UFM, School of Business, Master of Finance program, Guatemala, 21-April-2017

The predictive performance of AR plus GARCH versus QAR plus Beta-t-EGARCH for extreme observations

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Structure

- Objective of the thesis
- Summary of Chapters 1 and 2
- Econometric models:
  - AR plus GARCH
  - QAR plus Beta-t-EGARCH
- Definition of outliers
- Estimation results
- Comparison of forecast performance

Objective of the thesis

- We compare return and volatility **predictive performances** of two dynamic econometric models of asset returns:
- AR plus GARCH and QAR plus Beta-t-EGARCH.
- ***Which model is better to be used by practitioners?***

Chapter 1 (Applied Economics)

- The manner in which investors react to incoming market news can present a bias in their choice of algorithm as a means of effectively utilizing that news.
- The Chapter 1 shows a link between the trading algorithms and predictive performance for each model.
- We study whether to be **concerned** or **calm** after news on market value, and use either the **AR plus GARCH model** or the **QAR plus Beta-t-EGARCH model**, respectively.

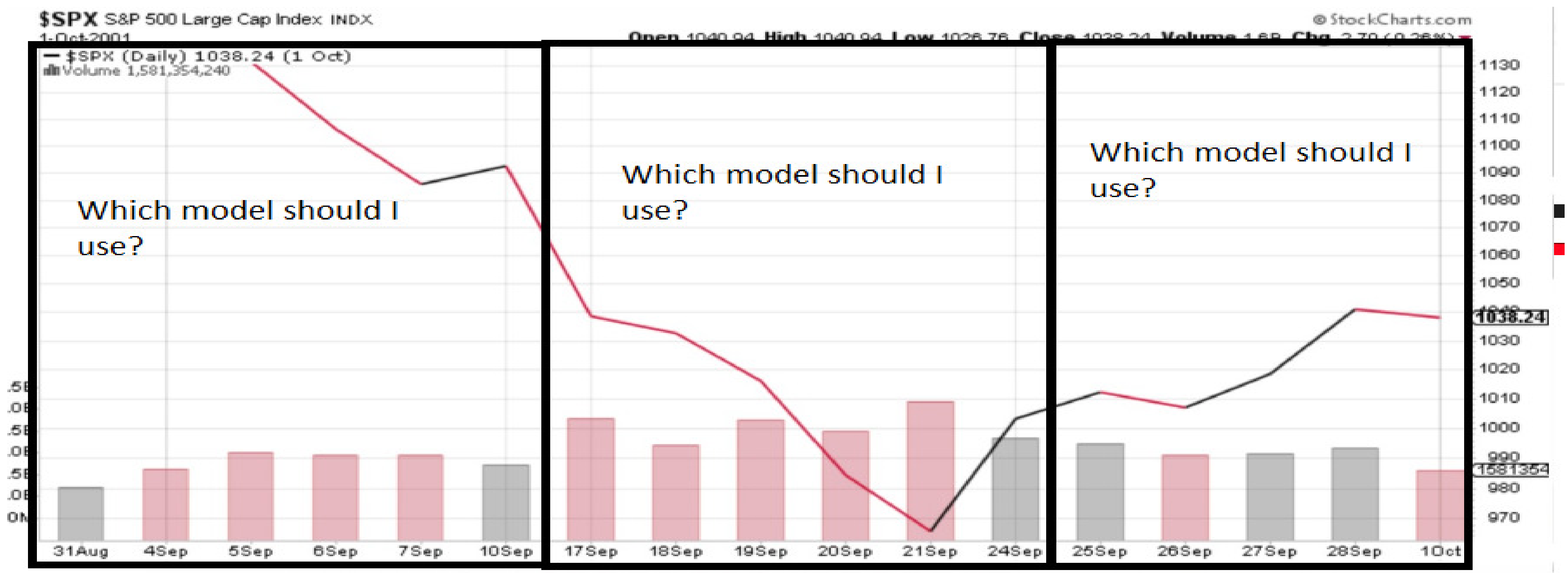
Chapter 1 (Applied Economics)

- The full data window is from the S&P 500 for period 2nd February 1950 to 17th October 2016.
- From the full data window, we use three datasets:
  - All days of the full data window (general conclusion)
  - Each day when outlier is observed
  - The trading day after each day when outlier is observed
- We determine outliers by using Chebyshev’s inequality.

Chapter 1 (Applied Economics)

- We observe the patterns for the trading days surrounding an outlier, in order to determine if an investor should be calm and use one trading model or be concerned and use another trading model:

Example: S&P 500, September 11, 2001



Chapter 2 (Applied Economics Letters)

- The full data window is from the Dow Jones Industrial Average (DJIA) index for period 26th May 1896 to 10th March 2017.
- We use an extended volatility model that also includes **leverage effects** (asymmetric effects of positive and negative returns on volatility).
- We study the following datasets:
  - Each day when outlier is observed
  - The trading day after each day when outlier is observed

Econometric models

Econometric models

- AR plus GARCH and QAR plus Beta-t-EGARCH are both recursive models of the mean and volatility of asset returns that are updated by new information **in a different way**.
- Each model includes two dynamic equations:
  - (i) Expected return (i.e. μ_t)
  - (ii) Volatility (driven by λ_t)

Econometric models

- μ_t is the time-varying conditional expected value of daily returns (i.e. a measure of **investment income**).
- λ_t is related to the time-varying conditional standard deviation of daily returns (i.e. volatility, a **measure of risk**).
- For every day, both equations are updated by using the new information that arrives to the market.
- That new information is represented by the ε_t error term.

AR plus GARCH model

$$y_t = \mu_t + v_t = \mu_t + \lambda_t^{1/2}\epsilon_t \text{ with } \epsilon_t \sim N(0,1) \text{ i.i.d.}$$

$$\mu_t = c + \sum_{j=1}^{p} \phi_j y_{t-j} = c + \sum_{j=1}^{p} \phi_j \left(\mu_{t-j} + \lambda_{t-j}^{1/2} \epsilon_{t-j} \right)$$

$$\lambda_t = \omega + [\alpha + \alpha^* \mathbb{1}(v_{t-1} < 0)] v_{t-1}^2 + \beta \lambda_{t-1}$$

$$= \omega + [\alpha + \alpha^* \mathbb{1}(\epsilon_{t-1} < 0)] \lambda_t \epsilon_t^2 + \beta \lambda_{t-1}$$

Updating terms

- **AR plus GARCH:**
- The updating term for expected return is proportional to the **linear function**.
- The updating term for volatility is proportional to the **quadratic function**.

QAR plus Beta-t-EGARCH model

$$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\epsilon_t \text{ with } \epsilon_t \sim t(\nu) \text{ i.i.d.}$$

$$\mu_t = c + \sum_{j=1}^{p} \phi_j \mu_{t-j} + \theta e_{t-1}$$

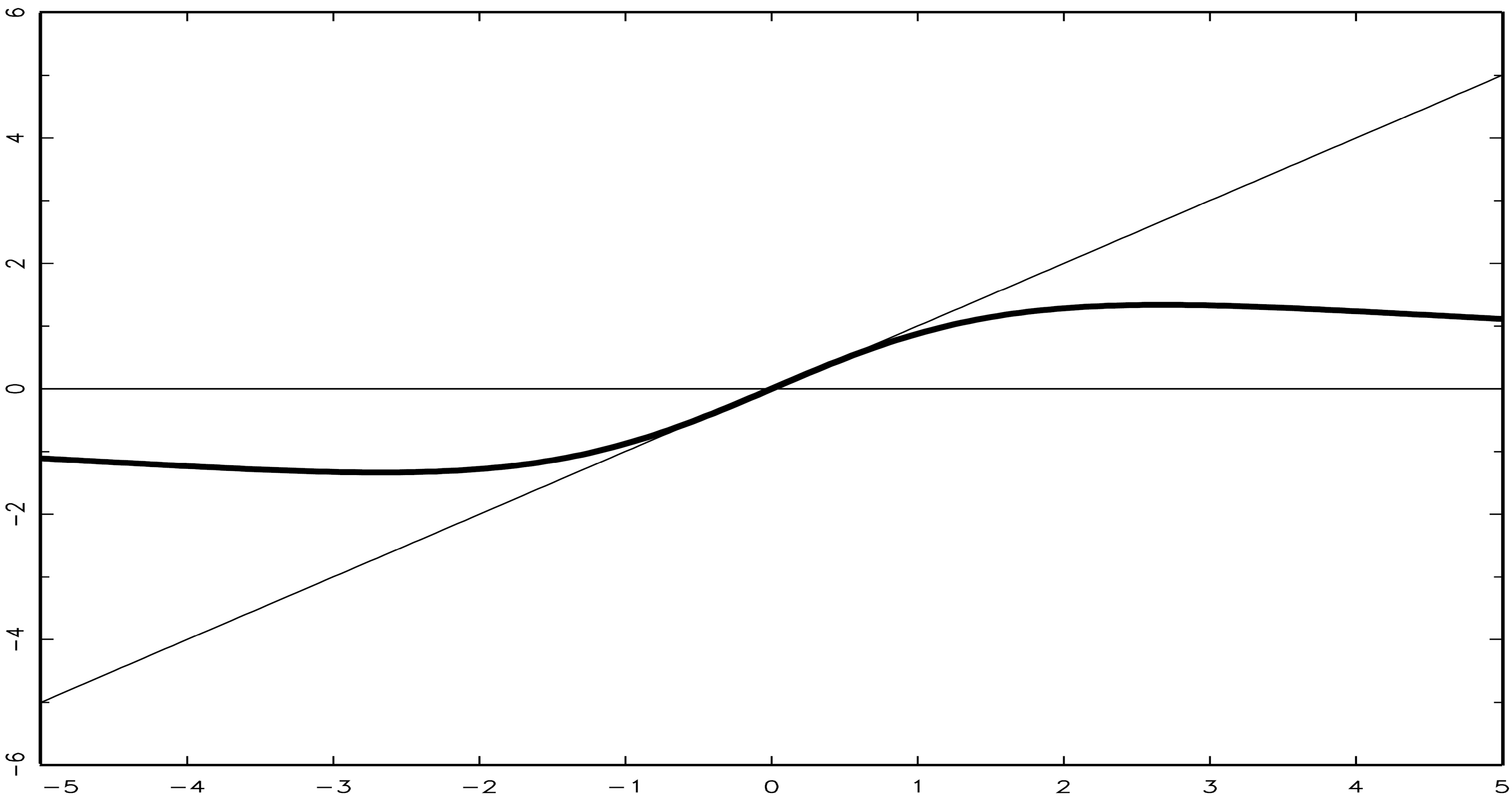
$$\lambda_t = \omega + \alpha u_{t-1} + \alpha^* \mathrm{sgn}(-v_{t-1})(u_{t-1} + 1) + \beta \lambda_{t-1}$$

where sgn() denotes the signum function.

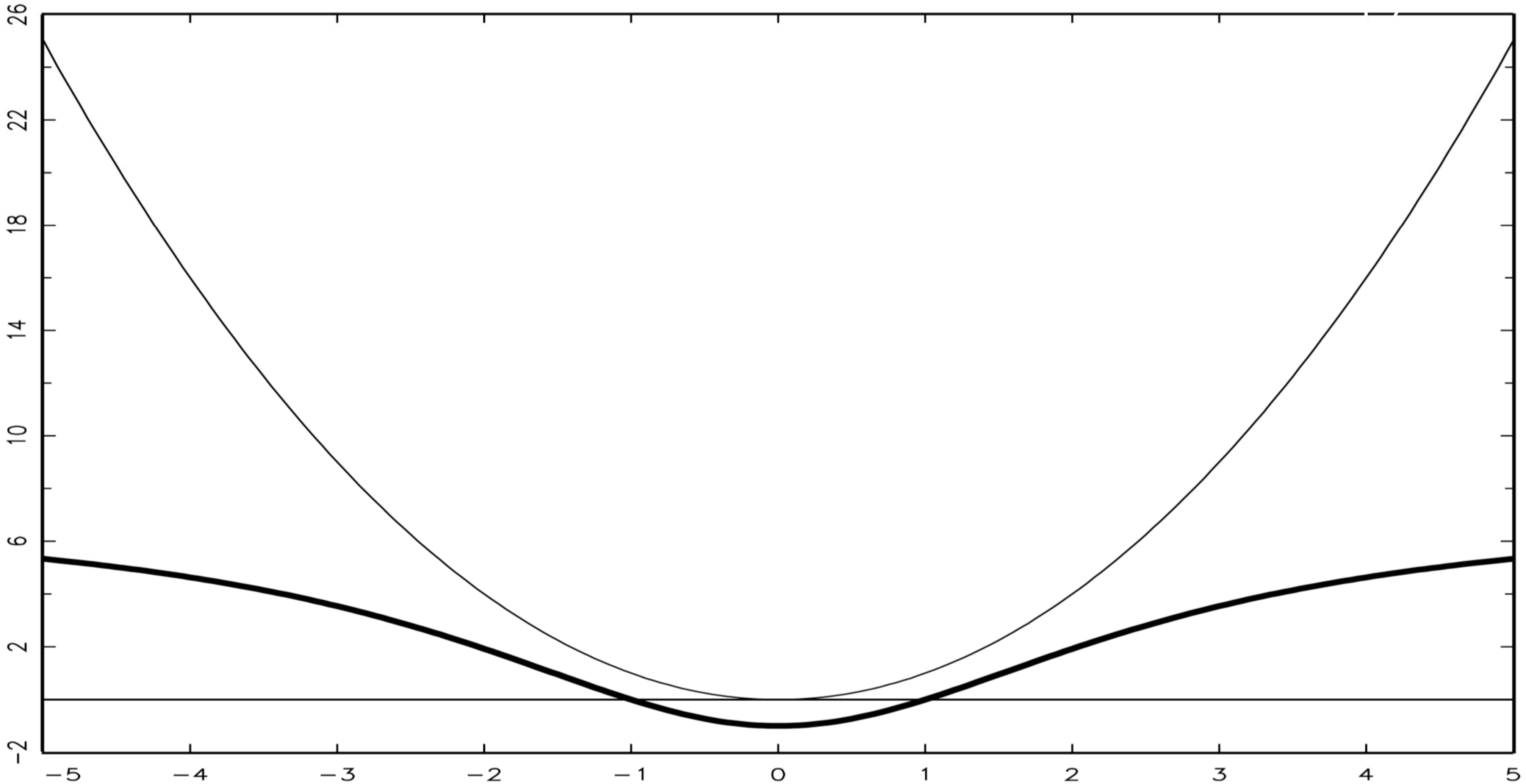
Updating terms

- **QAR plus Beta-t-EGARCH:**
- The updating term for expected return is proportional to
- $e_t = \frac{\nu \exp(\lambda_t)\varepsilon_t}{\nu+\varepsilon_t^2} \to \exp(\lambda_t)\varepsilon_t$ as $\nu \to \infty$ (i.e. linear in the limit)
- The updating term for volatility is proportional to
- $u_t = \frac{(\nu+1)\varepsilon_t^2}{\nu+\varepsilon_t^2} - 1 \to \varepsilon_t^2$ as $\nu \to \infty$ (i.e. quadratic in the limit)

Updating of μ_t: ϵ_t for AR (thin line), e_t for QAR (thick line, $\hat{\nu} = 7.1428$)



Updating of λ_t: ϵ_t^2 for GARCH (thin line), u_t for Beta-t-EGARCH (thick line, $\hat{\nu} = 7.1428$)



Difference between the two models

- For AR plus GARCH, the new information arriving to the market is not discounted. In fact, for GARCH, that information is accentuated due to the quadratic transformation. Hence, a **concerned investor** may prefer this model.
- For QAR plus Beta-t-EGARCH, the new information arriving to the market is discounted. Hence, a **calm investor** may prefer this model.

Outliers

Datasets, outliers

- We undertake forecast performance analysis for two datasets selected from the full data window:
- **(D1) each day for which an outlier is observed.**
- **(D2) the trading day after each day for which an outlier is observed.**

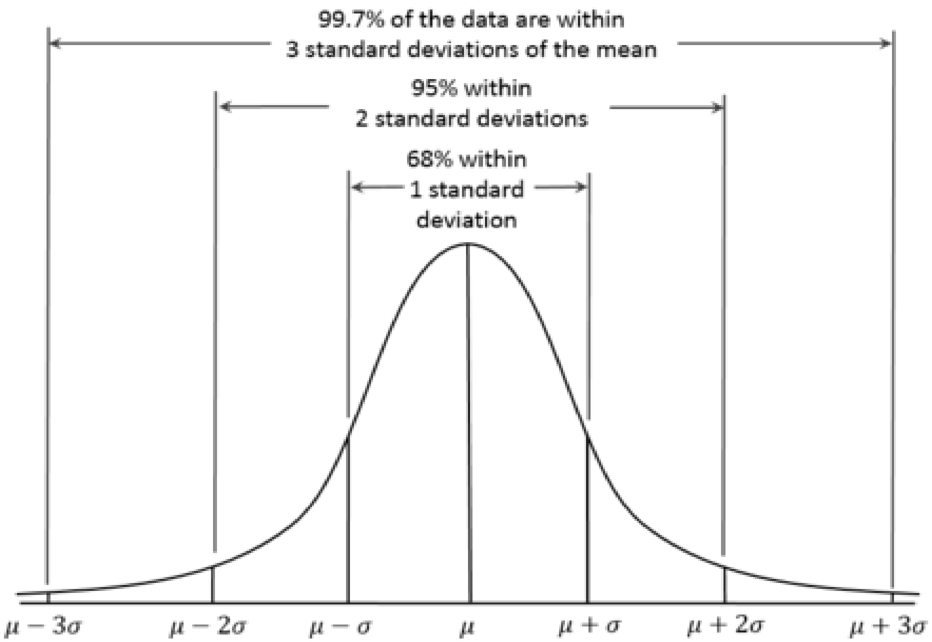
Definition of outliers

- To identify outliers, we use Chebyshev's inequality:

$$\Pr(|y_t - \mu| \geq k\sigma) \leq \frac{1}{k^2} \quad \text{for} \quad k > 1$$

- μ is estimated by the sample mean; σ is estimated by the sample standard deviation.
- We consider four alternative definitions of outliers:
- $k = 3,4,5,6$ which correspond to 11.11%, 6.25%, 4.00% and 2.78% upper bounds of probability, respectively.

Definition of outliers



Standard deviation	Inside Range	Outlier Probability
1	68.27%	31.73%
2	95.45%	4.55%
3	99.73%	0.27%
4	99.99%	0.01%
5	99.99%	0.00%

Chebyshev's Inequality (k)	Inside Range	Outlier Probability	Sample Size	% of total
1	0.00%	100.00%	32,865	
2	75.00%	25.00%		
3	88.89%	11.11%	516	1.57%
4	93.75%	6.25%	224	0.68%
5	96.00%	4.00%	100	0.30%
6	97.22%	2.78%	52	0.16%

Definition of outliers

- Outliers are extreme observations that are typically unpredictable. For the days when an outlier is observed, the S&P 500 level changes very significantly.
- **Some examples**:
- 26th June 1950: North Korean troops attack South Korea
- 26th September 1955: President Eisenhower's hearth attack

Definition of outliers

- **Some examples**:
- 23rd October 1957: Suez Canal crisis
- 17th April 1961: Bay of Pigs invasion
- 26th November 1963: assassination of President Kennedy
- 16th August 1971: President Nixon ends of the gold standard
- 19th October 1987: Black Monday
- 17th September 2001: September 11 attacks

Estimation results (full data window)

Models selection, parameter estimates

- **We estimate both models for the full data window.**
- **First**, to identify the lag structure of AR(p) and QAR(p), we estimate the partial autocorrelation function (PACF) of y_t up to 30 lags.
- We find significant φ_j parameters for several lags; we find that θ is significant for QAR; we find that α, α^* and β are all significantly different from zero for both GARCH and Beta-t-EGARCH.

Comparison of statistical performance

- **Second**, we use the following statistical performance metrics:

(i) mean LL = LL/T

(ii) mean Akaike information criterion (AIC) = 2K/T − 2LL/T

(iii) mean Bayesian information criterion (BIC) = ln(T)K/T − 2LL/T

(iv) mean Hannan-Quinn criterion (HQC) = 2K ln[ln(T)]/T − 2LL/T

- All metrics suggest that QAR-Beta-t-EGARCH is superior to AR-GARCH.

Comparison of statistical performance, Ljung-Box test

- **Third,** we use the likelihood-ratio (LR) test to study whether the mean LL values of AR-GARCH and QAR-Beta-t-EGARCH are significantly different. According to the LR test, we find that LL of QAR-Beta-t-EGARCH is superior to that of AR-GARCH.
- **Fourth,** we use the Ljung–Box test with the lag order 30 for the residual time-series. We find that independence of ε_t is supported for both AR-GARCH and QAR-Beta-t-EGARCH.

Comparison of forecast performance

Comparison of forecast performance

- Return forecast for both models: μ_t
- We compare μ_t with the realized return: y_t
- Volatility forecast for GARCH: $\sqrt{\lambda_t}$
- Volatility forecast for Beta-t-EGARCH: $\exp(\lambda_t)\sqrt{\frac{\nu}{\nu-2}}$
- We compare volatility forecasts with a proxy of true volatility $|y_t|$ (Day and Lewis 1992).

Comparison of forecast performance

- For each day, we define forecast precision by using the absolute value of the difference between the benchmark value and the forecast (absolute error, AE):
- $AE_t(\text{return}) = |y_t - \mu_t|$ (for both models)
- $AE_t(\text{volatility}) = ||y_t| - \sqrt{\lambda_t}|$ (for GARCH)
- $AE_t(\text{volatility}) = ||y_t| - \exp(\lambda_t)\sqrt{\frac{\nu}{\nu-2}}|$ (for Beta-t-EGARCH)

Comparison of forecast performance

- We compare the absolute forecast error for both return and volatility forecasts, and study whether the difference of the absolute forecast errors is significant:
- $d_t = AE_{t,AR-GARCH} - AE_{t,QAR-Beta-t-EGARCH}$
- $d_t = c + \varepsilon_t$ (OLS-HAC estimation of c)
- Positive c: QAR plus Beta-t-EGARCH is better predictor
- Negative c: AR plus GARCH is better predictor

(D1) $\lvert y_t - \mu \rvert \geq 3\sigma$:		
Expected return	4.5137E-06(2.2121E-04)	AR-GARCH and QAR-Beta-t-EGARCH are identical
Volatility	−1.4557E-03***(2.2121E-04)	AR-GARCH is superior to QAR-Beta-t-EGARCH
(D1) $\lvert y_t - \mu \rvert \geq 4\sigma$:		
Expected return	−2.2605E-04(1.5293E-04)	AR-GARCH and QAR-Beta-t-EGARCH are identical
Volatility	−2.5490E-03***(4.4906E-04)	AR-GARCH is superior to QAR-Beta-t-EGARCH
(D1) $\lvert y_t - \mu \rvert \geq 5\sigma$:		
Expected return	−4.0676E-04(2.7928E-04)	AR-GARCH and QAR-Beta-t-EGARCH are identical
Volatility	−4.0282E-03***(9.5615E-04)	AR-GARCH is superior to QAR-Beta-t-EGARCH
(D1) $\lvert y_t - \mu \rvert \geq 6\sigma$:		
Expected return	−8.4623E-04**(3.5905E-04)	AR-GARCH is superior to QAR-Beta-t-EGARCH
Volatility	−6.0126E-03***(1.9150E-03)	AR-GARCH is superior to QAR-Beta-t-EGARCH
(D2) $\lvert y_t - \mu \rvert \geq 3\sigma$:		
Expected return	5.9304E-04***(1.1380E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
Volatility	1.3087E-03***(3.5999E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
(D2) $\lvert y_t - \mu \rvert \geq 4\sigma$:		
Expected return	8.2777E-04***(2.3840E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
Volatility	1.7352E-03***(5.6340E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
(D2) $\lvert y_t - \mu \rvert \geq 5\sigma$:		
Expected return	1.4739E-03***(5.5474E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
Volatility	1.8081E-03*(1.0236E-03)	QAR-Beta-t-EGARCH is superior to AR-GARCH
(D2) $\lvert y_t - \mu \rvert \geq 6\sigma$:		
Expected return	2.3238E-03**(8.8787E-04)	QAR-Beta-t-EGARCH is superior to AR-GARCH
Volatility	3.5994E-03(2.1960E-03)	AR-GARCH and QAR-Beta-t-EGARCH are identical

Thank you!

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