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Model stability and forecast performance of Beta-*t*-EGARCH

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Motivation

Standard financial time series model (Box and Jenkins, 1970; Bollerslev, 1987)

ARMA(1,1) plus *t*-GARCH(1,1):

$y_t = \mu_t + v_t = \mu_t + \sqrt{\lambda_t}\varepsilon_t$ where $\varepsilon_t \sim t(\nu)$ i.i.d.

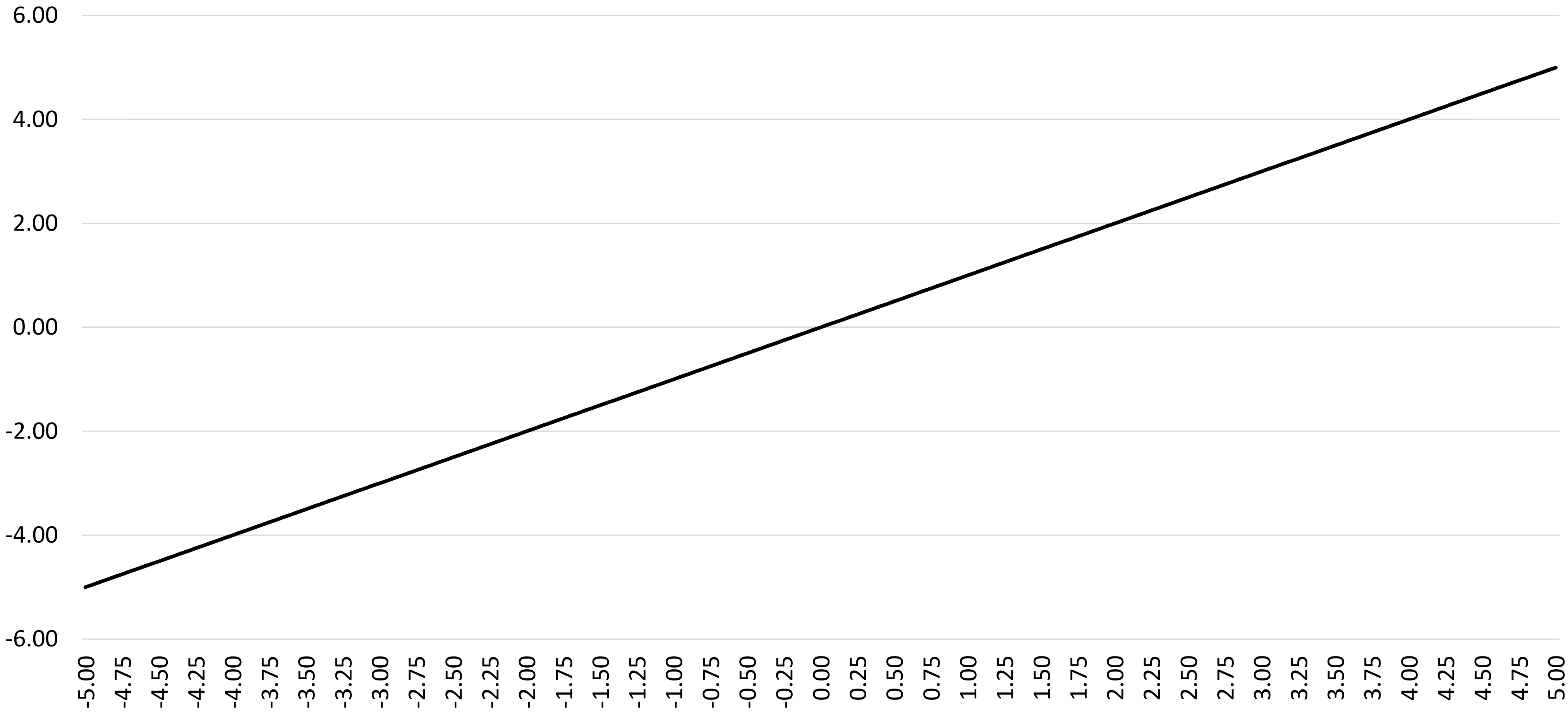
$\mu_{t+1} = c + \varphi y_t + \theta v_t = c + \varphi y_t + \theta\sqrt{\lambda_t}\varepsilon_t$

$\lambda_{t+1} = \omega + \beta\lambda_t + \alpha(v_t)^2 = \omega + \beta\lambda_t + \alpha\lambda_t(\varepsilon_t)^2$

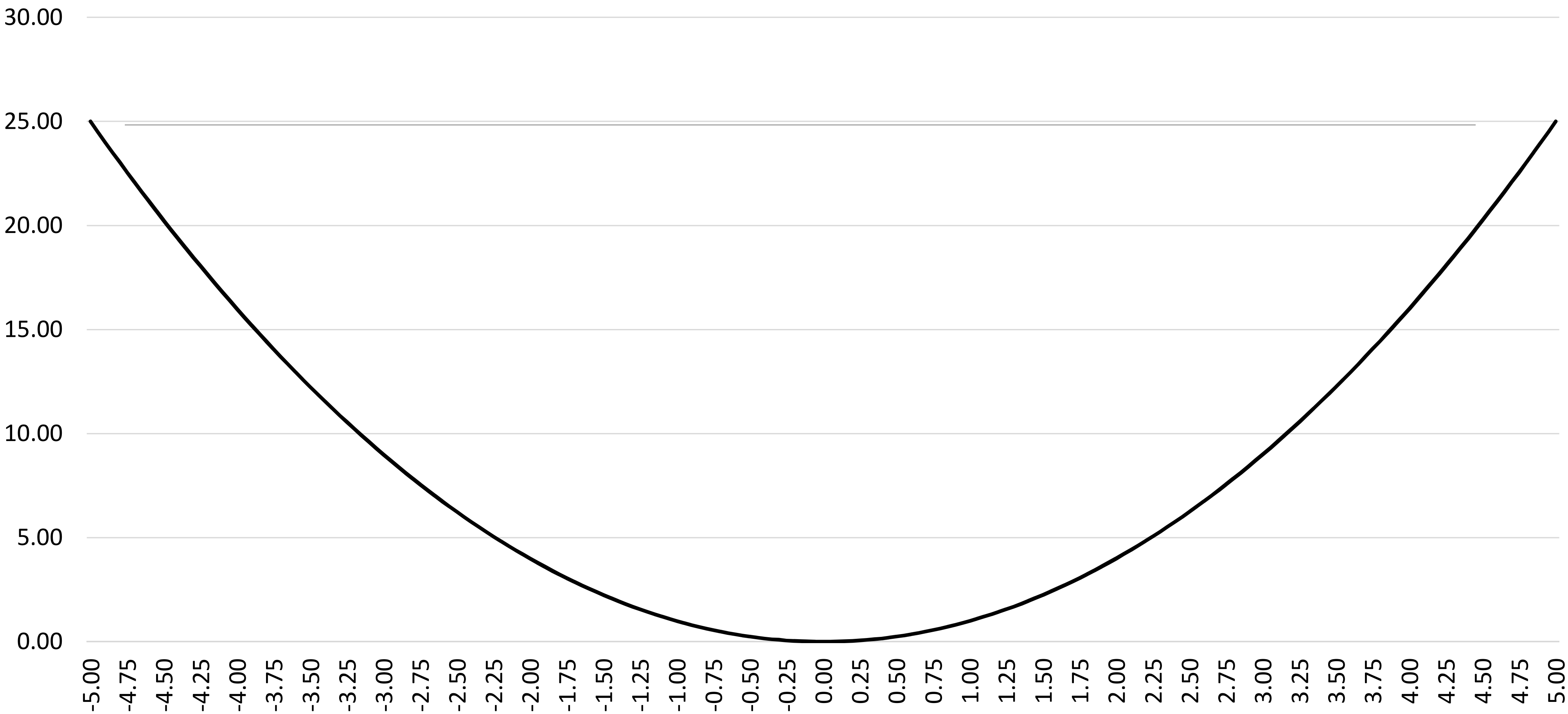
ε_t represents the new information on asset value arriving to the market on day t.

This model updates μ_t (expected return) and λ_t (volatility) by a transformation of ε_t (see the equations).

ν as a function of ε (expected return)



v-squared as a function of ε (volatility)



Dynamic conditional score model (Harvey, 2013; Harvey and Chakravarty, 2008)

QAR(1) plus Beta-*t*-EGARCH(1,1):

$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\varepsilon_t$ where $\varepsilon_t \sim t(\nu)$ i.i.d.

$\mu_{t+1} = c + \varphi\mu_t + \theta u_t$

$\lambda_{t+1} = \omega + \beta\lambda_t + \alpha e_t$

where u_t and e_t are nonlinear transformations of ε_t.

u_t and e_t update expected return and volatility, respectively.

Dynamic conditional score model

u_t and e_t are proportional to the conditional score of the log-likelihood of y_t.

These conditional scores are

$\partial \ln f(y_t|y_1, \dots, y_{t-1}) / \partial \mu_t$ (this is proportional to u_t)

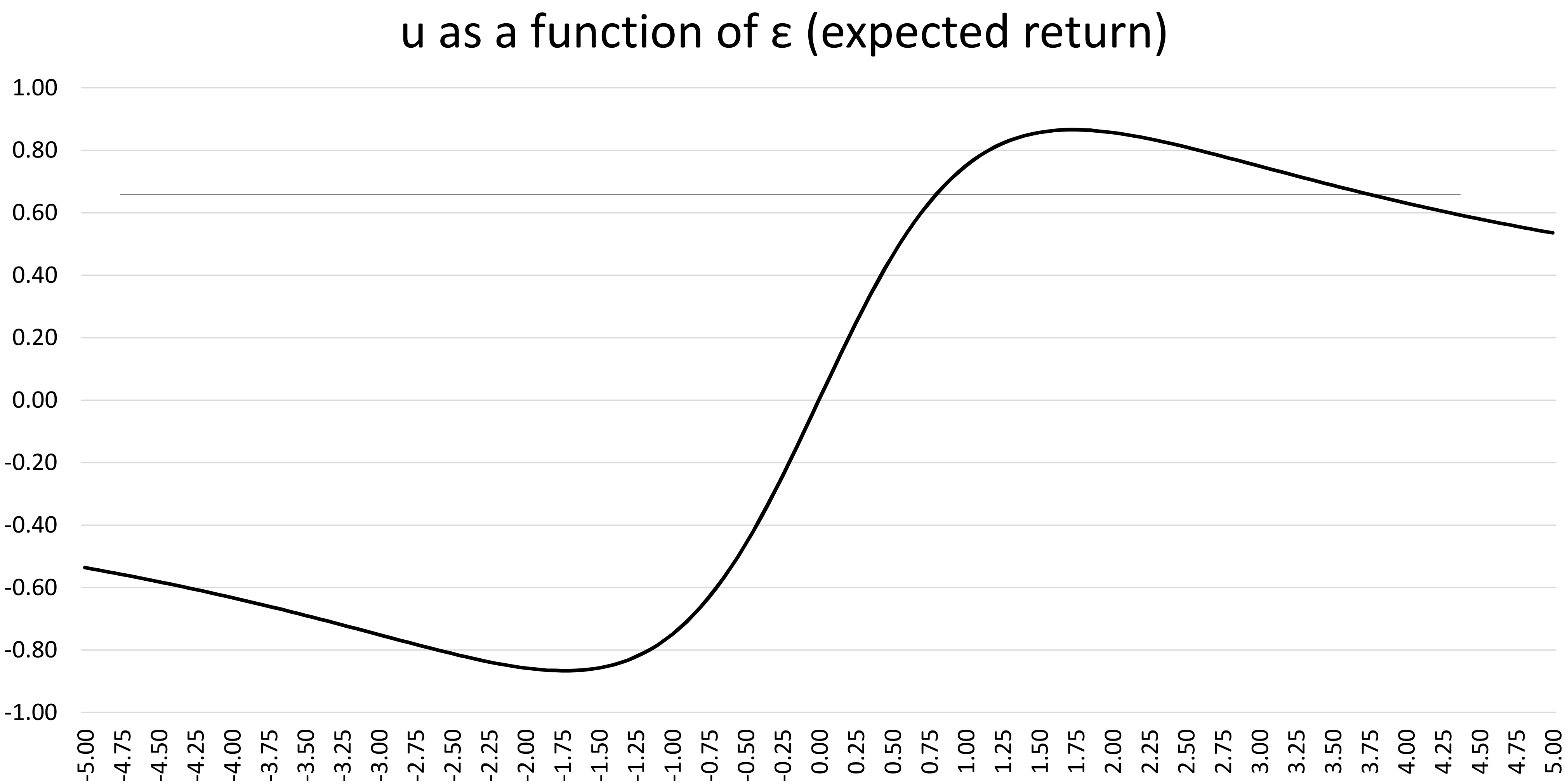
$\partial \ln f(y_t|y_1, \dots, y_{t-1}) / \partial \lambda_t$ (this is proportional to e_t)

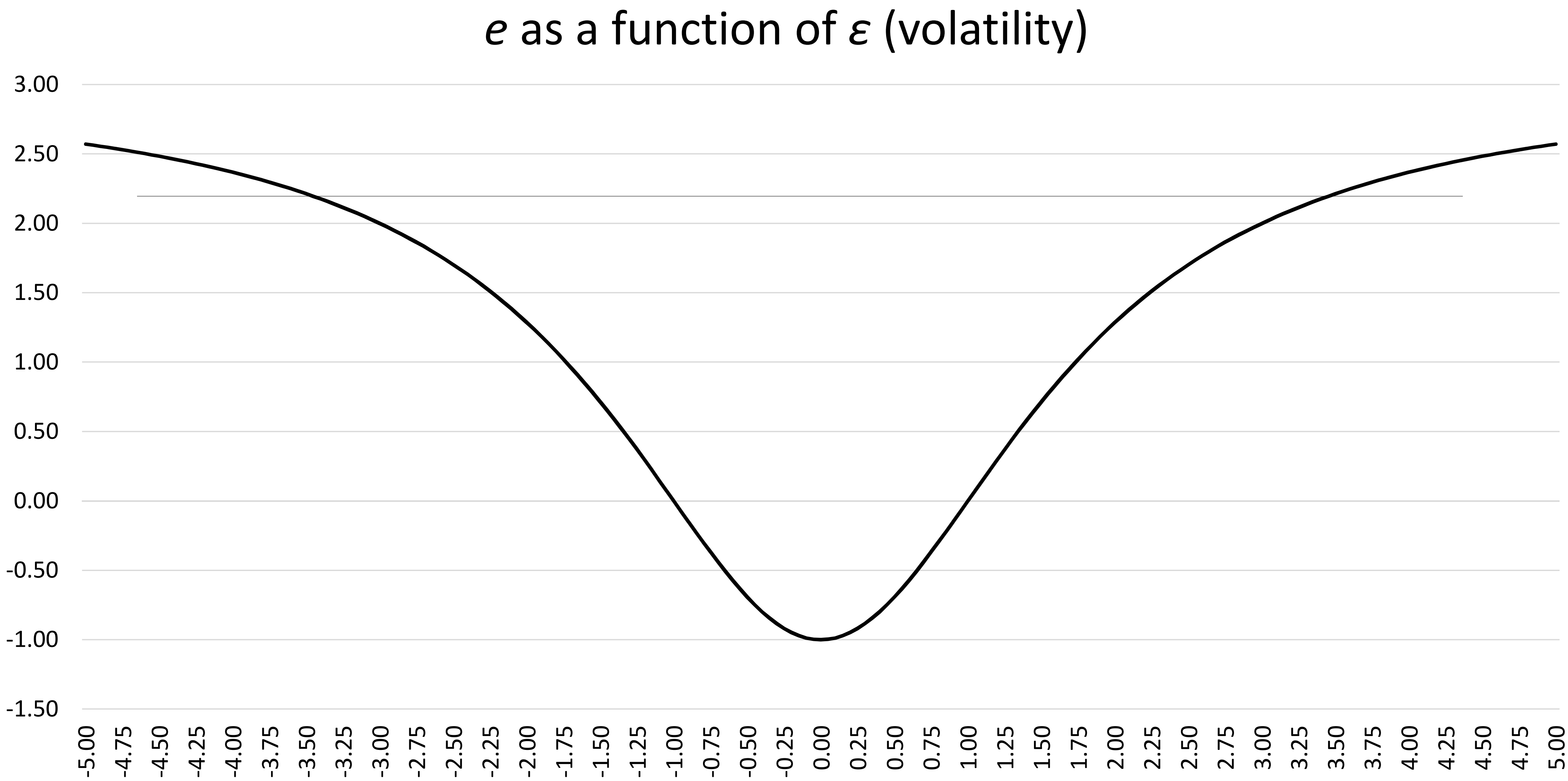
Dynamic conditional score model

u_t and e_t are given by

$$u_t = \left[1 + \frac{(v_t)^2}{\nu\exp(2\lambda_t)}\right]^{-1} v_t = \frac{\nu\exp(\lambda_t)\varepsilon_t}{\nu+\varepsilon_t{}^2}$$

$$e_t = \frac{(\nu+1)(v_t)^2}{(v_t)^2+\nu\exp(2\lambda_t)} - 1 = \frac{(\nu+1)(\varepsilon_t)^2}{(\varepsilon_t)^2+\nu} - 1$$


u as a function of ε (expected return)
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
-5.00
-4.75
-4.50
-4.25
-4.00
-3.75
-3.50
-3.25
-3.00
-2.75
-2.50
-2.25
-2.00
-1.75
-1.50
-1.25
-1.00
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
4.25
4.50
4.75
5.00



e as a function of ε (volatility)
3.00
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-5.00
-4.75
-4.50
-4.25
-4.00
-3.75
-3.50
-3.25
-3.00
-2.75
-2.50
-2.25
-2.00
-1.75
-1.50
-1.25
-1.00
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
3.25
3.50
3.75
4.00
4.25
4.50
4.75
5.00

Standard model versus DCS

In the standard model, extreme observations (outliers) are not discounted in the dynamic equations of expected return and volatility.

ARMA involves a linear transformation; it does not discount.

GARCH accentuates the effect of extreme observations, due to squaring them.

On the other hand, for DCS extreme observations are discounted in the dynamic equations of expected return and volatility.

Standard model versus DCS

If GARCH is persistent ($\beta + \alpha$ is significant), then it may overestimate volatility after an extreme observation, given that it is not followed by subsequent extreme observations.

If GARCH is persistent, then it may predict volatility correctly after an extreme observation, given that it is followed by subsequent extreme observations.

Standard model versus DCS

If Beta-t-EGARCH is persistent (β is significant), then it may underestimate volatility after an extreme observation, given that it is followed by subsequent extreme observations.

If Beta-t-EGARCH is persistent, then it may predict volatility correctly after an extreme observation, given that it is not followed by subsequent extreme observations.

(Q1) Which model predicts better stock returns?

Standard model versus DCS

In practice investors update the parameters of forecasting models of financial asset prices, by re-estimating them for the most recent data available.

The discounting property of DCS and non-discounting property of the standard model may imply that DCS models are more stable than the standard model.

(Q2) Is DCS a more stable model than the standard model?

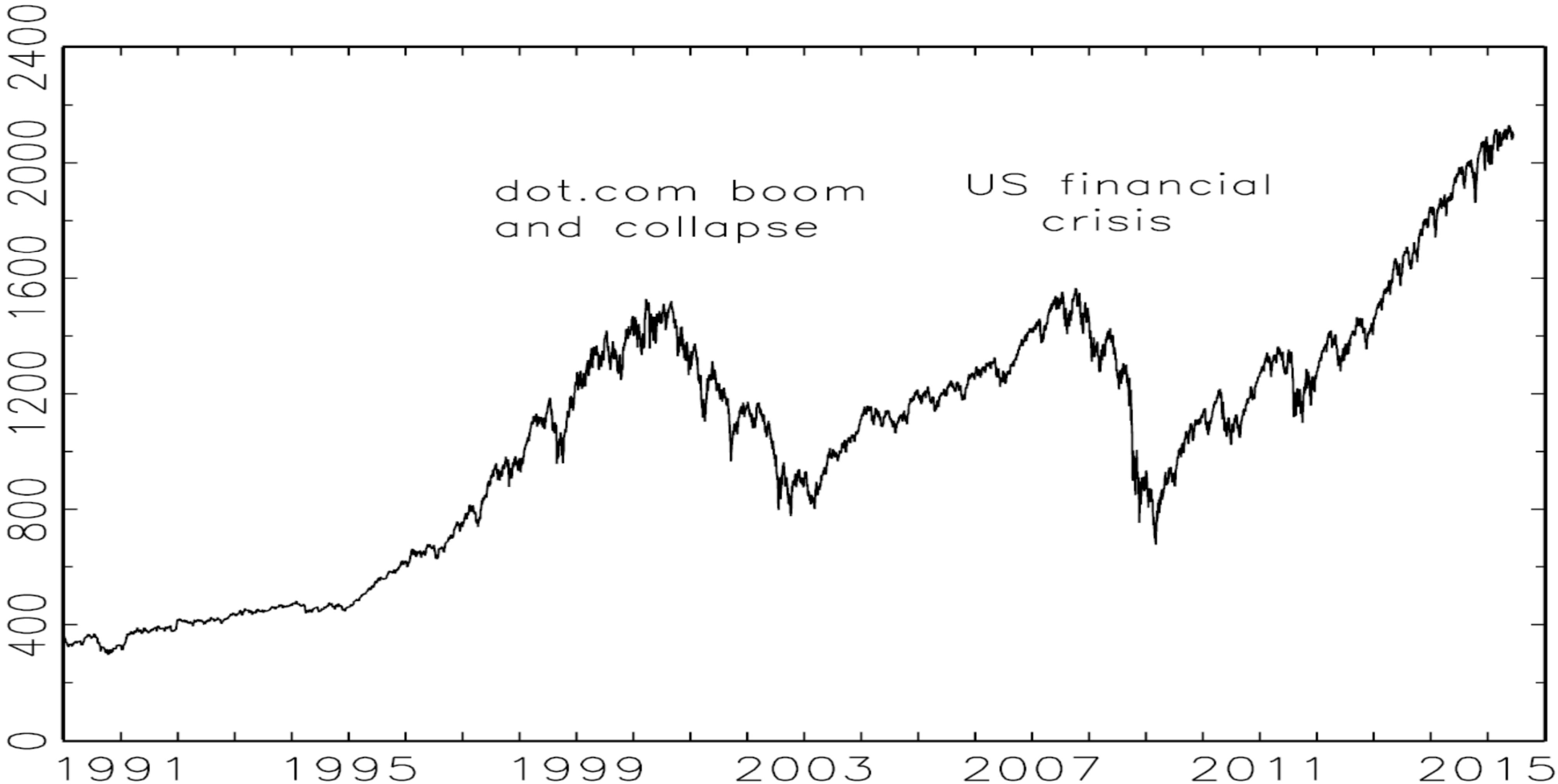
Data

Dataset

We use data on the S&P 500 index for period 2nd January 1990 to 17th June 2015 ($T = 6{,}416$ days) (full data window).

We estimate models for the daily percentage change of S&P 500, denoted by y_t.

We estimate the standard and DCS models several times for a set of rolling windows.



dot.com boom
and collapse
US financial
crisis
0
400
800
1200
1600
2000
2400
1991
1995
1999
2003
2007
2011
2015

Dataset

We start with the first 2,500 days of the sample.

Then, we shift this data window by excluding the first observation and adding the new observation.

We repeat this procedure until the end of the total sample period.

This gives 6,416-2,500=3,916 rolling windows, each with a sample size of 2,500.

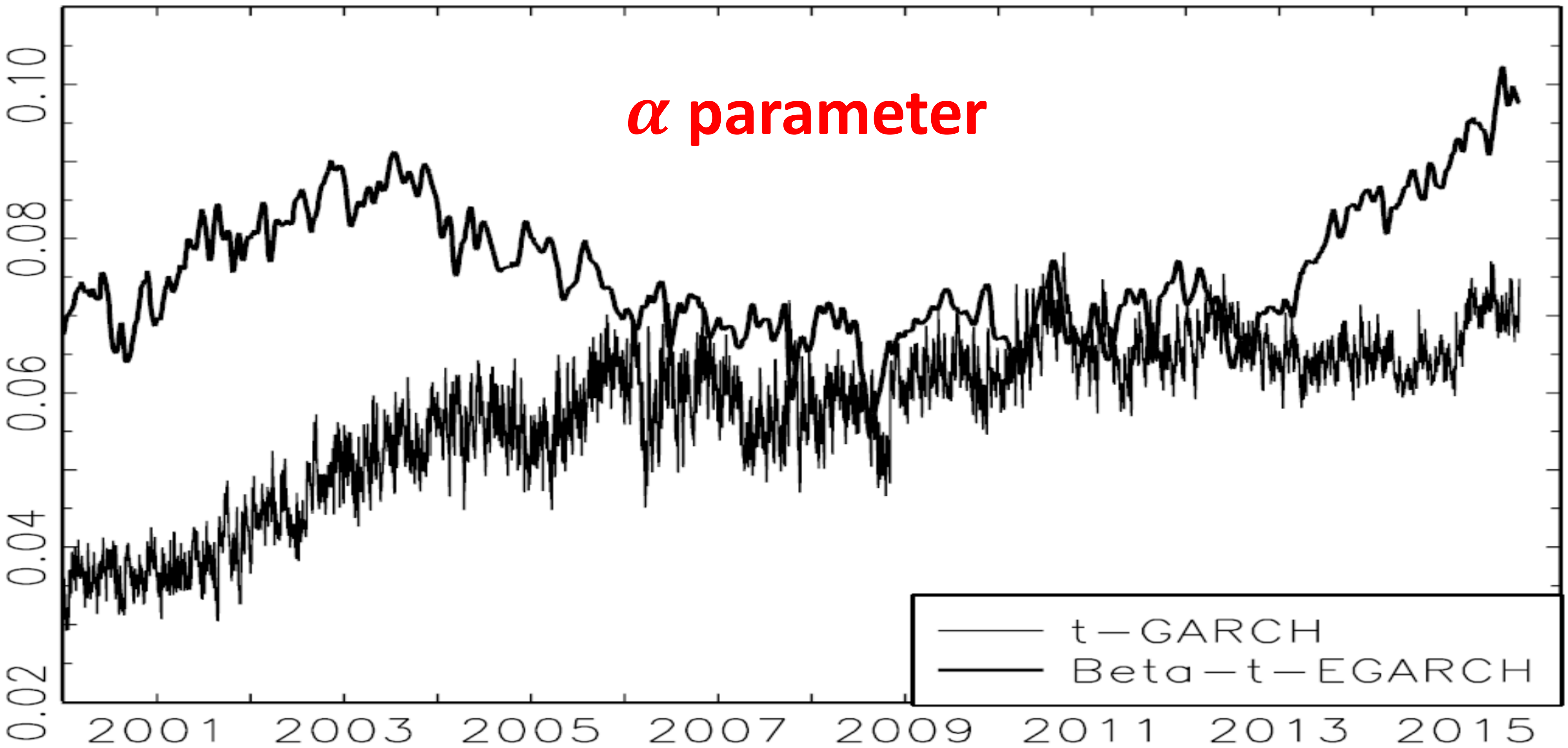
For each of these rolling windows, we estimate a different set of parameters by the maximum likelihood method.

Model stability

Model stability

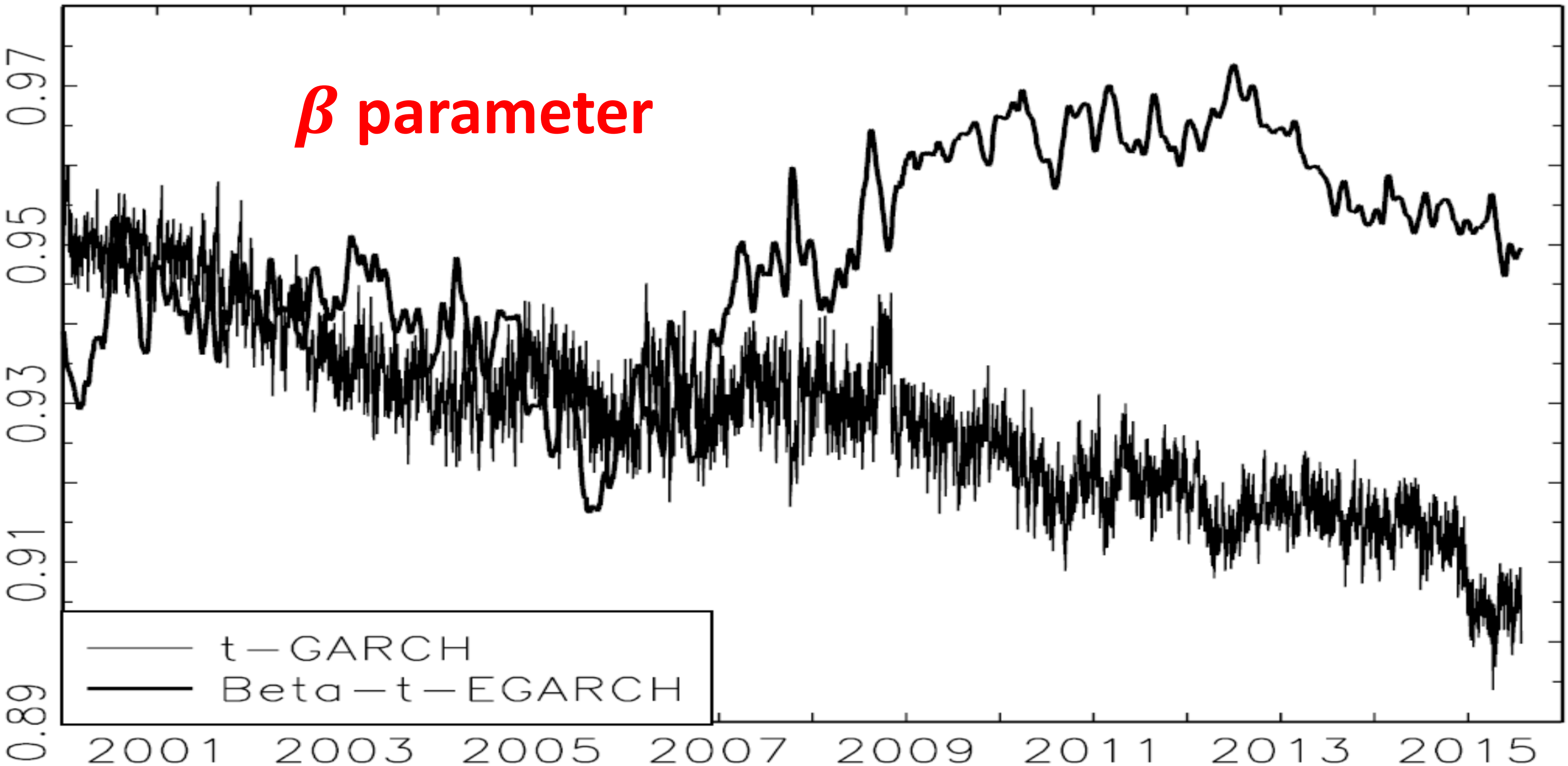
For the parametric standard financial and DCS models, model stability is equivalent with parameter stability.

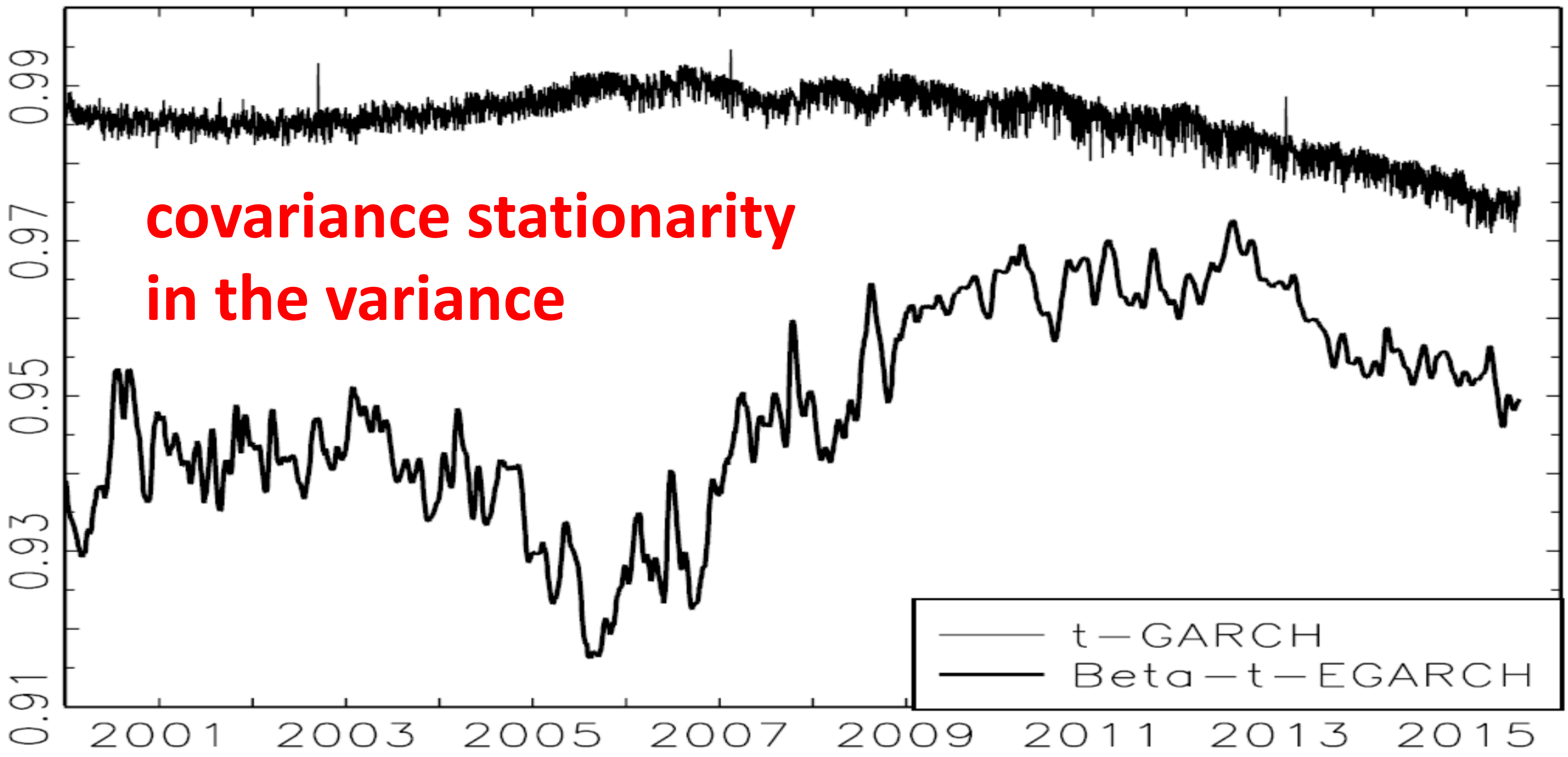
We present the evolution of α, β and the covariance stationarity in the variance statistics ($\alpha + \beta$ and $|\beta|$ for GARCH and Beta-t-EGARCH, respectively), in the following figures:



α parameter
0.02
0.04
0.06
0.08
0.10
2001
2003
2005
2007
2009
2011
2013
2015
t−GARCH
Beta−t−EGARCH

β parameter




covariance stationarity
in the variance
0.99
0.97
0.95
0.93
0.91
2001
2003
2005
2007
2009
2011
2013
2015
t−GARCH
Beta−t−EGARCH

Model stability

These figures suggest that DCS is more stable than the standard financial time series model.

This is a very elegant property for DCS.

However, does stability imply superior predictive performance?

Forecast performance

Forecast performance

We compare the density forecast performance of both models.

We focus on **density forecasts** (i.e. forecasts of the entire probability distribution) instead of **point forecasts** (i.e. forecasts of the expected return), due to the results of Granger and Pesaran (2000a; 2000b).

These authors demonstrate that point or interval forecasts cannot provide higher realized economic benefit to decision makers than density forecasts. Granger and Pesaran (2000a) also present that density forecasts can be applied by different users of forecasts with different nonlinear and asymmetric cost functions.

Forecast performance

For the forecasts performance analysis, we consider the **rolling data-window approach** used for the model stability analysis.

We divide each rolling data window with 2,500 observations into an estimation window with 2,250 observations and a forecast evaluation window with 250 observations.

For each of the estimation windows we estimate the parameters of ARMA plus t-GARCH and QAR plus Beta-t-EGARCH, and given those parameters we evaluate the log-density of y_t for each day of the corresponding forecast evaluation window.

Forecast performance

Let $\ln f(y_t|y_1, \dots, y_{t-1})$ and $\ln g(y_t|y_1, \dots, y_{t-1})$ denote the log-densities for ARMA plus *t*-GARCH and QAR plus Beta-*t*-EGARCH, respectively.

We represent the out-of-sample density forecast performance of ARMA plus *t*-GARCH with respect to QAR plus Beta-*t*-EGARCH by the mean log-density difference metric

$$\bar{d}_T = \frac{1}{T}\sum_{t=1}^{T} d_t = \frac{1}{T}\sum_{t=1}^{T}[\ln f(y_t|y_1, \dots, y_{t-1}) - \ln g(y_t|y_1, \dots, y_{t-1})]$$

Forecast performance

We use the Amisano-Giacomini (2007) (AG) density forecast performance test to evaluate the significance of $\bar{d}_T$.

The null hypothesis of the AG test is $E(d_t) = 0$ for all periods of the forecast evaluation window.

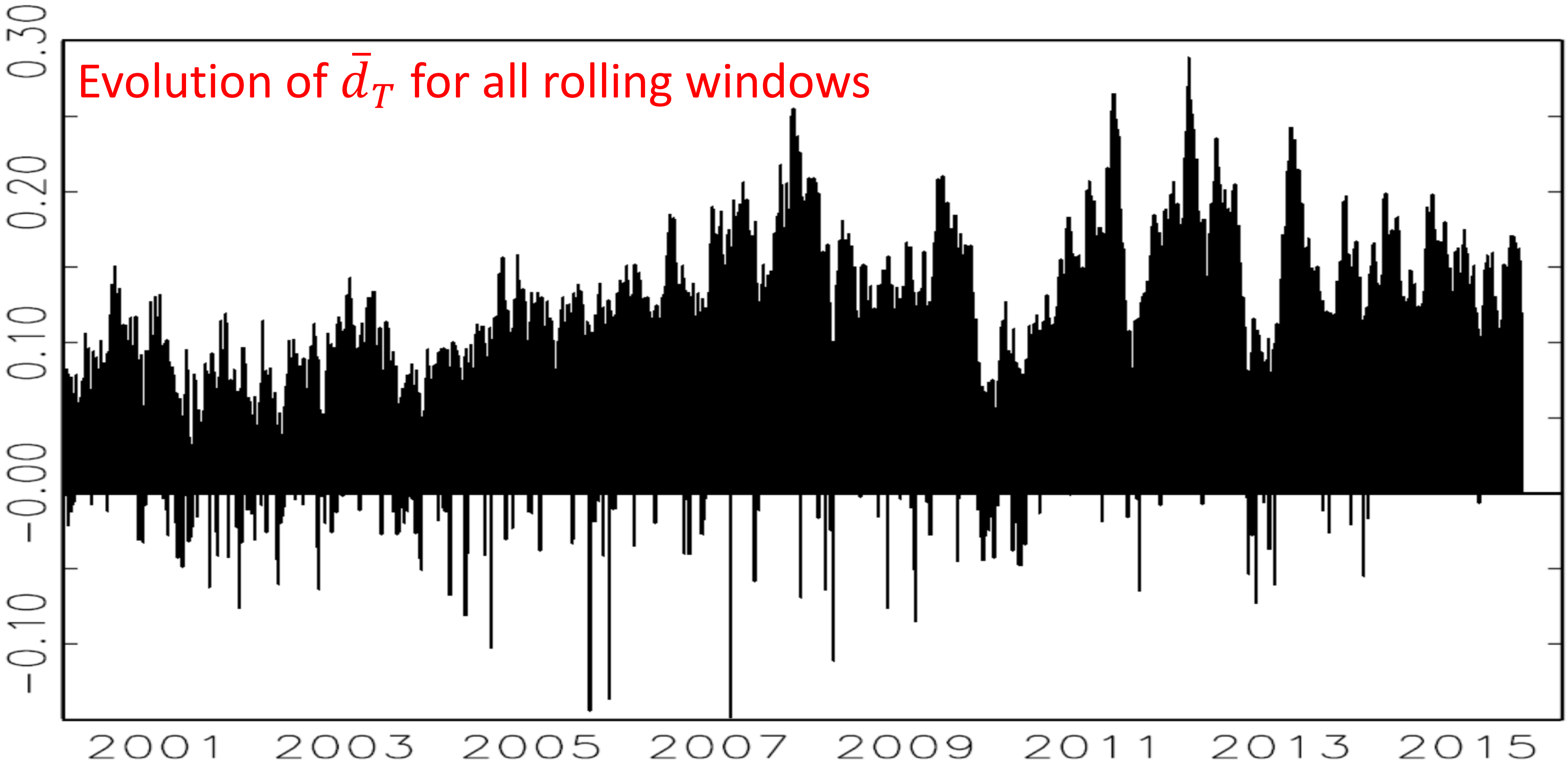
AG (2007) demonstrate that

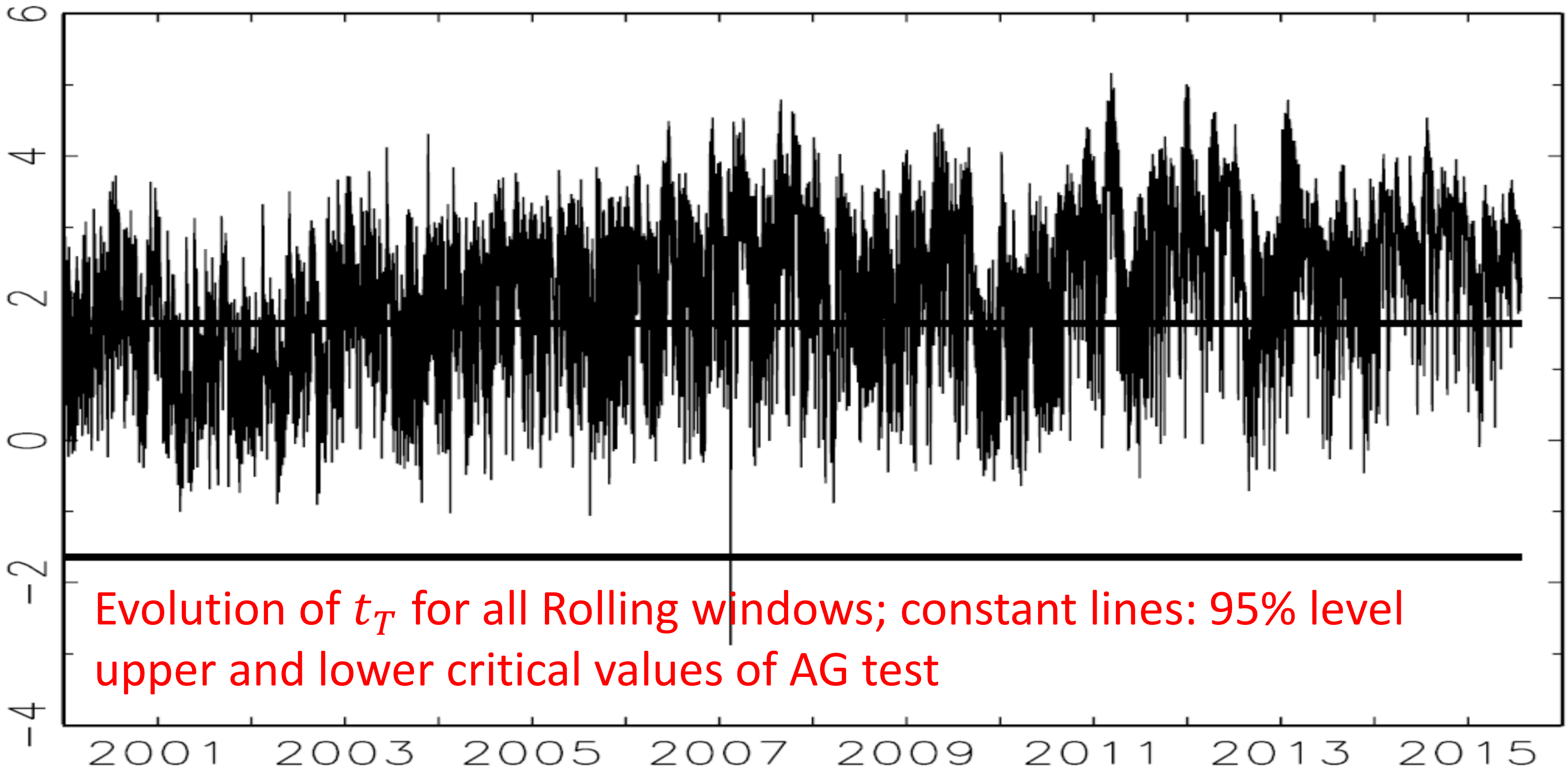
$$t_T = \frac{\bar{d}_T}{s_T} \sim N(0,1)$$

where s_T is heteroscedasticity and autocorrelation consistent estimator of the asymptotic standard deviation of $\bar{d}_T$.

Forecast performance

We present the AG test results for all rolling data-windows in the following figures:


Evolution of $\bar{d}_T$ for all rolling windows
0.30
0.20
0.10
0.00
−0.10
2001
2003
2005
2007
2009
2011
2013
2015


6
4
2
0
−2
−4
2001
2003
2005
2007
2009
2011
2013
2015
Evolution of t_T for all Rolling windows; constant lines: 95% level upper and lower critical values of AG test

Forecast performance

We find that

(i) the forecast performance of ARMA plus *t*-GARCH is superior to QAR plus Beta-*t*-EGARCH at the 5% level of significance for 64.58% of the rolling windows;

(ii) there is no significant difference in the predictive performance of these models for 35.39% of the rolling windows;

(iii) QAR plus Beta-*t*-EGARCH is superior to ARMA plus *t*-GARCH only for 0.03% of the rolling windows.

Summary of results for S&P 500

These results suggest that although QAR plus Beta-*t*-EGARCH(1,1) is a more stable model than ARMA plus *t*-GARCH(1,1), the out-of-sample density forecast performance of ARMA plus *t*-GARCH(1,1) for the investor loss function is superior.

This corresponds to the findings of Hansen and Lunde (2005) related to the out-of-sample forecast performance of GARCH(1,1).

Robustness analysis

Robustness analysis

We also study the robustness of model stability and forecast performance results obtained for the S&P 500, by repeating the same analysis for a Monte Carlo simulation experiment.

We simulate 6,416 values for y_t from the following stochastic volatility model:

Robustness analysis

$y_t = c + \varphi y_{t-1} + \exp(\lambda_t/2)\varepsilon_t$ where $\varepsilon_t \sim t(\nu)$ i.i.d.

$\lambda_{t+1} = \omega + \beta\lambda_t + \sigma_u u_t$ where $u_t \sim N(0,1)$ i.i.d.

(Harvey, Ruiz and Shephard, 1994; Harvey and Shephard, 1996)

For the Monte Carlo simulation we use empirically reasonable values of parameters: $c = 0.001$; $\varphi = 0.1$; $\nu = 8$; $\omega = -0.05$; $\beta = 0.995$; $\sigma_u = 0.1$.

Robustness analysis

The model stability and forecast performance results for the Monte Carlo simulation experiment coincide with the results for S&P 500.

References

- Amisano, G. and Giacomini, R. (2007) Comparing density forecasts via weighted likelihood ratio tests, Journal of Business & Economic Statistics, 25, 177-190.
- Bollerslev, T. (1987) A conditionally heteroskedastic time series model for security prices and rates of return data, The Review of Economics and Statistics, 59, 542-547.
- Box, G. E. P. and Jenkins, G. M. (1970) Time Series Analysis, Forecasting and Control, Holden-Day, San Francisco.
- Granger, C. W. J. and Pesaran, M. H. (2000a) A decision theoretic approach to forecast evaluation. In Statistics and Finance: An Interface, edited by Chan, W. S., Li, W. K. and Tong, W., 261-278. Imperial College Press, London.
- Granger, C. W. J. and Pesaran, M. H. (2000b) Economic and statistical measures of forecast accuracy, Journal of Forecasting, 19, 537-560.

- Hansen, P. R. and Lunde, A. (2005) A forecast comparison of volatility models: does anything beat a GARCH(1,1)?, Journal of Applied Econometrics, 20, 873-889.
- Harvey, A. C. (2013) Dynamic Models for Volatility and Heavy Tails, Cambridge University Press, Cambridge.
- Harvey, A. C. and Chakravarty, T. (2008) Beta-t-(E)GARCH. Cambridge Working Papers in Economics 0840, Faculty of Economics, University of Cambridge, Cambridge.
- Harvey, A. C., Ruiz, E. and Shephard, N. (1994) Multivariate stochastic variance models, The Review of Economic Studies, 61, 247-264.
- Harvey, A. C. and Shephard, N. (1996) Estimation of an asymmetric stochastic volatility model for asset returns, Journal of Business & economic Statistics, 14, 429-434.

Thank you for your attention!

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