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Forecast performance of dynamic conditional score copula models

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Motivation and contribution

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We study the relationship between returns on two financial assets.

Relationships between asset returns influence the risk of the portfolio (i.e. portfolio volatility) formed by those assets.

From a practical point of view, our paper may be useful for the construction of optimal financial portfolios of two assets or Monte Carlo simulation of returns of those assets.

We suppose that relationships may be time-varying and suggest new models, with respect to the body of literature, that can measure and forecast those time-varying relations.

Relationships between random variables

Relationships between random variables

The standard metric for measuring the relationship between two random variables is the **correlation coefficient**.

Consider two random variables: X and Y.

The correlation coefficient of (X, Y) is

$$\rho(X, Y) = \frac{\mathrm{cov}(X, Y)}{\sigma_X \sigma_Y}$$

where $\mathrm{cov}(X, Y)$ is the covariance of X and Y, σ_X is the standard deviation of X, and σ_Y is the standard deviation of Y.

Relationships between random variables

Hence, the relationship is measured by the **covariance**,

$$\text{cov}(X, Y) = E\{[X - E(X)][Y - E(Y)]\}$$

As this formula shows, the correlation coefficient represents by a single value the **average relationship** between X and Y.

Thus, $\rho(X, Y)$ is an aggregate measure of relationship between X and Y.

Relationships between random variables

Nevertheless, by aggregation, we loose information about different levels of relationship existing at different levels of X and Y.

For example:

If both X and Y are low then the relationship may be strongly positive.

If both X and Y are near zero then the relationship may be near zero.

If both X and Y are high then the relationship may be medium positive.

All these specific relationships that correspond to different levels of X and Y are averaged out by $\rho(X,Y)$, hence they are lost.

Relationships between random variables

In the present work, we use a more precise measure of relationship that allows to measure different levels of relationship for different levels of X and Y.

This measure is the **copula function** (Joe, 2015).

Relationships between random variables

We use ten pairs of daily return time series of financial assets.

For all time series, we divide the full data window into the estimation and forecast evaluation windows.

The **estimation window** starts in the beginning of the full data window and ends on the last day of 2013. (*In-sample analysis*)

The **forecast evaluation window** is for the period of January 2014 to June 2015. (*Out-of-sample forecast analysis*)

The ten pairs of financial assets are as follows:

Pair	Identifiers	Correlation	τ	β
1	DOW and NASDAQ	0.7514	0.5640	0.5649
2	CAC and DAX	0.8321	0.6190	0.6005
3	FTSE and HF	0.7146	0.5626	0.5949
4	EMER and US	0.3946	0.2206	0.2055
5	CAPIT and CONS	0.7780	0.5539	0.5330
6	AAPL and GOOG	0.4525	0.3156	0.2963
7	KO and PEP	0.5195	0.3519	0.3610
8	CHEV and EXX	0.6808	0.4541	0.4553
9	GOLD and SILVER	0.6979	0.5138	0.5140
10	NATGAS and OIL	0.2124	0.1535	0.1569

Relationships between random variables

Let X and Y denote the daily return for each financial asset.

For each return observation of X, we compute the proportion of returns of the full data window that are lower than the observation.

For each return observation of Y, we compute the proportion of returns of the full data window that are lower than the observation.

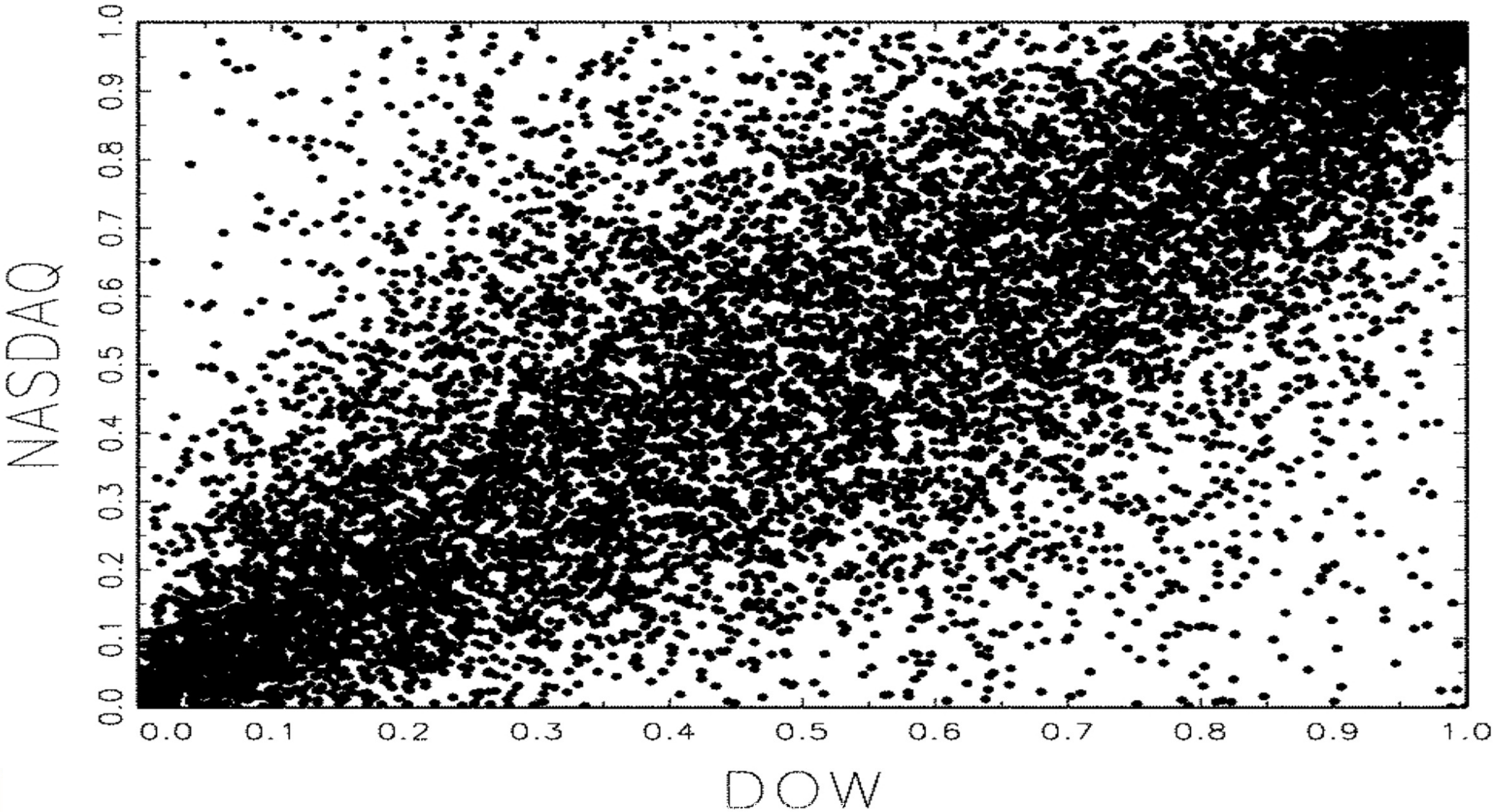
These are named **percentile ranks**.

Relationships between random variables

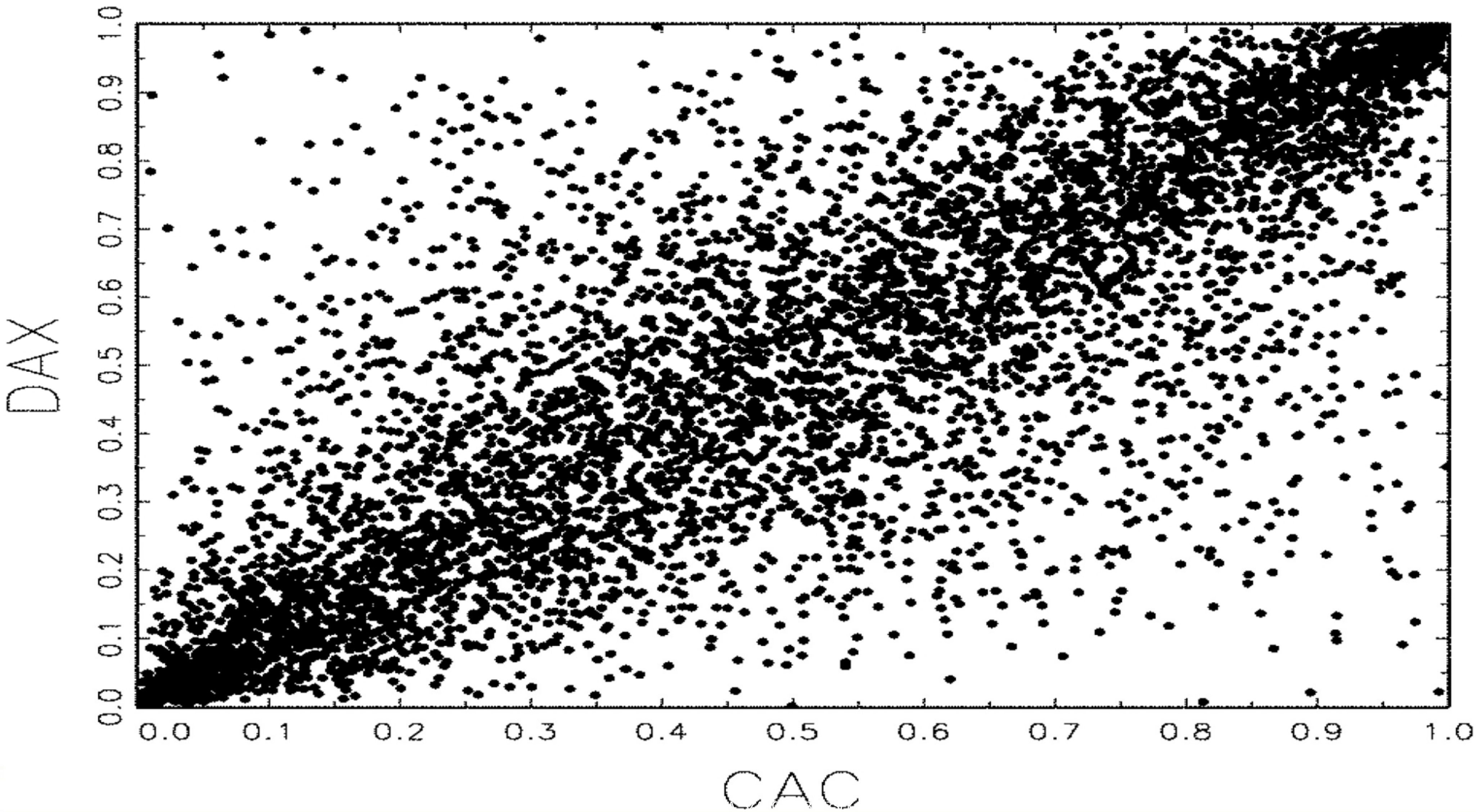
We support the idea that for different levels of X and Y we may have different levels of relationship by showing scatter plots of the percentile ranks of X and Y.

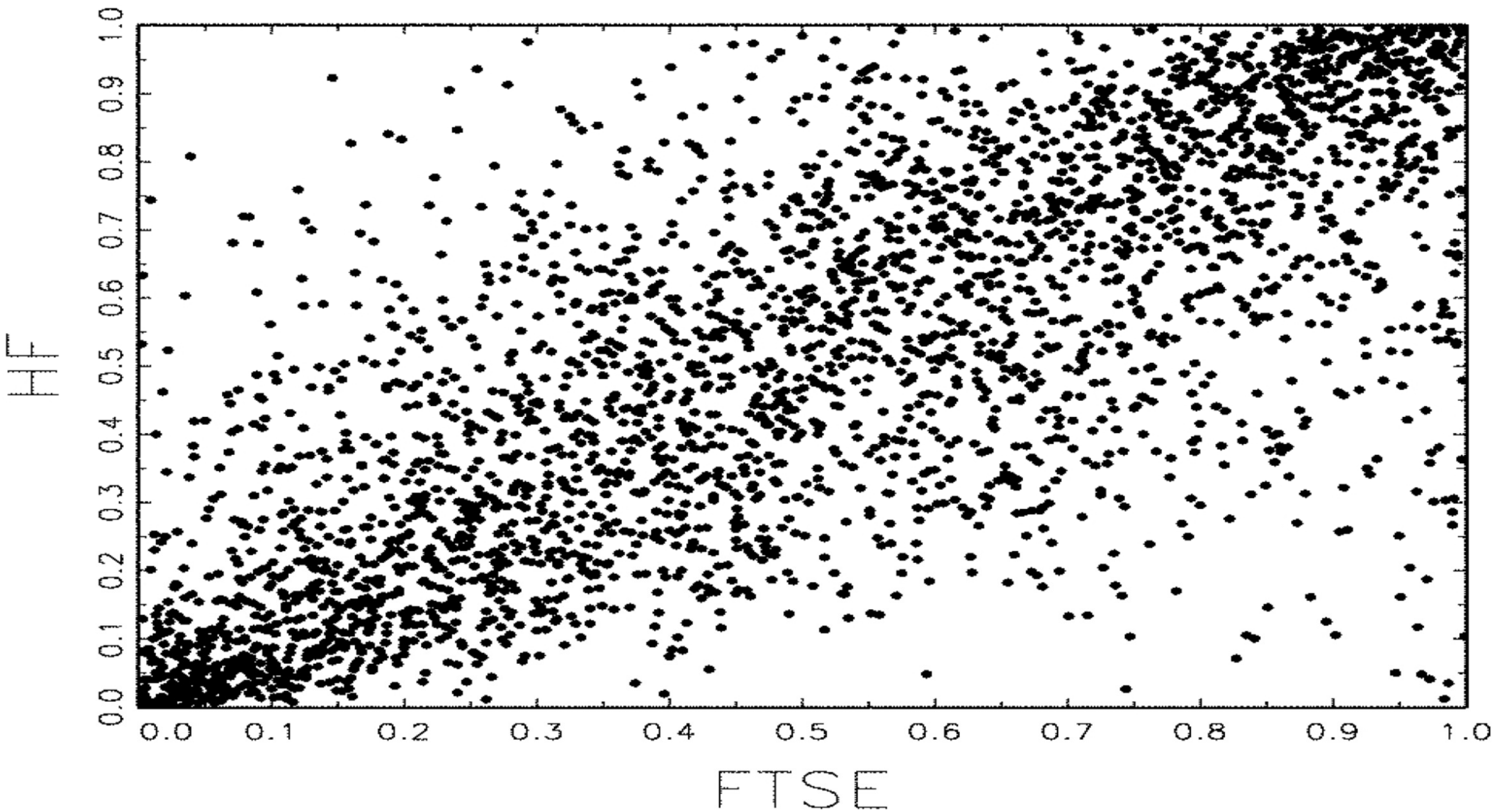
Each point on that scatterplot corresponds to particular day, and the corresponding value shows the proportion of other observations of the full data window that are lower than the return observed for that day.

See the following figures on percentile ranks of X and Y for some pairs of assets:

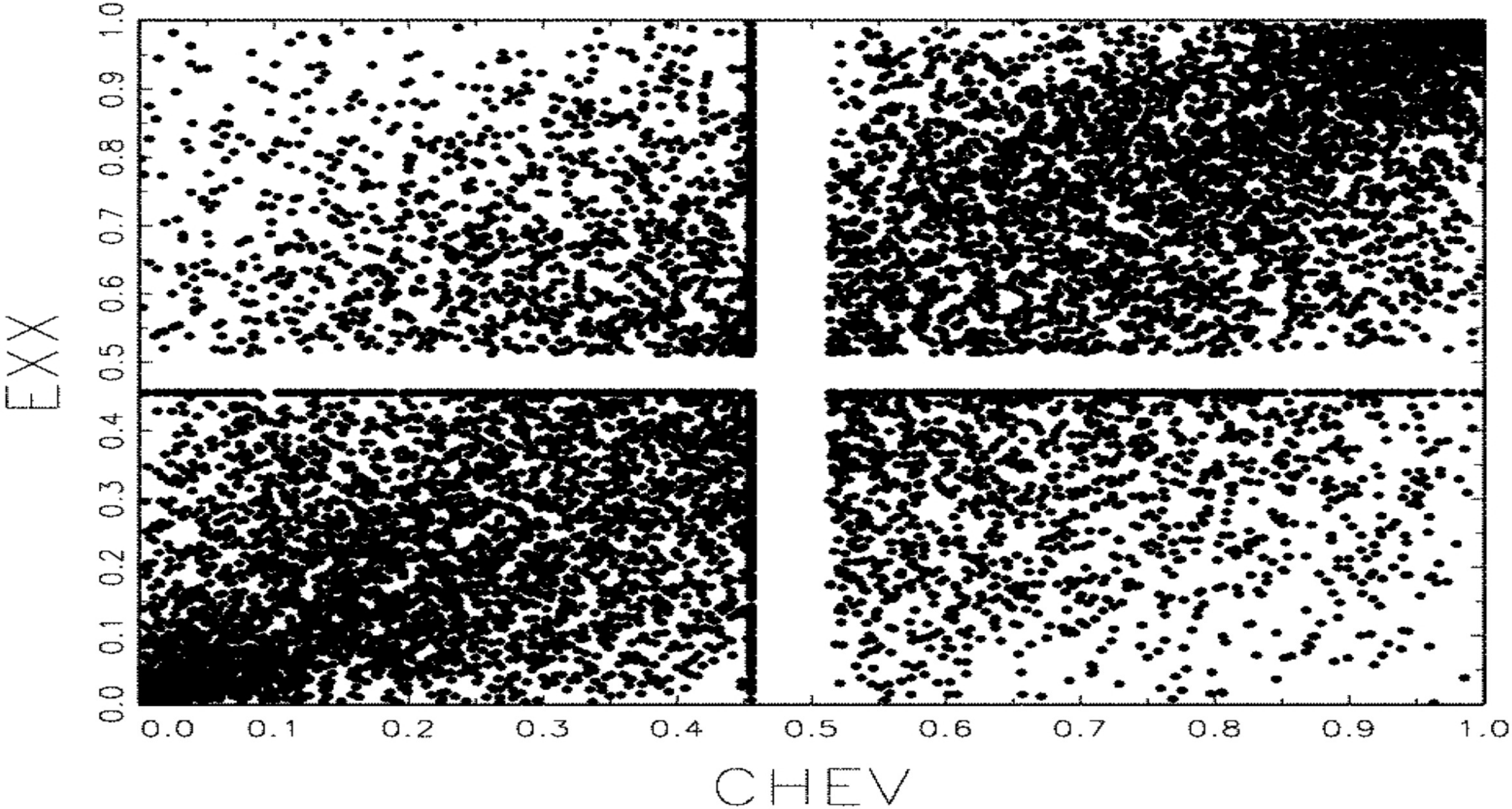


NASDAQ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DOW


DAX
CAC
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0



HF
FTSE
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0



EXX
CHEV
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Relationships between random variables

Some points about these figures:

(i) Positive association (all figures)

(ii) Relatively high concentration of points in the lower left and upper right corners (all figures)

(iii) Possible asymmetry of concentration of points (HF-FTSE)

(iv) Empty region in the center of figure (EXX-CHEV)

Relationships between random variables

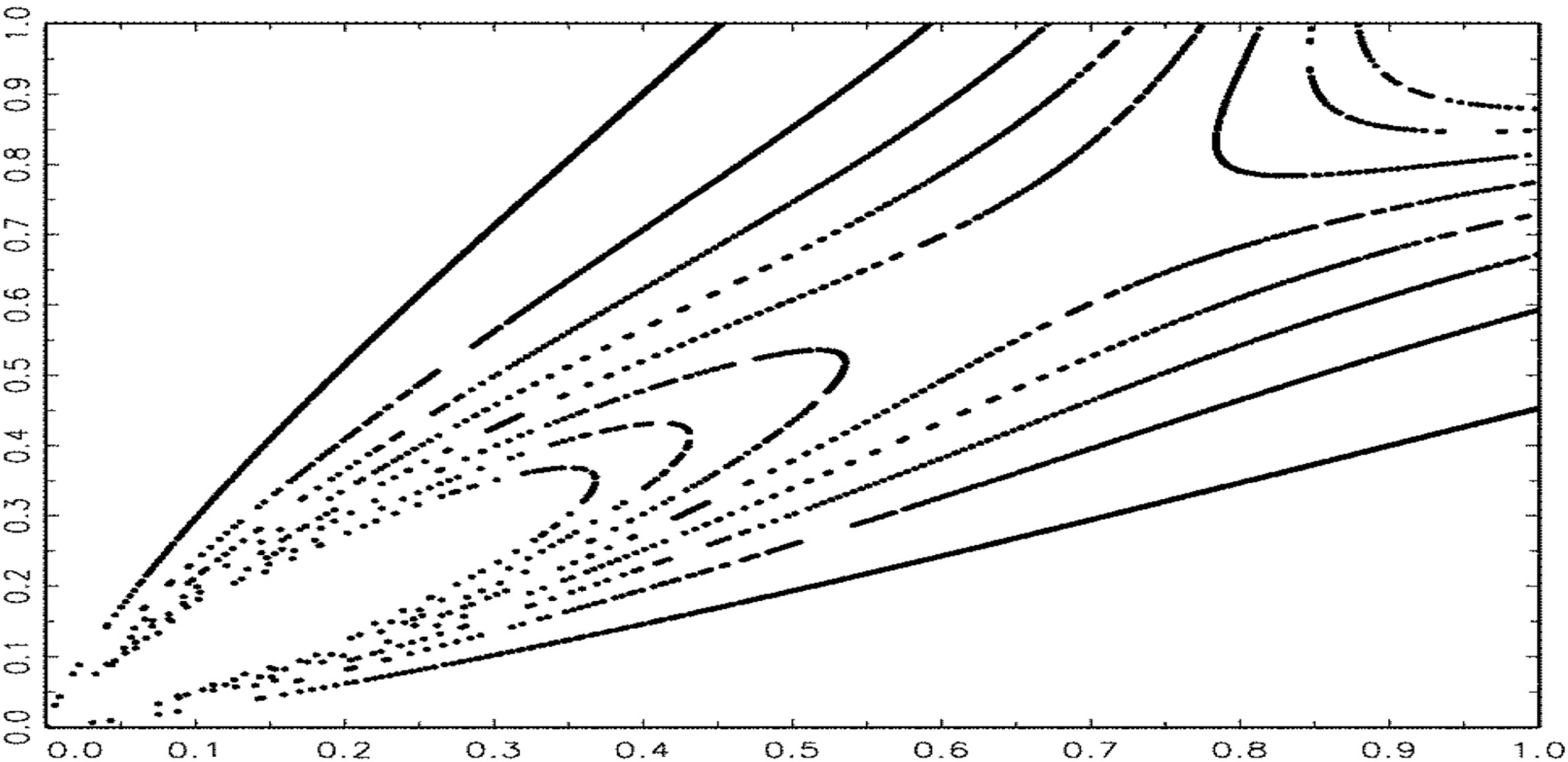
We use the copula function in order to capture the different patterns of relationship observed on the percentile rank scatterplots.

There are a number of different parametric copula functions available, and each of them implies different patterns of relationship (Joe, 2015).

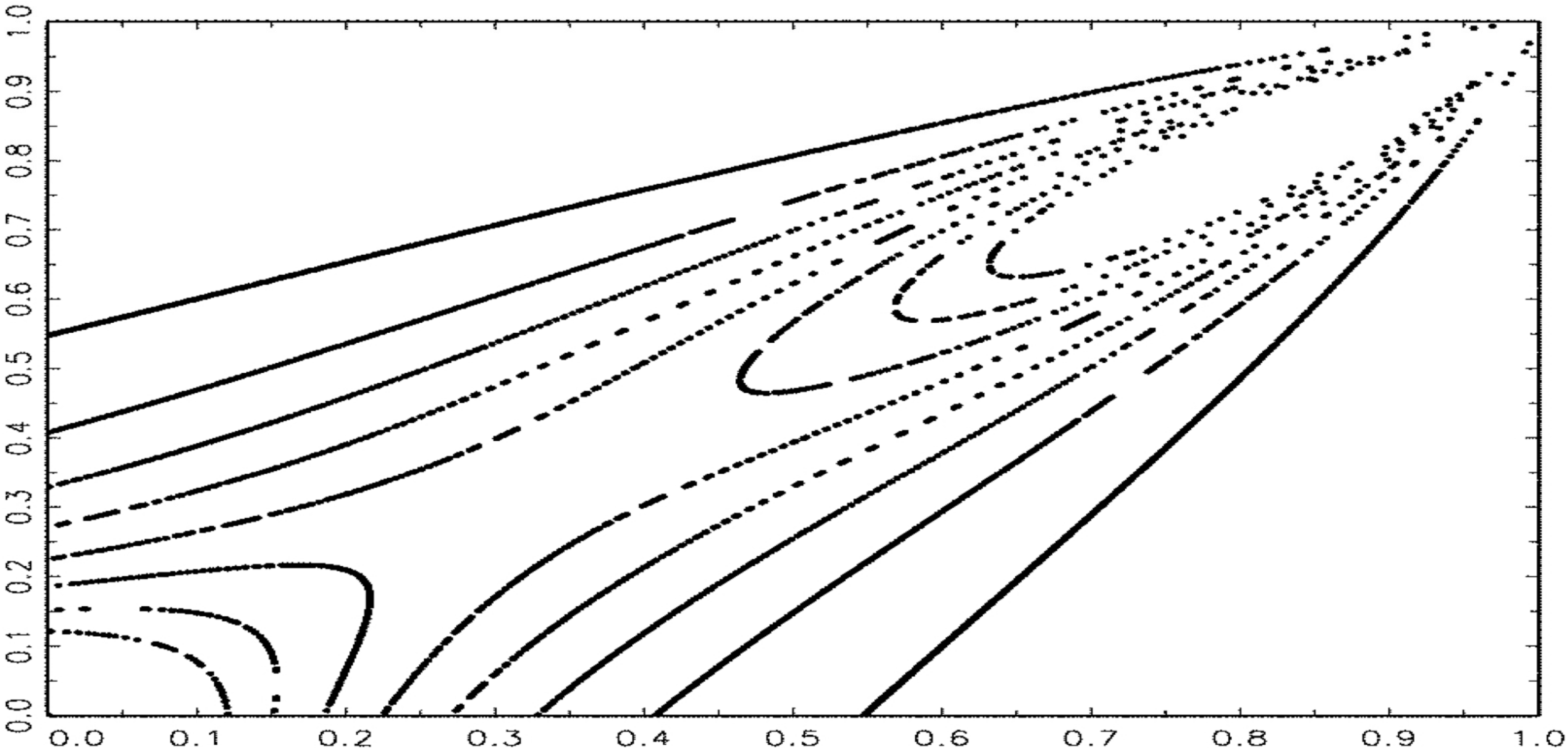
We use eight alternative copula functions.

In the following figures, we present contour plots for the copula density associated with each of the copula functions:

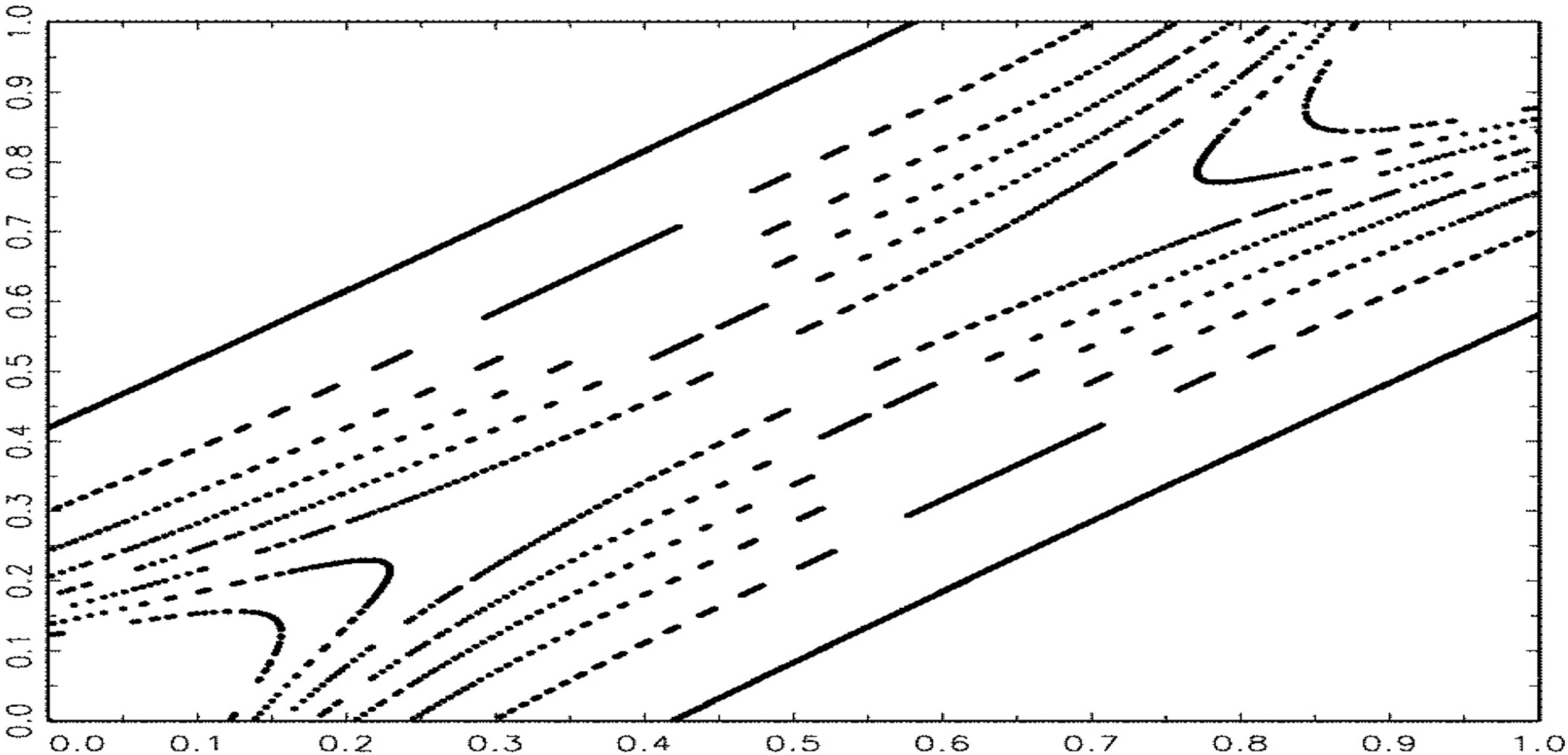
Clayton



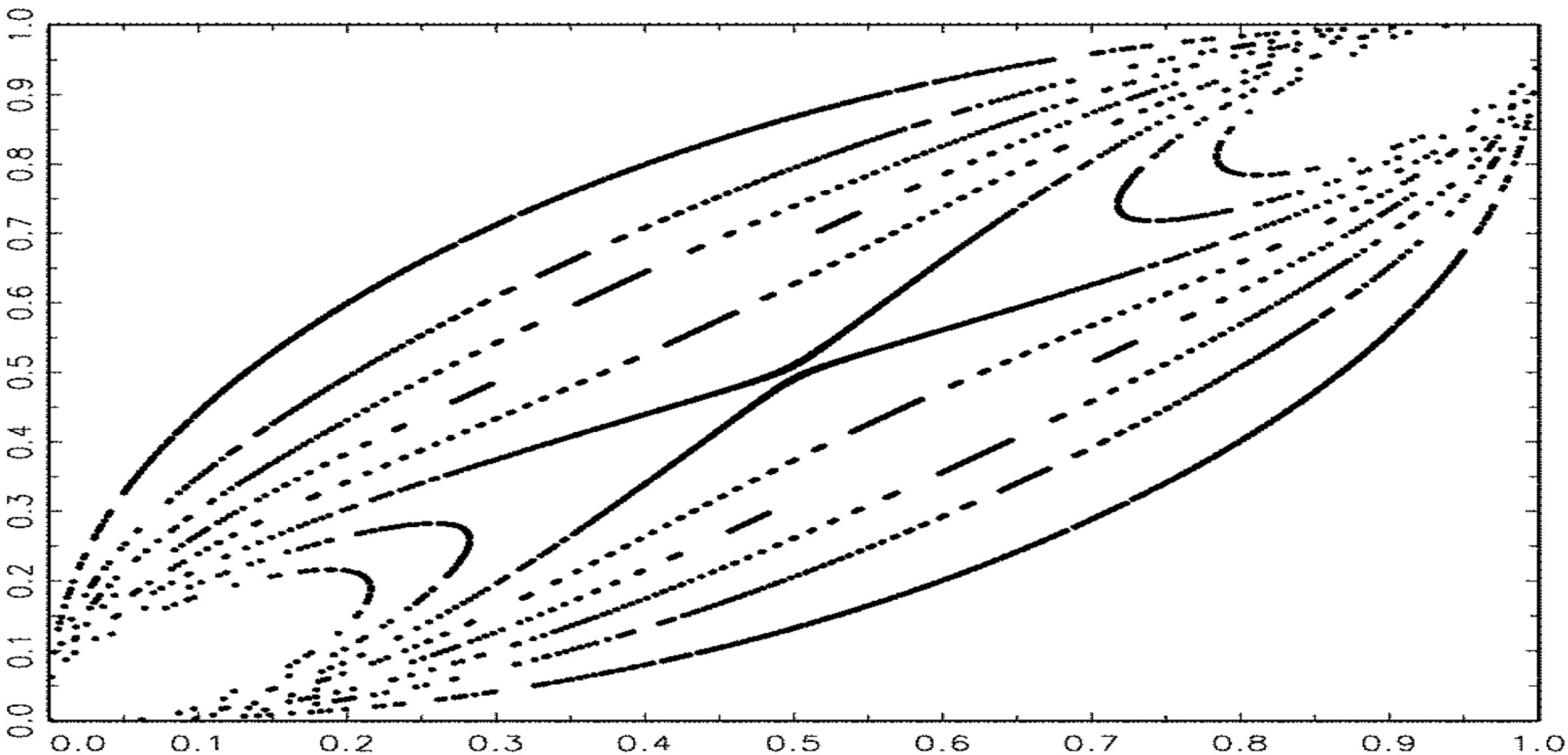
Rotated Clayton



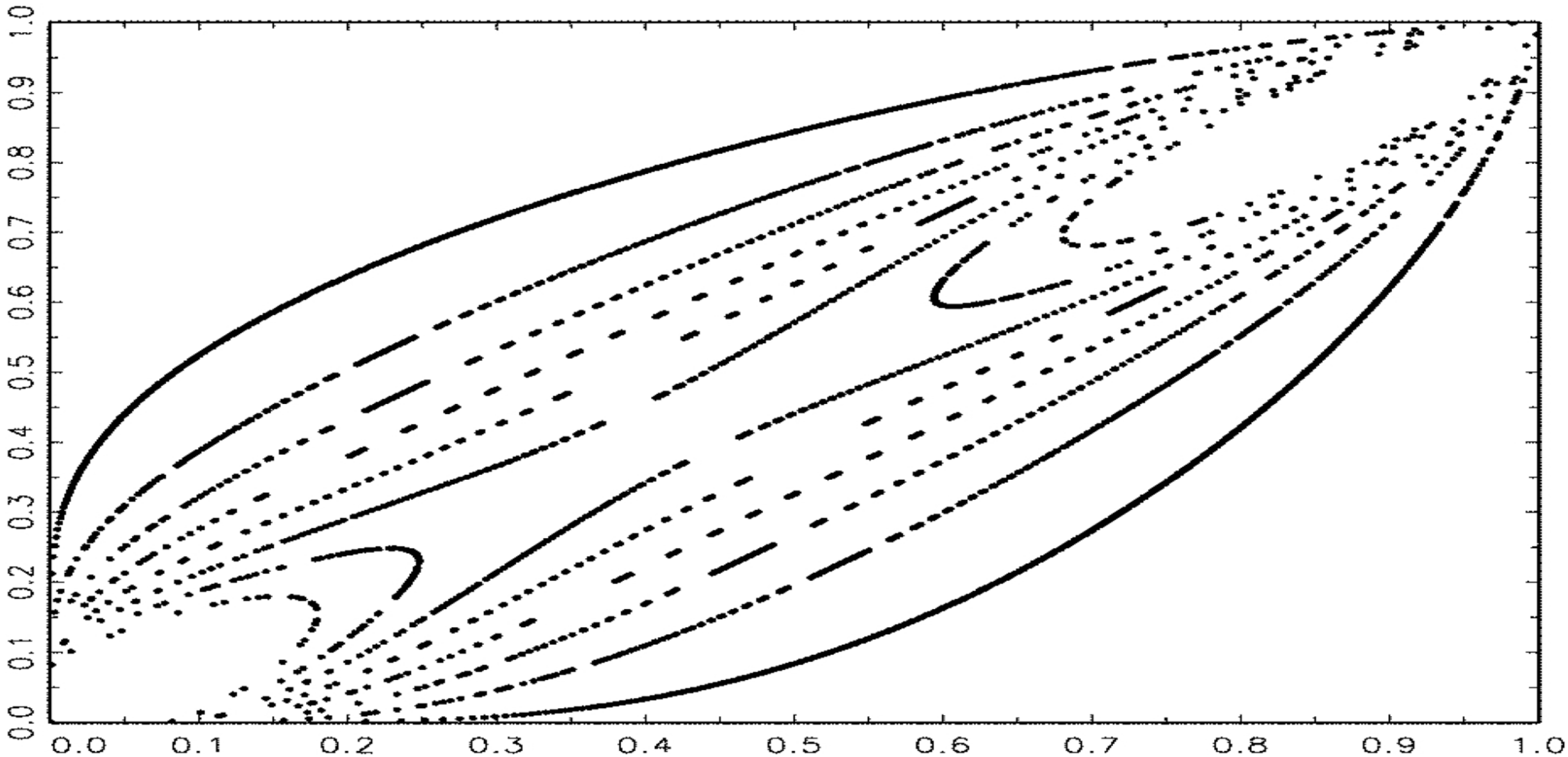
Frank



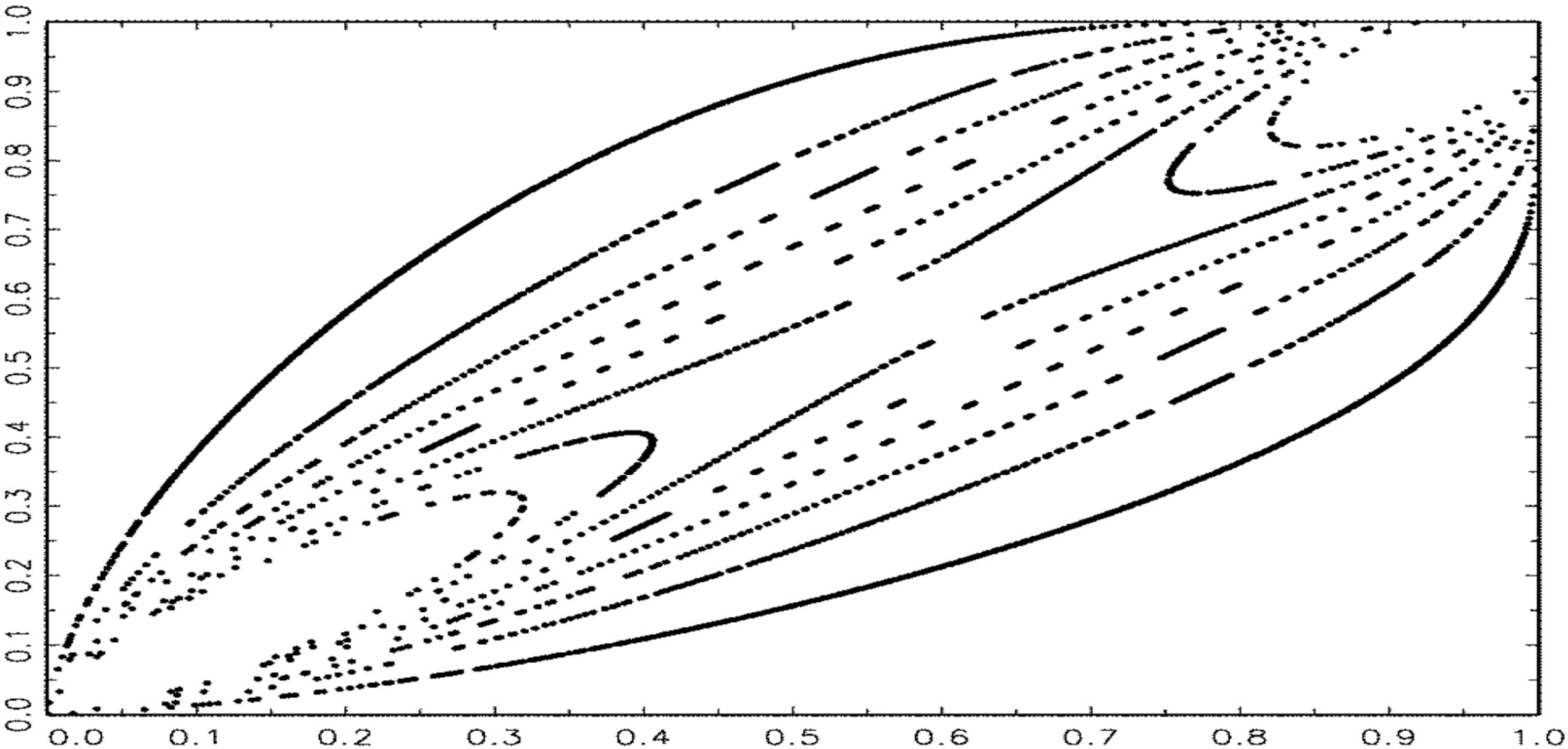
Gaussian



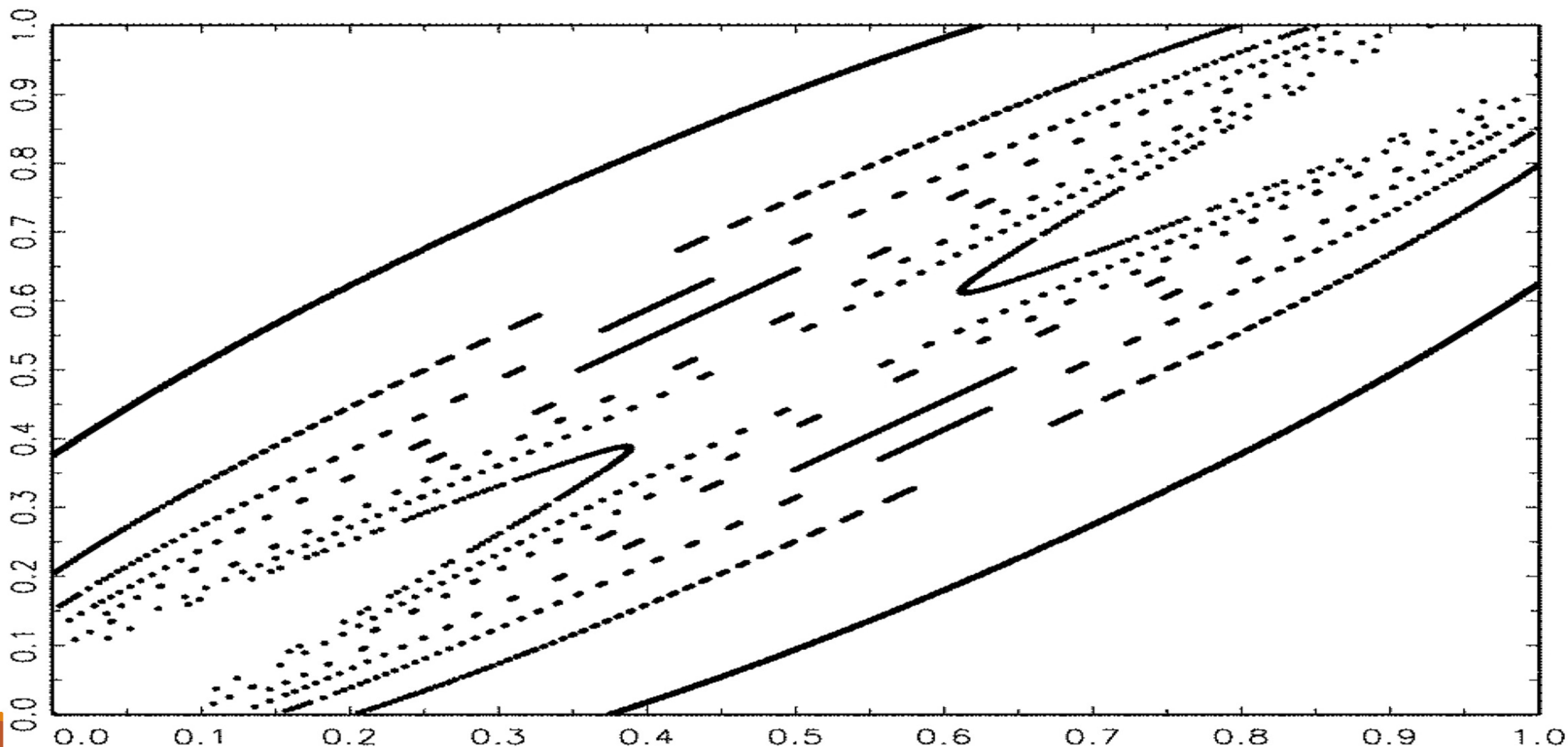
Gumbel



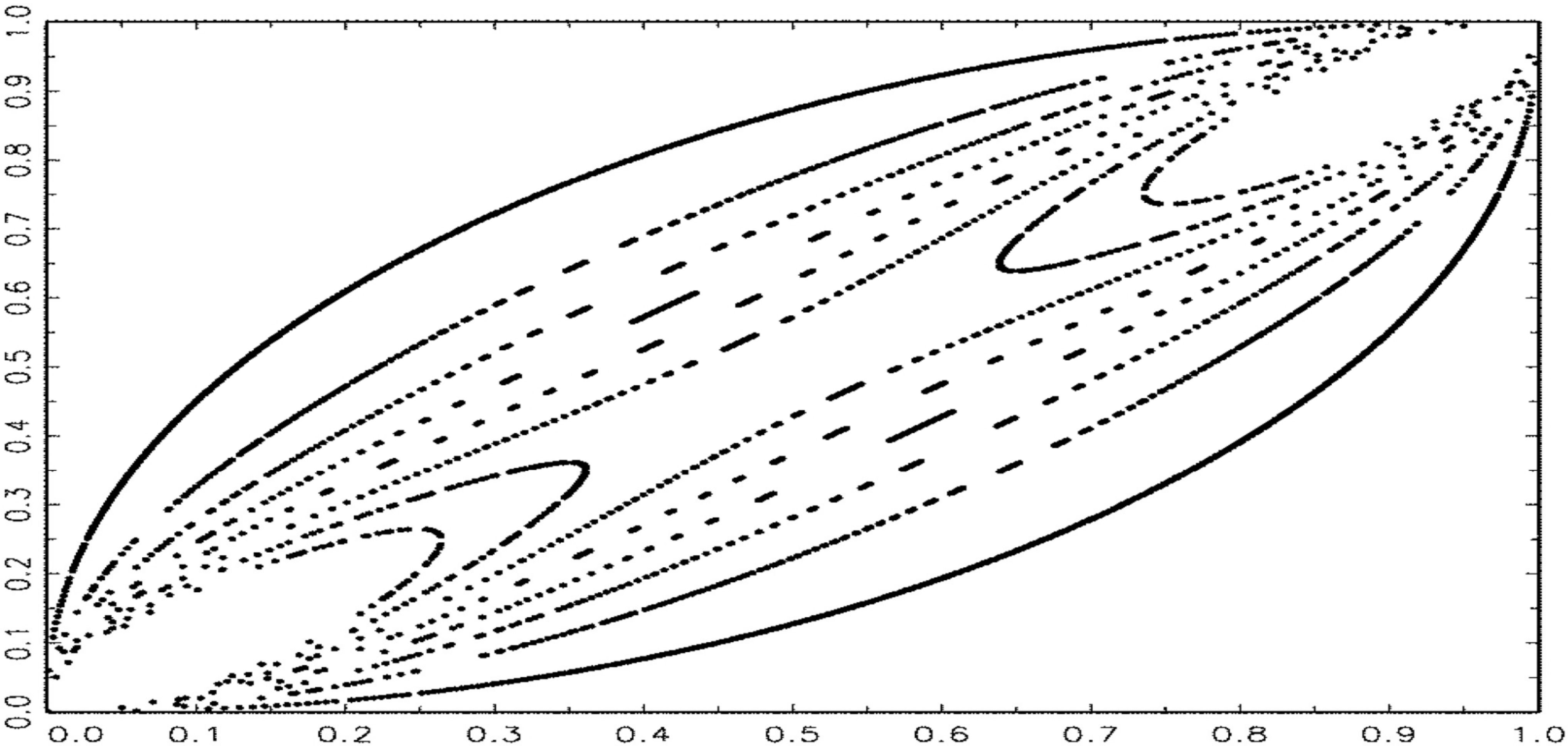
Rotated Gumbel



Plackett



Student's t



Relationships between random variables

In all these copula densities, the parameter ρ measures the level of the association between X and Y.

For all the eight parametric copulas (with the exception of the Student's t), there is only one parameter (ρ) in the copula density.

For the Student's t-copula, in addition to ρ, there is also the degrees of freedom parameter ν.

Relationships between random variables

The level of relationships among financial asset returns is **not constant over time**.

For example, during periods of financial crisis (e.g. the 2008 US financial crisis), the association among different assets usually is high as the common (systematic) factor influences more asset prices.

Relationships between random variables

We consider the possibility of time-varying relationships by modeling the ρ parameter of the copula as time-varying.

We consider a new dynamic model, the ***dynamic conditional score driven copula model***, in order to update ρ by the new information for every day. We consider two dynamic specifications:

$\tilde{\rho}_t = \tilde{\rho}_{t-1} + \kappa\xi_{t-1}$ (random walk type specification)

$\tilde{\rho}_t = \delta + \gamma\tilde{\rho}_{t-1} + \kappa\xi_{t-1}$ with $|\gamma| < 1$ (QAR(1) type specification)

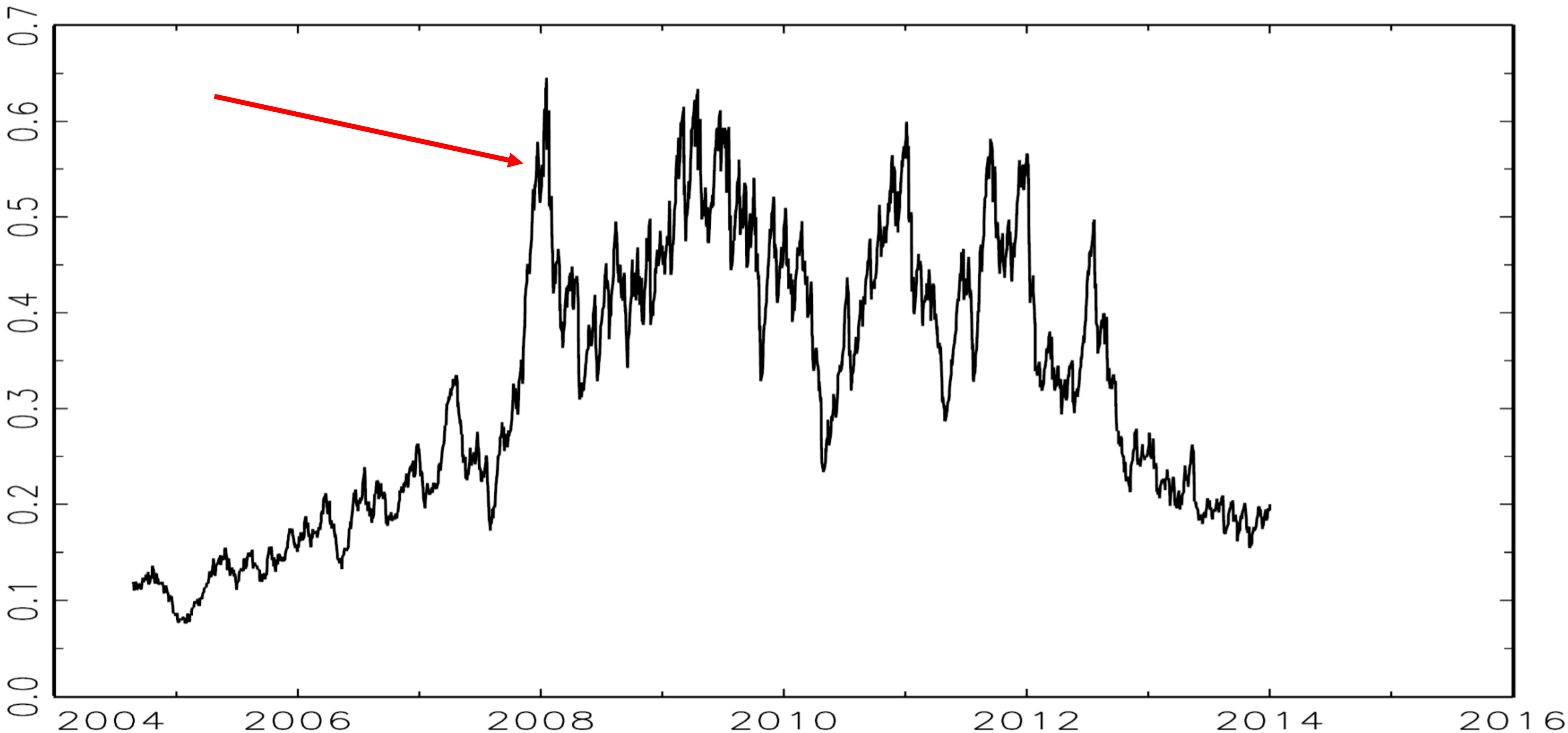
where ξ_{t-1} is the conditional score with respect to ρ_{t-1}.

Relationships between random variables

In the following figure we present the evolution of the Kendall's tau measure of association implied by $\tilde{\rho}_t$ for

AAPL-GOOG

AAPL and GOOG, RW



Statistical inference and results

Statistical inference

An advantage of the use of copula-based models of association for multivariate time series data is that, once we have selected the parametric copula function, we can model separately the marginal distribution of each asset return time series.

Statistical inference

For each asset we consider three competing models of the marginal distribution:

(i) ARMA(1,1) plus *t*-GARCH(1,1) with leverage effects (Box and Jenkins, 1970; Bollerslev, 1987; Glosten et al., 1993)

(ii) *t*-QAR(1) plus Beta-*t*-EGARCH(1,1) with leverage effects (Harvey and Chakravarty, 2008; Harvey, 2013)

(iii) Gen-*t*-QAR(1) plus Beta-Gen-*t*-EGARCH(1,1) with leverage effects (Harvey and Lange, 2015)

Statistical inference

In the first step, we estimate these univariate models for each asset and use the *Bayesian Information Criterion (BIC)* to choose the most parsimonious model of the marginal distribution:

	DOW	NASDAQ	CAC	DAX
ARMA plus t-GARCH	NA	−6.5915	−5.9633	−6.0116
t-QAR plus Beta-t-EGARCH	−6.5964	−6.5925	**−5.9662**	**−6.0160**
Gen-t-QAR plus Beta-Gen-t-EGARCH	**−6.5967**	**−6.6025**	−5.9643	−6.0145
	FTSE	HF	EMER	US
ARMA plus t-GARCH	NA	NA	−6.5206	−6.5972
t-QAR plus Beta-t-EGARCH	**−6.7707**	**−9.6600**	**−6.5255**	−6.5992
Gen-t-QAR plus Beta-Gen-t-EGARCH	−6.7625	−9.6508	−6.5134	**−6.6032**
	CAPIT	CONS	AAPL	GOOG
ARMA plus t-GARCH	−6.2327	−6.8122	−4.9023	−5.2618
t-QAR plus Beta-t-EGARCH	−6.2331	**−6.8137**	**−4.9028**	**−5.2738**
Gen-t-QAR plus Beta-Gen-t-EGARCH	**−6.2332**	−6.8113	−4.9001	−5.2721
	KO	PEP	CHEV	EXX
ARMA plus t-GARCH	−5.7999	−5.7359	**−5.6217**	**−5.9015**
t-QAR plus Beta-t-EGARCH	**−5.8043**	**−5.7426**	−5.6195	−5.9002
Gen-t-QAR plus Beta-Gen-t-EGARCH	−5.8024	−5.7387	−5.6192	−5.8968
	GOLD	SILVER	NATGAS	OIL
ARMA plus t-GARCH	**−6.4457**	−5.4027	NA	−4.9268
t-QAR plus Beta-t-EGARCH	−6.4426	**−5.4035**	**−4.1487**	**−4.9305**
Gen-t-QAR plus Beta-Gen-t-EGARCH	−6.4378	−5.4022	−4.1475	−4.9287

Statistical inference

In the second step, we estimate jointly the optimal model of the marginal distribution and each of the parametric copulas.

We estimate the parameters by using the **maximum likelihood method**,

$$\widehat{\Theta}_{\mathrm{ML}} = \underset{\Theta}{\operatorname{argmax}}\, LL[(y_{11}, y_{21}), \ldots, (y_{1T}, y_{2T}); \Theta]$$

We undertake robust estimation of the standard errors of parameters.

Statistical inference

The Log-Likelihood (LL) function is given by

$$LL[(y_{11}, y_{21}), \dots, (y_{1T}, y_{2T}); \Theta] = \sum_{t=1}^{T} \ln f[y_{1t}, y_{2t} | (y_{11}, y_{21}), \dots (y_{1t-1}, y_{2t-1})]$$

where

$$\ln f[y_{1t}, y_{2t} | (y_{11}, y_{21}), \dots (y_{1t-1}, y_{2t-1})] =$$

$$\ln c[F(y_{1t} | y_{11}, \dots, y_{1t-1}), F(y_{2t} | y_{21}, \dots, y_{2t-1}) | (y_{11}, y_{21}), \dots (y_{1t-1}, y_{2t-1})] +$$

$$\ln f(y_{1t} | y_{11}, \dots, y_{1t-1}) + \ln f(y_{2t} | y_{21}, \dots, y_{2t-1})$$

where $c(x, y)$ is the bivariate copula density, $F(x)$ is the cumulative distribution function and $f(x)$ is the probability density function.

In-sample statistical performance

We compare the statistical performance of different specifications by using the BIC metric:

Pair	Identifiers	$\tilde{\rho}_t$	Clayton	R Clayton	Frank	Gaussian	Gumbel	R Gumbel	Plackett	Student's t
1	DOW and NASDAQ	RW	−14.0615	−14.0227	−14.1646	−14.2405	−13.9405	−13.9555	−14.1833	−14.2609
1	DOW and NASDAQ	QAR	−14.0614	−14.0236	−14.1644	−14.2419	−14.1924	−14.1981	−14.1821	**−14.2612**
2	CAC and DAX	RW	−13.0805	−13.0335	−13.2081	−13.2939	−12.8958	−12.9203	−13.2342	**−13.3233**
2	CAC and DAX	QAR	−13.0811	−13.0351	−13.2086	−13.2962	−13.2370	−13.2576	−13.2338	−13.3218
3	FTSE and HF	RW	−17.2641	−17.1956	−17.3564	−17.4260	−17.1603	−17.1616	−17.3925	−17.4487
3	FTSE and HF	QAR	−17.2671	−17.2022	−17.3610	−17.4334	−17.3888	−17.3987	−17.3956	**−17.4501**
4	EMER and US	RW	−13.2410	−13.2374	−13.2515	−13.2712	−13.2367	−13.2354	−13.2558	**−13.2748**
4	EMER and US	QAR	−13.2425	−13.2372	−13.2517	−13.2724	−13.2612	−13.2560	−13.2550	−13.2740
5	CAPIT and CONS	RW	−13.8078	−13.7430	−13.8746	−13.9494	−13.7014	−13.7280	−13.9030	−13.9745
5	CAPIT and CONS	QAR	−13.8115	−13.7461	−13.8779	−13.9522	−13.9022	−13.9313	−13.9033	**−13.9745**
6	AAPL and GOOG	RW	−10.4489	−10.3625	−10.4460	−10.4443	−10.3749	−10.4308	−10.4729	**−10.4866**
6	AAPL and GOOG	QAR	−10.4554	−10.3653	−10.4484	−10.4463	−10.4320	−10.4818	−10.4713	−10.4815
7	KO and PEP	RW	−11.8188	−11.7888	−11.8559	−11.8732	−11.7863	−11.8007	−11.8666	**−11.8890**
7	KO and PEP	QAR	−11.8197	−11.7912	−11.8574	−11.8749	−11.8470	−11.8685	−11.8673	−11.8882
8	CHEV and EXX	RW	−12.0128	−12.0034	−12.0807	−12.1253	−11.9594	−11.9612	−12.0957	**−12.1439**
8	CHEV and EXX	QAR	−12.0130	−12.0037	−12.0810	−12.1258	−12.0947	−12.1019	−12.0958	−12.1425
9	GOLD and SILVER	RW	−12.4503	−12.4248	−12.5665	−12.5935	−12.3798	−12.3977	−12.5757	**−12.6069**
9	GOLD and SILVER	QAR	−12.4521	−12.4264	−12.5677	−12.5953	−12.5434	−12.5637	−12.5760	−12.6064
10	NATGAS and OIL	RW	−9.1322	−9.1337	−9.1497	−9.1526	−9.1309	−9.1340	−9.1478	**−9.1554**
10	NATGAS and OIL	QAR	−9.1329	−9.1343	−9.1506	−9.1530	−9.1399	−9.1457	−9.1491	−9.1529

In-sample statistical performance

The in-sample estimation shows that the Student's *t*-copula is the most parsimonious copula model.

Out-of-sample density forecast performance

We also undertake an out-of-sample density forecast performance analysis by using the **Amisano-Giacomini (2007) test**.

We show the results for

DOW-NASDAQ

CAC-DAX:

DOW and NASDAQ	mean ln f	AG statistic	CAC and DAX	mean ln f	AG statistic
Student's t	7.5280	NA	Student's t	7.0546	NA
Gaussian	7.5260	0.2329	Plackett	7.0472	0.4804
Gumbel	7.4947	2.5680**	Gaussian	7.0334	3.7754***
R Gumbel	7.4777	3.9945***	Frank	7.0315	1.2097
Plackett	7.4724	4.0608***	Clayton	6.9415	4.9306***
Frank	7.4571	4.1862***	R Clayton	6.9225	5.4495***
R Clayton	7.3784	6.0719***	R Gumbel	6.8419	8.5207***
Clayton	7.3767	7.1028***	Gumbel	6.8339	8.4559***

Out-of-sample density forecast performance

For 8 out of 10 pairs, the bivariate model with the Student's *t*-copula has the highest mean log density estimate.

With respect to the Amisano-Giacomini test statistic, the group of the best predicting models includes other parametric copulas besides the Student's *t*-copula (for example, the Plackett and Gaussian copulas).

For most of the cases, the asymmetric Clayton and Gumbel copulas (and their rotated versions) are less precise predictors.

The robustness of the Student's *t*-copula is supported since this copula is within the group of the best predicting models for 9 out of 10 cases.

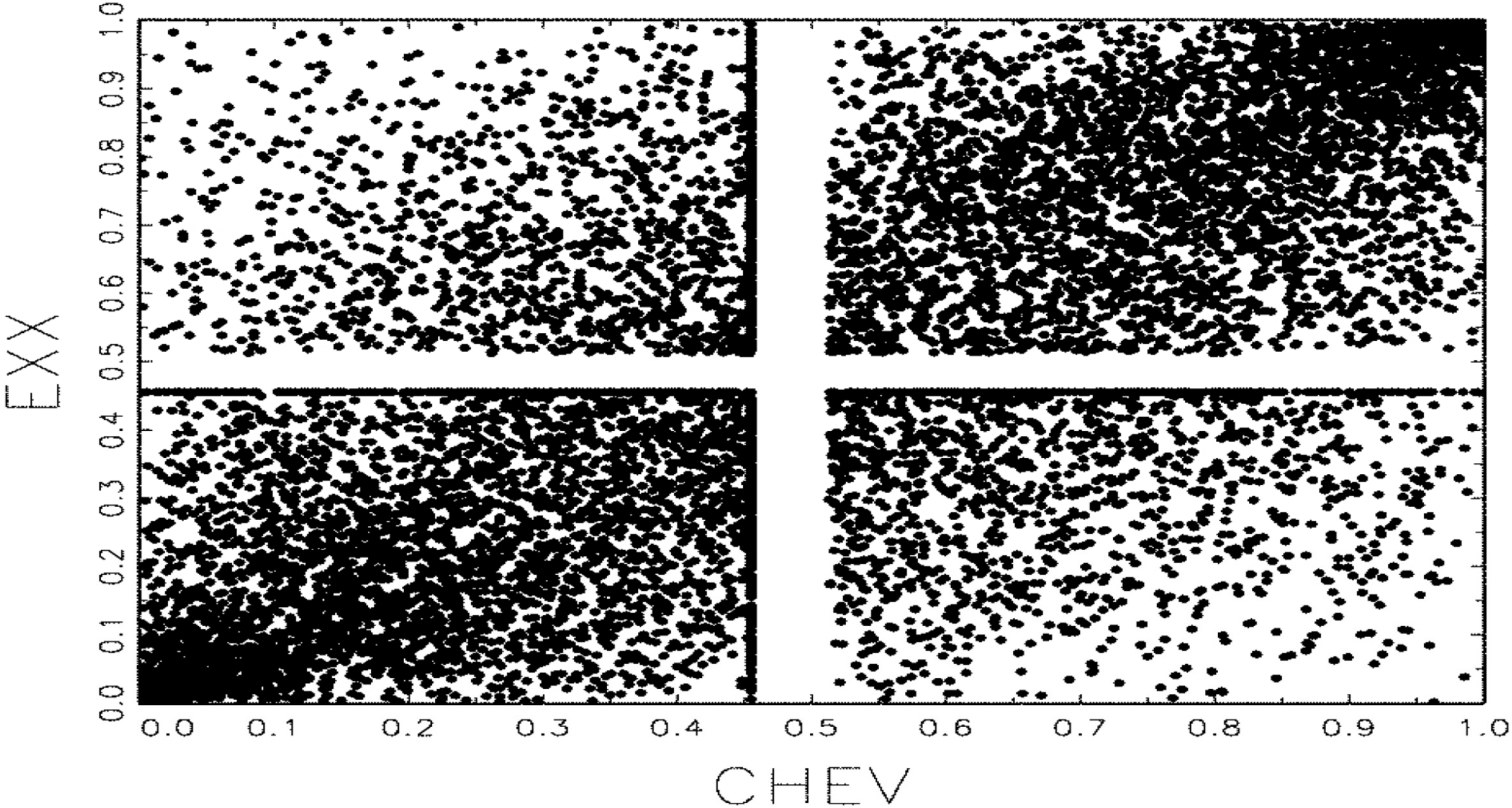
Monte Carlo VaR application

Copulas can be used to undertake Monte Carlo simulation experiments for future returns.

We apply the most parsimonious Student's *t*-copula model in order to estimate the Monte Carlo Value-at-Risk (VaR) for different portfolios of FTSE and HF.

Robustness analysis

Recall the **percentile rank** scatterplot for CHEV-EXX:



EXX
CHEV
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Robustness analysis

The empty region in the center of this figure is due to the significant proportion of zero return observations over the sample period. (This is 5.8% for CHEV and 5.7% for EXX.)

The copula models presented until now assume that the probability of zero return observations is zero. This is contradictory to the CHEV-EXX percentile rank scatter plot.

Robustness analysis

We consider extended copula specifications for which the probability of a zero return observation is not necessarily zero.

We use the model of Sucarrat and Grønneberg (2016).

We find that the models for which the probability of the zero return observation is specified are always less parsimonious than those models for which the probability of the zero return observation is zero (i.e. the estimation results presented previously).

Thank you for your attention!

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