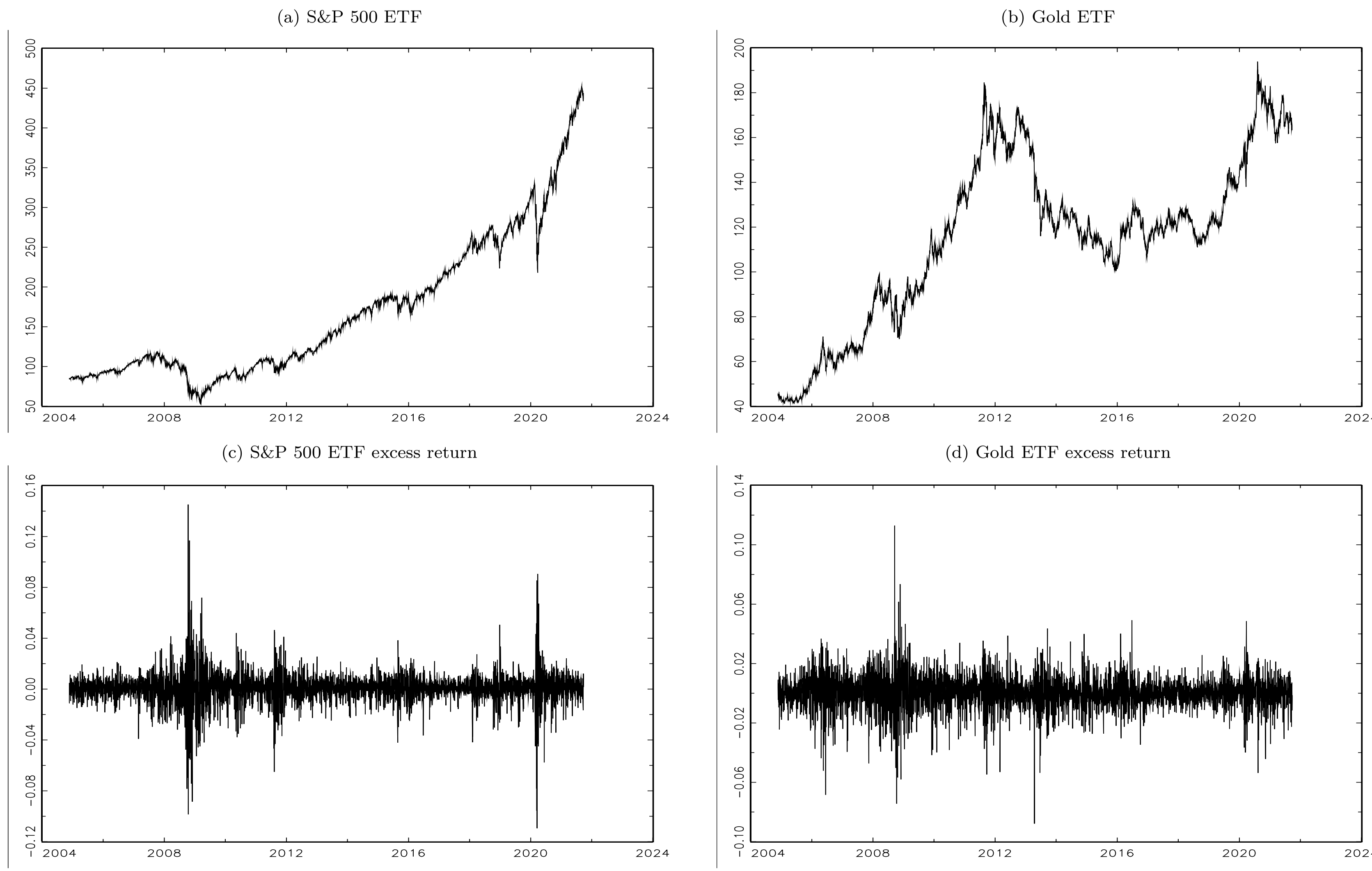


**Fig. 1.** Evolution of S&P 500 ETF and Gold ETF prices and daily excess returns (from November 19, 2004 to September 24, 2021).

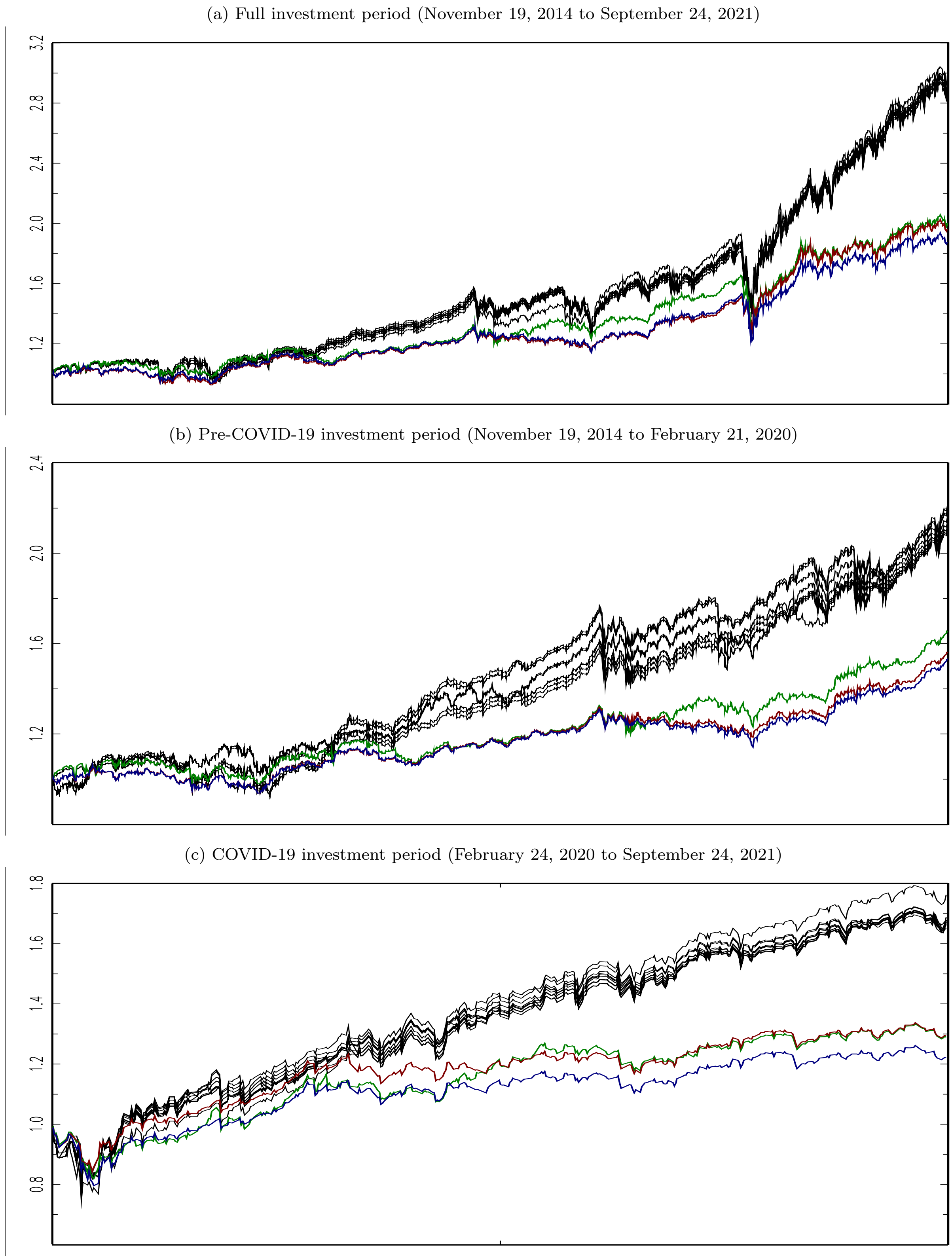


**Fig. 2.** Value of 1 USD investments for the naïve portfolio (blue), the best-performing Gaussian-DCC portfolio (green), the best-performing $t$-DCC portfolio (red), and the 10 best-performing score-driven portfolios (black).