Equity market neutral hedge funds and the stock market: an application of score-driven copula models

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Equity market neutral hedge funds

- Equity market neutral hedge fund strategies include **factor-based strategies**, in which investment strategies are constructed based on common relationships among financial assets.
- For the **factor-based strategies**, *portfolios are constructed to be neutral to the stock market* (i.e. with $\beta = 0$).
- Equity market neutral hedge fund strategies also include the **statistical arbitrage-trading strategy**, in which *investment strategies are constructed based on pricing anomalies*.
- For the **statistical arbitrage-trading strategy**, high frequency techniques and technical analysis are frequently used.

Research question

- A common property of equity market neutral hedge funds is that their value is not sensitive to movements in the value of the equity market.
- The value of the equity market can be represented by using a stock market index.
- Research question of the present paper:
- *Are equity market neutral hedge funds market neutral?*

Patton (2009): April 1993 to April 2003

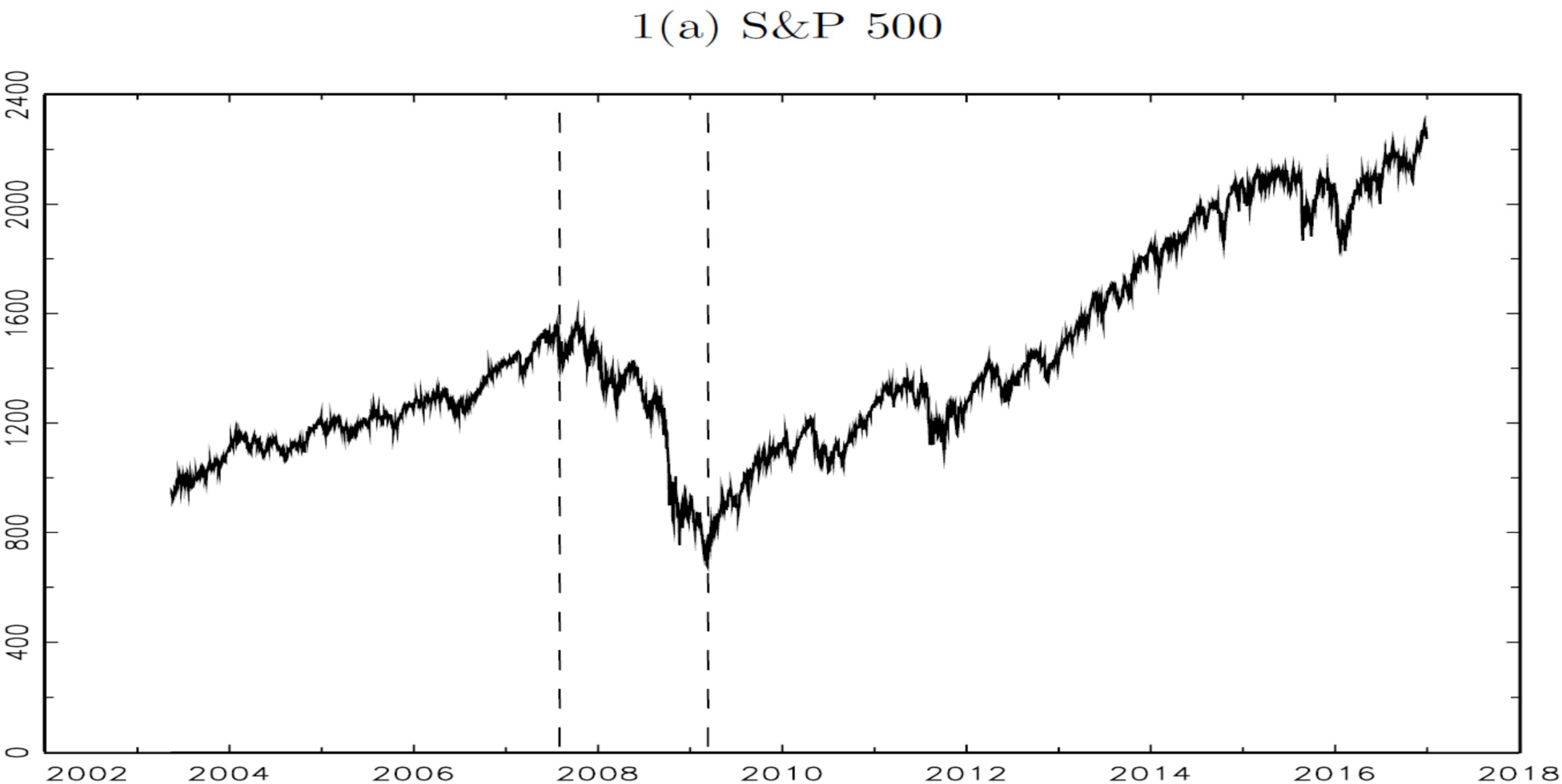
- Our paper is closely related to the work of Patton (2009), who proposes different market neutrality concepts and statistical tests to answer the question: "*Are market neutral hedge funds really market neutral?*" (Patton 2009, p. 2495).
- Patton (2009) uses two datasets:
- Firstly, a combined dataset of 1,423 individual hedge funds with monthly return data for period April 1993 to April 2003.
- For these data, approximately 25% of the hedge funds exhibit significant non-market neutrality (i.e. *75% are market neutral*).

Patton (2009): April 1993 to April 2003

- Secondly, Patton (2009) also uses monthly return data from **Hedge Fund Research Equity Market Neutral Index (HFRX EH)**.
- For these data, the *market neutrality null hypothesis of the statistical tests is not rejected*.
- According to Patton (2009, p. 2515), this is due to the fact that **the market exposures of individual hedge funds offset each other in the cross section**.

Contribution of the present paper

- In the present paper, we focus on the **correlation neutrality** concept of Patton (2009), and for each day of the data window, *May 2003 to December 2016*, we measure different levels of association of the Standard & Poor's 500 (S&P 500) index and HFRX EH.
- We use HFRX EH data, since individual hedge fund data is not available to us.
- We estimate different average levels of association for the periods before, during and after the US financial crisis of 2008.


1(a) S&P 500
2400
2000
1600
1200
800
400
0
2002
2004
2006
2008
2010
2012
2014
2016
2018

Data: time-series variables

We use daily log-return data for period *May 2003 to December 2016* from the S&P 500 index and HFRX EH index.

We denote log-returns by using y_{1t} and y_{2t}, respectively.

The HFRX EH index represents the performance of a portfolio of individual equity market neutral hedge funds, which in most cases maintain net stock market exposure that is no greater than 10% long or short.

S&P 500 represents the value of the equity market portfolio.

Data: descriptive statistics

Variable	S&P 500 log-return	HFRX EH log-return
Start date	31st March 2003	31st March 2003
End date	30th December 2016	30th December 2016
Sample size T	$3,464$	$3,464$
Minimum	-0.0903	-0.0309
Maximum	0.1158	0.0310
Mean	0.0004	0.0000
SD	0.0118	0.0025
Skewness	-0.0990	-0.1628
Excess kurtosis	11.8409	16.5357

Data: partial autocorrelation for S&P 500 → AR and QAR lag selection

PACF(1) (S&P)	−0.1017***	PACF(11) (S&P)	−0.0181	PACF(21) (S&P)	−0.0275
PACF(2) (S&P)	−0.0683***	PACF(12) (S&P)	0.0277	PACF(22) (S&P)	0.0271
PACF(3) (S&P)	0.0200	PACF(13) (S&P)	0.0121	PACF(23) (S&P)	−0.0012
PACF(4) (S&P)	−0.0234	PACF(14) (S&P)	−0.0289*	PACF(24) (S&P)	−0.0042
PACF(5) (S&P)	−0.0522***	PACF(15) (S&P)	−0.0549***	PACF(25) (S&P)	0.0099
PACF(6) (S&P)	−0.0008	PACF(16) (S&P)	0.0456***	PACF(26) (S&P)	−0.0072
PACF(7) (S&P)	−0.0318*	PACF(17) (S&P)	0.0190	PACF(27) (S&P)	0.0295*
PACF(8) (S&P)	0.0150	PACF(18) (S&P)	−0.0566***	PACF(28) (S&P)	−0.0198
PACF(9) (S&P)	−0.0181	PACF(19) (S&P)	0.0033	PACF(29) (S&P)	−0.0046
PACF(10) (S&P)	0.0314*	PACF(20) (S&P)	0.0355**	PACF(30) (S&P)	0.0132

Data: partial autocorrelation for HFRX EH → AR and QAR lag selection

PACF(1) (HFRX)	0.0798***	PACF(11) (HFRX)	−0.0155	PACF(21) (HFRX)	0.0211
PACF(2) (HFRX)	−0.0347**	PACF(12) (HFRX)	−0.0259	PACF(22) (HFRX)	0.0093
PACF(3) (HFRX)	−0.0285*	PACF(13) (HFRX)	−0.0175	PACF(23) (HFRX)	−0.0024
PACF(4) (HFRX)	−0.0104	PACF(14) (HFRX)	−0.0126	PACF(24) (HFRX)	0.0499***
PACF(5) (HFRX)	−0.0482***	PACF(15) (HFRX)	−0.0073	PACF(25) (HFRX)	0.0073
PACF(6) (HFRX)	−0.0273	PACF(16) (HFRX)	0.0153	PACF(26) (HFRX)	0.0183
PACF(7) (HFRX)	0.0035	PACF(17) (HFRX)	−0.0264	PACF(27) (HFRX)	0.0073
PACF(8) (HFRX)	−0.0065	PACF(18) (HFRX)	−0.0050	PACF(28) (HFRX)	−0.0130
PACF(9) (HFRX)	0.0353**	PACF(19) (HFRX)	−0.0006	PACF(29) (HFRX)	−0.0162
PACF(10) (HFRX)	−0.0056	PACF(20) (HFRX)	0.0247	PACF(30) (HFRX)	−0.0298*

Methodology: DCS models

- Motivated by the fact that hedge fund returns are non-linear (Fung and Hsieh 2001) and also by the recent development of the score-driven time-series models of association, we use a non-linear **dynamic conditional score (DCS) model of location, scale and copula (Harvey 2013)**.
- *To the best of our knowledge, this model has not yet been applied in the body of literature on hedge funds.*

Methodology: univariate models

- We consider three alternative models for location and scale.
- Firstly, *AR(p) plus t-GARCH(1,1) with leverage effects* (Box and Jenkins 1970; Glosten et al. 1993).
- Secondly, *t-QAR(p) plus Beta-t-EGARCH(1,1) with leverage effects* (Harvey and Chakravary 2008; Harvey 2013; Harvey and Sucarrat 2014).
- Thirdly, *Gen-t-QAR(p) plus Beta-Gen-t-EGARCH(1,1) with leverage effects* (Harvey and Lange 2016).

AR(p) plus t-GARCH(1,1)

$$y_{kt} = \mu_{kt} + v_{kt} = \mu_{kt} + \sqrt{\lambda_{kt}}\epsilon_{kt} \text{ with } \epsilon_{kt} \sim t(\nu_k) \text{ i.i.d.}$$

$$\mu_{kt} = c_k + \sum_{j=1}^{p} \phi_{k,j} y_{kt-j}$$

$$\lambda_{kt} = \omega_k + [\alpha_k + \alpha_k^* \mathbb{1}(v_{kt-1} < 0)] v_{kt-1}^2 + \beta_k \lambda_{kt-1}$$

t-QAR(p) plus Beta-t-EGARCH(1,1)

$$y_{kt} = \mu_{kt} + v_{kt} = \mu_{kt} + \exp \lambda_{kt} \epsilon_{kt} \text{ with } \epsilon_{kt} \sim t(\nu_k) \text{ i.i.d.}$$

$$\mu_{kt} = c_k + \sum_{j=1}^{p} \phi_{k,j} \mu_{kt-j} + \theta_k u_{\mu,kt-1}$$

$$\lambda_{kt} = \omega_k + \alpha_k u_{\lambda,kt-1} + \alpha_k^* \text{sgn}(-v_{kt-1})(u_{\lambda,kt-1} + 1) + \beta_k \lambda_{kt-1}$$

$$u_{\mu,kt} = [\nu_k \exp(\lambda_{kt}) \epsilon_{kt}]/[\nu_k + \epsilon_{kt}^2]$$

$$u_{\lambda,kt} = [(\nu_k + 1)\epsilon_{kt}^2]/[\nu_k + \epsilon_{kt}^2] - 1$$

Gen-t-QAR(p) plus Beta-Gen-t-EGARCH(1,1)

$$y_{kt} = \mu_{kt} + v_{kt} = \mu_{kt} + \exp \lambda_{kt} \epsilon_{kt} \text{ with } \epsilon_{kt} \sim \text{Gen-}t(\nu_k, \eta_k) \text{ i.i.d.}$$

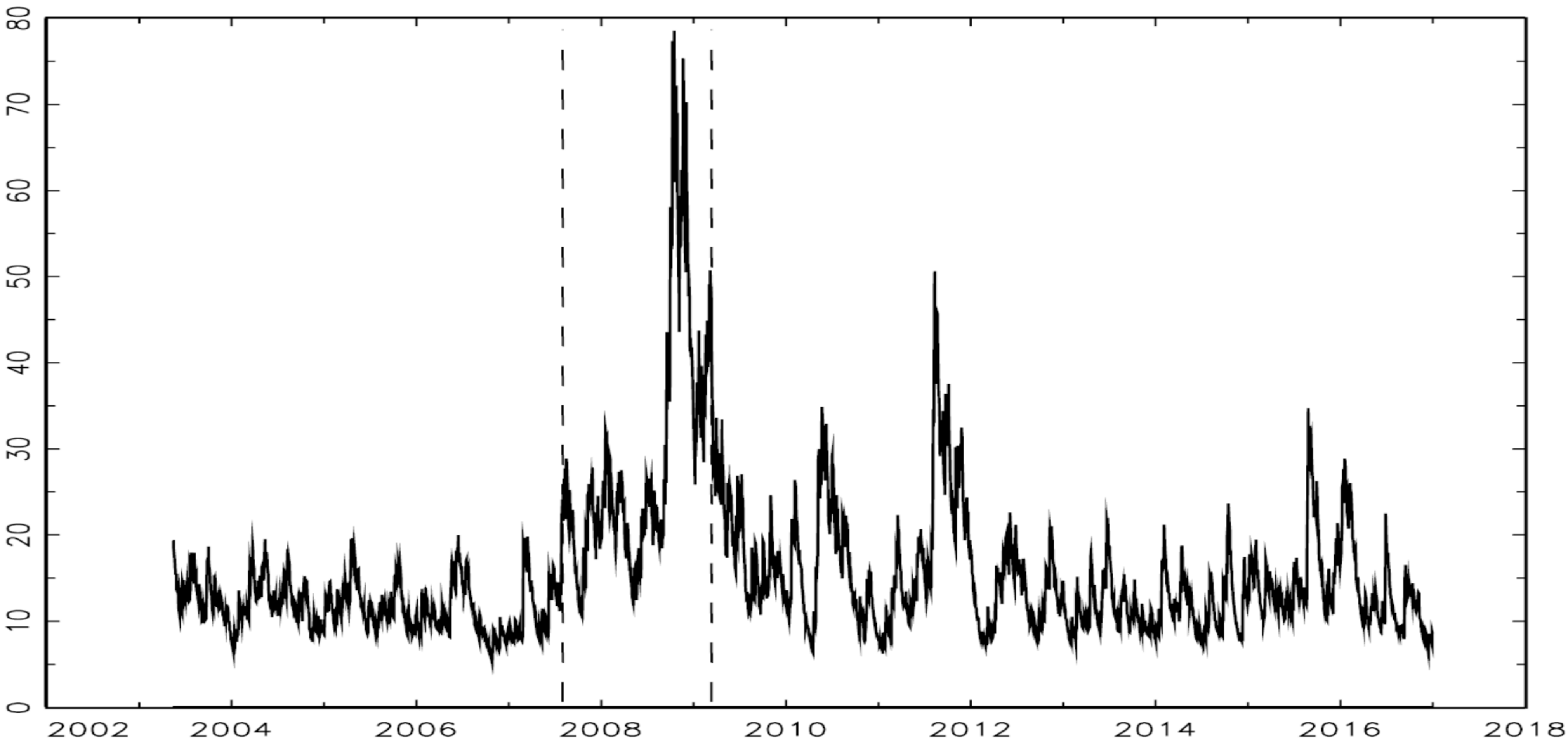
$$\mu_{kt} = c_k + \textstyle\sum_{j=1}^{p} \phi_{k,j} \mu_{kt-j} + \theta_k u_{\mu,kt-1}$$

$$\lambda_{kt} = \omega_k + \alpha_k u_{\lambda,kt-1} + \alpha_k^* \text{sgn}(-v_{kt-1})(u_{\lambda,kt-1} + 1) + \beta_k \lambda_{kt-1}$$

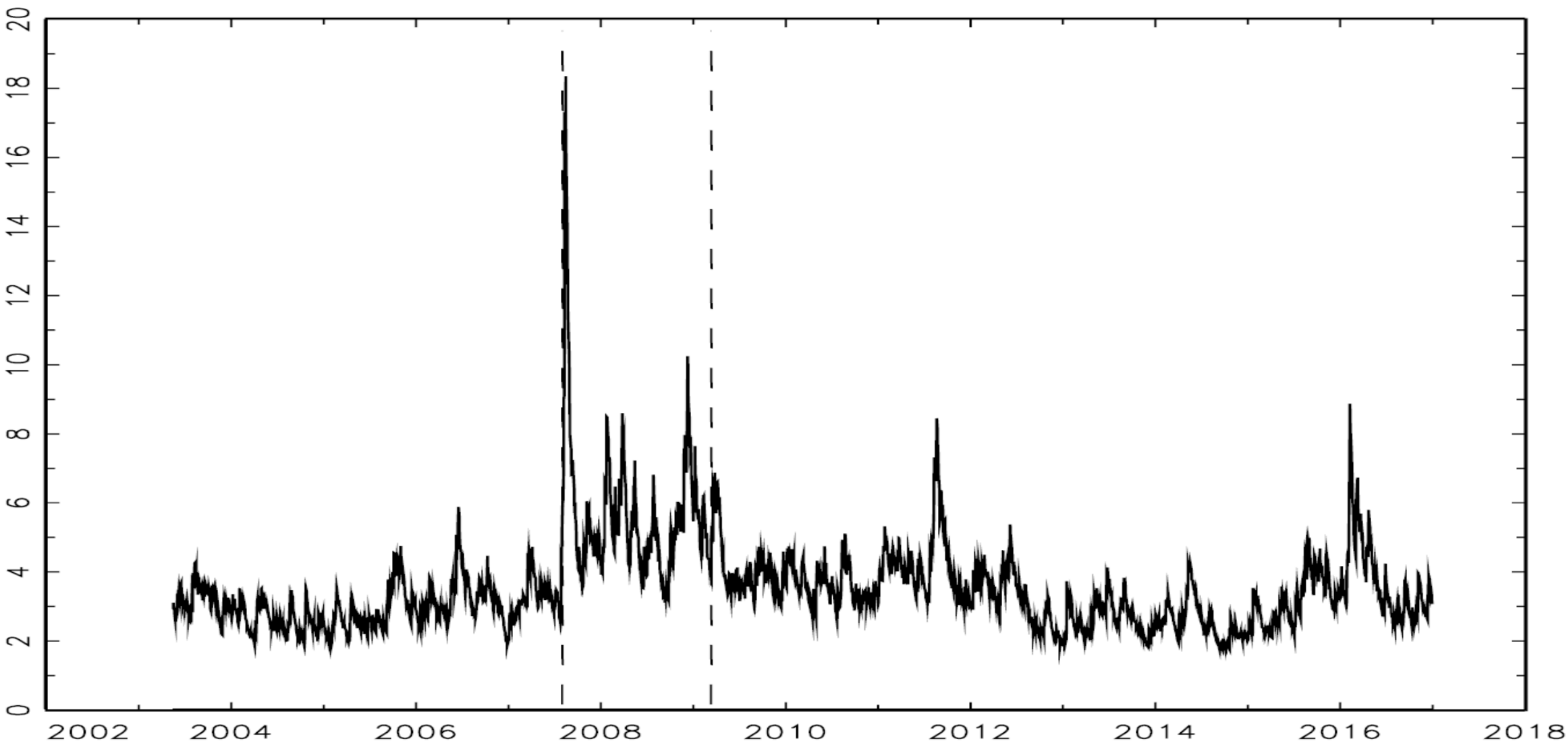
$$u_{\mu,kt} = [\nu_k \exp(\lambda_{kt}) \epsilon_{kt} |\epsilon_{kt}|^{\eta_k - 2}] / [\nu_k + |\epsilon_{kt}|^{\eta_k}]$$

$$u_{\lambda,kt} = [(\nu_k + 1)|\epsilon_{kt}|^{\eta_k}] / [\nu_k + |\epsilon_{kt}|^{\eta_k}] - 1$$

1(c) Volatility of S&P 500 (% points)



1(d) Volatility of HFRX EH (% points)



Methodology: association of y_{1t} and y_{2t}

Body of literature on dynamic association:

- BEKK model (Baba, Engle, Kraft and Kroner 1991; Engle and Kroner 1995)
- DCC (dynamic conditional correlation) model (Engle 2002)
- Dynamic copula models (Patton 2006)
- *DCS copula models (Harvey 2013)*

Methodology: association of y_{1t} and y_{2t}

- We model the dynamic association of the two assets by using a *DCS model with the Student's t-copula*.
- (Boudt et al. 2012; Avdulaj and Barunik 2013, 2015; De Lira Salvatierra and Patton 2015; Harvey and Thiele 2016; Koopman et al. 2016)

Methodology: association of y_{1t} and y_{2t}

- We model the association between y_{1t} and y_{2t}, by using the **Student's t copula function**, denoted as $C(u, v; \rho_t, \nu)$ (Joe 2015).
- Function $C(u, v; \rho_t, \nu)$ measures the association between two uniform random variables, where u and v are realizations of $U \sim U(0,1)$ and $V \sim U(0,1)$, respectively.
- In this paper, we use conditional distribution functions $F(y_{1t} | y_{11}, \ldots, y_{1t-1})$ and $F(y_{2t} | y_{21}, \ldots, y_{2t-1})$ for *u* and *v*, respectively.

Methodology: association of y_{1t} and y_{2t}

- In $C(u, v; \rho_t, \nu)$, ρ_t is a time-varying correlation coefficient of U and V that we transform into a more robust measure of association: the time-varying Blomqvist's beta $\beta_t = 2\arcsin(\rho_t)/\pi \in (-1,1)$ (Joe 2015).

$$\tilde{\rho}_t = \delta + \gamma\tilde{\rho}_{t-1} + \kappa u_{\rho,t-1}$$

$$\rho_t = \tanh(\tilde{\rho}_t) \in (-1, 1)$$

- $u_{\rho,t}$ is the conditional score of the log-likelihood with respect to ρ_t.
- Furthermore, the time-constant degrees of freedom parameter ν, measures the tail heaviness of the copula.

Results: selection of univariate model

AR-t-GARCH:		t-QAR-Beta-t-EGARCH:		Gen-t-QAR-Beta-Gen-t-EGARCH:	
LL (S&P)	3.3227	LL (S&P)	3.3240	LL (S&P)	**3.3276**
AIC (S&P)	−6.6344	AIC (S&P)	−6.6370	AIC (S&P)	**−6.6435**
BIC (S&P)	−6.6004	BIC (S&P)	−6.6030	BIC (S&P)	**−6.6077**
HQC (S&P)	−6.6222	HQC (S&P)	−6.6249	HQC (S&P)	**−6.6307**
LL (HFRX)	4.7454	LL (HFRX)	4.7484	LL (HFRX)	**4.7492**
AIC (HFRX)	−9.4821	AIC (HFRX)	−9.4880	AIC (HFRX)	**−9.4891**
BIC (HFRX)	−9.4553	BIC (HFRX)	**−9.4611**	BIC (HFRX)	−9.4604
HQC (HFRX)	−9.4725	HQC (HFRX)	−9.4784	HQC (HFRX)	**−9.4788**

c_1	$0.0011^{***}(0.0002)$	c_2	$0.0000(0.0001)$
$\phi_{1,1}$	$0.3497^{***}(0.0657)$	$\phi_{2,1}$	$-0.0259(0.0415)$
$\phi_{1,2}$	$-0.3387^{***}(0.0402)$	$\phi_{2,2}$	$-0.0674(0.0487)$
$\phi_{1,5}$	$-0.4277^{***}(0.0414)$	$\phi_{2,3}$	$0.0835(0.0516)$
$\phi_{1,7}$	$-0.0439(0.0379)$	$\phi_{2,5}$	$-0.7016^{***}(0.0463)$
$\phi_{1,10}$	$-0.4055^{***}(0.0342)$	$\phi_{2,9}$	$0.0807(0.0605)$
$\phi_{1,14}$	$-0.1374^{***}(0.0387)$	$\phi_{2,24}$	$0.2351^{***}(0.0250)$
$\phi_{1,15}$	$-0.2207^{***}(0.0521)$	$\phi_{2,30}$	$-0.2692^{***}(0.0370)$
$\phi_{1,16}$	$-0.0573(0.0561)$	θ_2	$0.0210^{*}(0.0127)$
$\phi_{1,18}$	$-0.1642^{***}(0.0338)$	ω_2	$-0.1460^{***}(0.0477)$
$\phi_{1,20}$	$-0.3052^{***}(0.0425)$	α_2	$0.0445^{***}(0.0072)$
$\phi_{1,27}$	$0.3659^{***}(0.0415)$	α_2^*	$0.0158^{***}(0.0041)$
θ_1	$-0.0531^{***}(0.0076)$	β_2	$0.9768^{***}(0.0076)$
ω_1	$-0.0869^{***}(0.0225)$	$\lambda_{0,2}$	$-6.4136^{***}(0.3127)$
α_1	$0.0303^{***}(0.0056)$	ν_2	$11.1285^{***}(2.7018)$
α_1^*	$0.0476^{***}(0.0056)$	η_2	$1.9219^{***}(0.0892)$
β_1	$0.9830^{***}(0.0045)$	δ	$0.0034^{*}(0.0018)$
$\lambda_{0,1}$	$-4.5932^{***}(0.2796)$	γ	$0.9721^{***}(0.0120)$
ν_1	$17.1628^{***}(5.2530)$	κ	$0.0376^{***}(0.0096)$
η_1	$1.5189^{***}(0.0834)$	ν	$13.8495^{***}(3.4815)$

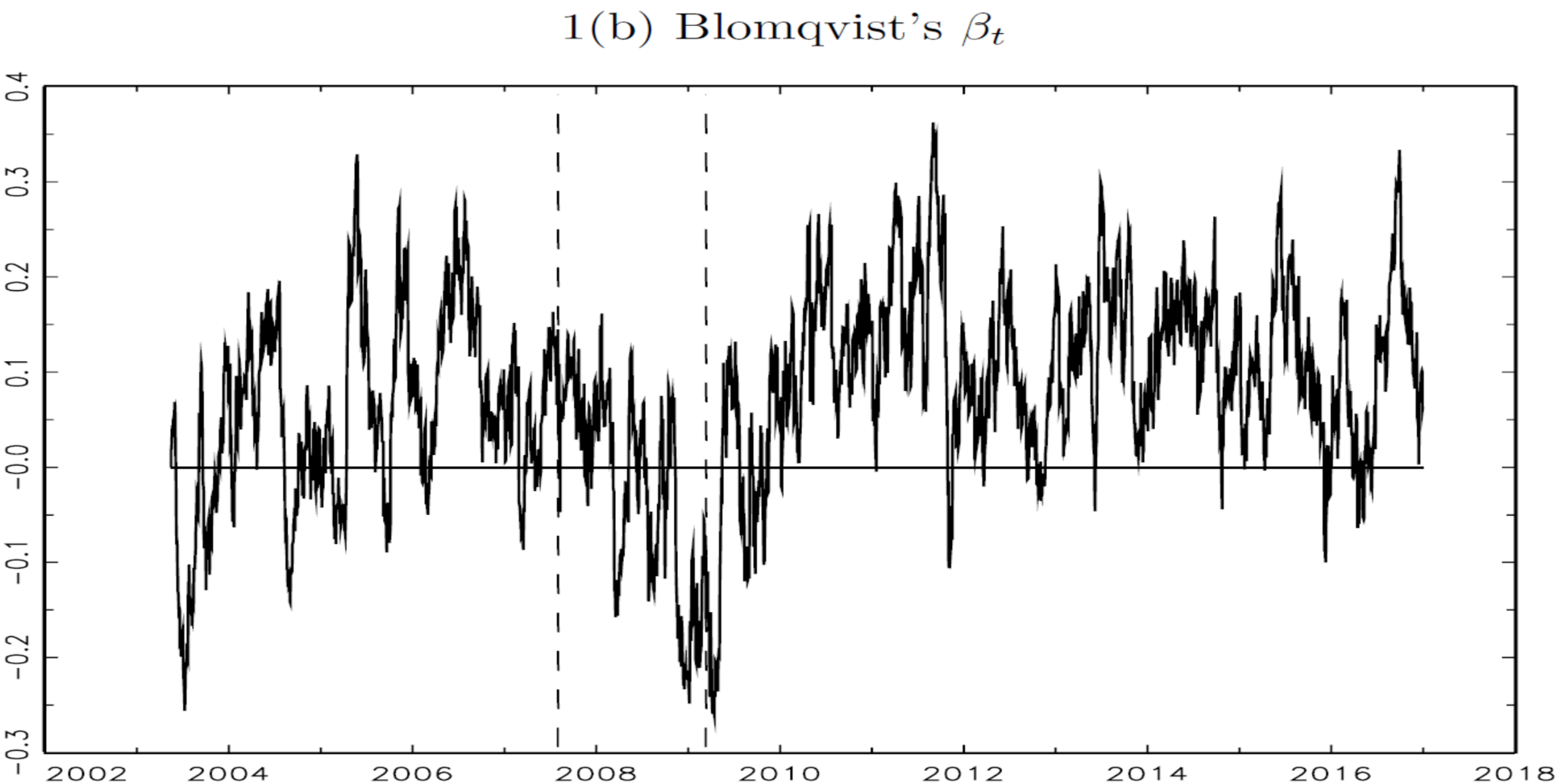
Results

- Parameter estimates by using the maximum likelihood (ML) method.
- Robust ML standard errors are reported in parentheses.
- ***, ** and * indicate significance at the 1%, 5% and 10% levels, respectively.

Residual diagnostics:	
LB p-value ϵ_{1t}	0.1473
LB p-value ϵ_{2t}	0.6663
Consistency and asymptotic normality of ML:	
GCLT λ_{1t}	0.8929
GCLT λ_{2t}	0.8269
GCLT $\tilde{\rho}_t$	0.8763
Likelihood-based metrics:	
LL	8.0980
AIC	-16.1728
BIC	-16.1013
HQC	-16.1472
OLS–HAC estimates of $\beta_t = c + \epsilon_t$:	
Pre-crisis (14th May 2003 to 19th Sep 2007)	
c	$0.0585^{***}(0.0082)$
During crisis (20th Sep 2007 to 27th Feb 2009)	
c	$-0.0305^{**}(0.0126)$
Post-crisis (2nd Mar 2009 to 30th Dec 2016)	
c	$0.1045^{***}(0.0063)$

Results

- Ljung-Box (1978) (LB) test of independence
- Gaussian central limit theory (GCLT) for ML estimates
- Log-likelihood (LL); Akaike, Bayesian, Hannan-Quinn criteria (AIC; BIC; HQC).
- Ordinary least squares (OLS); heteroscedasticity and autocorrelation consistent (HAC).
- *** indicates significance at the 1%, levels.


1(b) Blomqvist's β_t
0.4
0.3
0.2
0.1
−0.0
−0.1
−0.2
−0.3
2002
2004
2006
2008
2010
2012
2014
2016
2018

Conclusion

- As hypothesized by Patton (2009, p. 2515), these findings may be due to the fact that *the market exposures of the individual equity market neutral hedge funds are relatively constant*, while ***the non-market risk exposures of those hedge funds are offsetting***.
- The positive association for the pre- and post- periods of the financial crisis may be due to the ***relatively constant long positions*** of the individual equity market neutral hedge funds in the stock market, and the negative association for the period during the financial crisis may be due to ***relatively constant short positions*** of the individual equity market neutral hedge funds in the stock market.

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Thank you for your attention!

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