# Score-driven Markov-switching EGARCH models

*SZABOLCS BLAZSEK (FRANCISCO MARROQUIN UNIVERSITY)*

*HAN-CHIANG HO (WENZHOU-KEAN UNIVERSITY)*

*SU-PING LIU (WENZHOU-KEAN UNIVERSITY)*

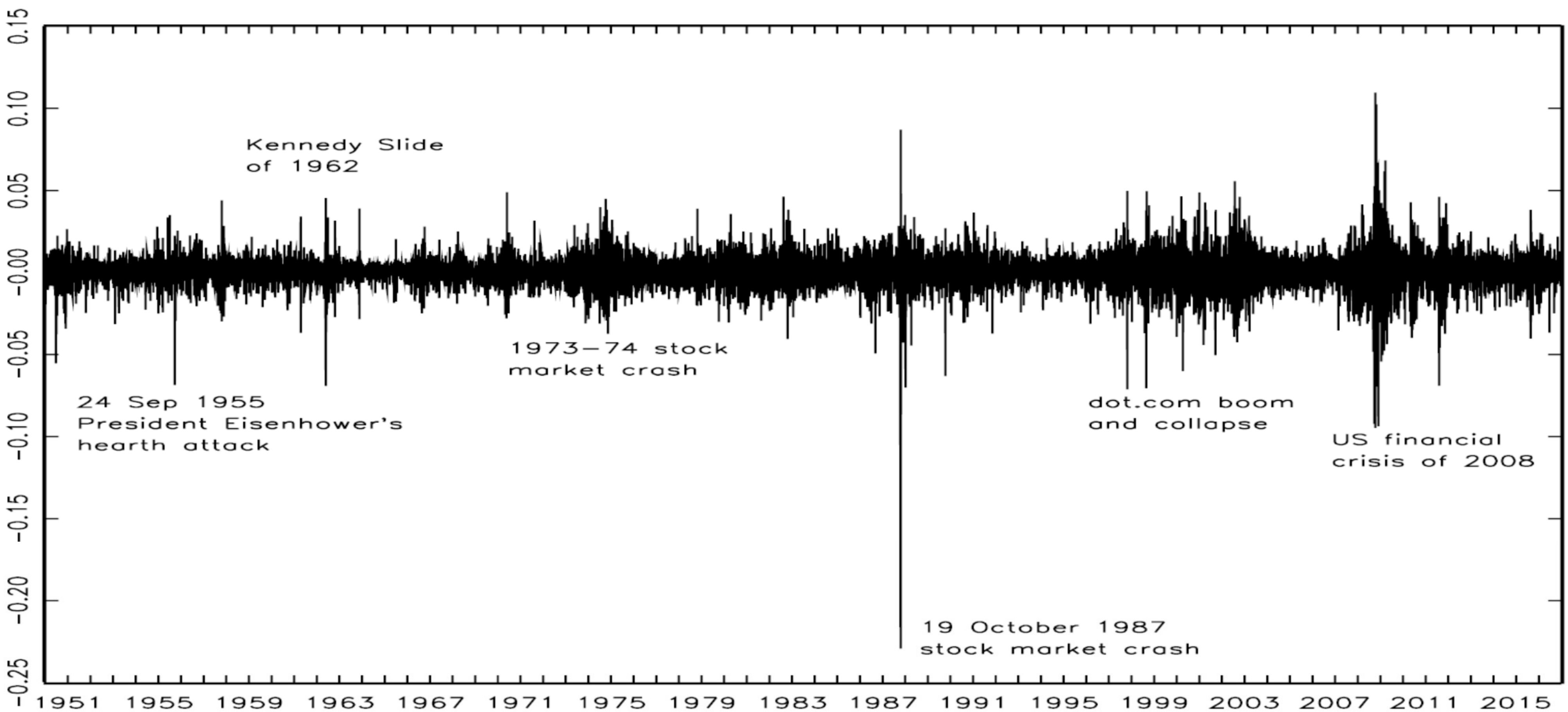
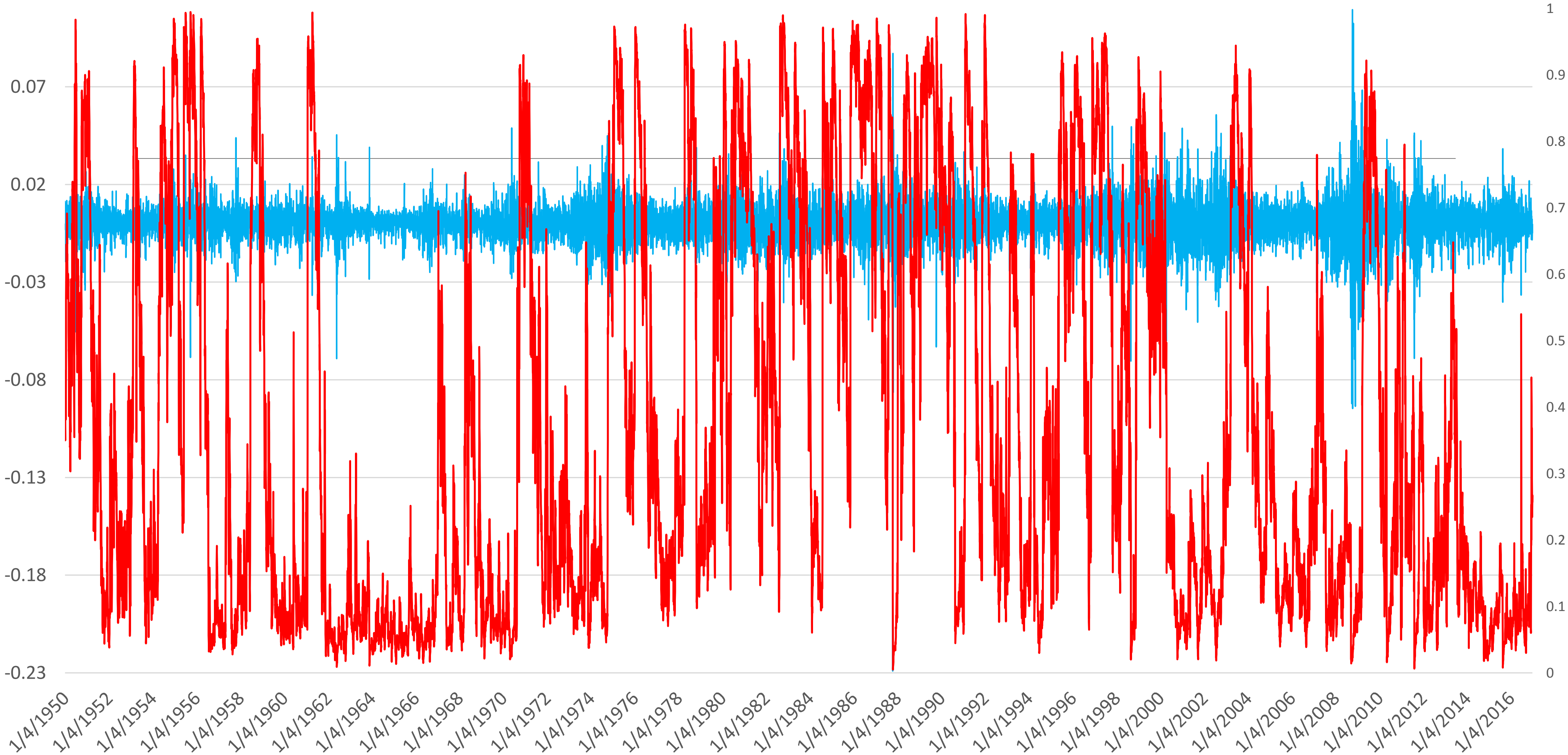



**Fig. 1. S&P 500 daily return for period 3rd January 1950 to 30th December 2016.**

(i) S&P 500 daily return; (ii) Probability of being in high volatility regime



# Motivation

The previous figure shows the daily log-returns on the S&P 500 index.

As the figure shows, *low-volatility* and *high-volatility* periods, each with random duration, follow each other.

We use a class of dynamic models that can replicate those data:

*Markov regime-switching (MS) volatility models*

Those models can be used to predict and also to simulate future log-returns on the S&P 500.

# Contribution of the present work

*We introduce new MS DCS-EGARCH models.*

MS (Markov regime-Switching) (Hamilton 1989 *Econometrica*; Kim and Nelson 1999 *The MIT Press*)

DCS (Dynamic Conditional Score) (Harvey 2013, *Cambridge University Press*)

DCS-EGARCH (Exponential Generalized Autoregressive Conditional Heteroscedasticity) (Harvey 2013, *Cambridge University Press*) *RECENT DYNAMIC VOLATILITY MODELS*

# Contribution of the present work

Blazsek and Ho (2017, *Applied Economics*) introduced the MS Beta-t-EGARCH model that is a particular MS DCS-EGARCH model.

Blazsek and Ho (2017) show that the statistical performance and the predictive performance of MS Beta-t-EGARCH are superior to those of single-regime Beta-t-EGARCH.

For MS Beta-t-EGARCH, the error term has the *Student's t distribution*:

$$\epsilon_t \sim t\{\exp[\delta_1(s_t)] + 2\}$$

# Contribution of the present work

We introduce:

(i) *MS GED-EGARCH* (*Generalized Error Distribution*)

(ii) *MS Gen-t-EGARCH* (*Generalized t distribution*)

(iii) *MS Skew-Gen-t-EGARCH* (*Skewed Generalized t distribution*)

(iv) *MS EGB2-EGARCH* (*Exponential Generalized Beta distribution of the second kind*)

(v) *MS NIG-EGARCH* (*normal-inverse Gaussian distribution*)

We include **leverage effects** for all volatility models.

# Contribution of the present work

We compare the statistical performance of those models with that of the benchmark

*MS t-GARCH with leverage effects*

(Bollerslev 1987; Glosten, Jagannathan and Runkle 1993)

and the recent

*MS Beta-t-GARCH with leverage effects*

(Harvey and Chakravarty 2008).

For each MS model, we also consider the corresponding single-regime alternative.

# Benchmark MS t-GARCH(1,1)

$$y_t = \mu_t(s_t) + v_t(s_t) = \mu_t(s_t) + \lambda_t^{1/2}(s_t)\varepsilon_t(s_t)$$

$$\mu_t(s_t) = c(s_t)$$

$$\lambda_t(s_t) = \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + \alpha(s_t)v_{t-1}^2(s_t)$$
$$+\alpha^*(s_t)1[v_{t-1}(s_t) < 0]v_{t-1}^2(s_t)$$

where $1(\cdot)$ is the indicator function.

This volatility model is updated by the square of the first lag of the regime-dependent unexpected return $v_t(s_t)$.

# DCS-EGARCH(1,1) models

$$y_t = \mu_t(s_t) + v_t(s_t) = \mu_t(s_t) + \exp[\lambda_t(s_t)]\varepsilon_t(s_t)$$

$$\mu_t(s_t) = c(s_t)$$

$$\lambda_t(s_t) = \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + \alpha(s_t)u_{\lambda,t-1}(s_t)$$
$$+\alpha^*(s_t)\mathrm{sgn}[-v_{t-1}(s_t)][u_{\lambda,t-1}(s_t) + 1]$$

where $\mathrm{sgn}(\cdot)$ is the signum function.

$u_{\lambda,t}(s_t)$ denotes the regime-dependent score function with respect to $\lambda_t(s_t)$.

# Markov regime-switching (MS) model

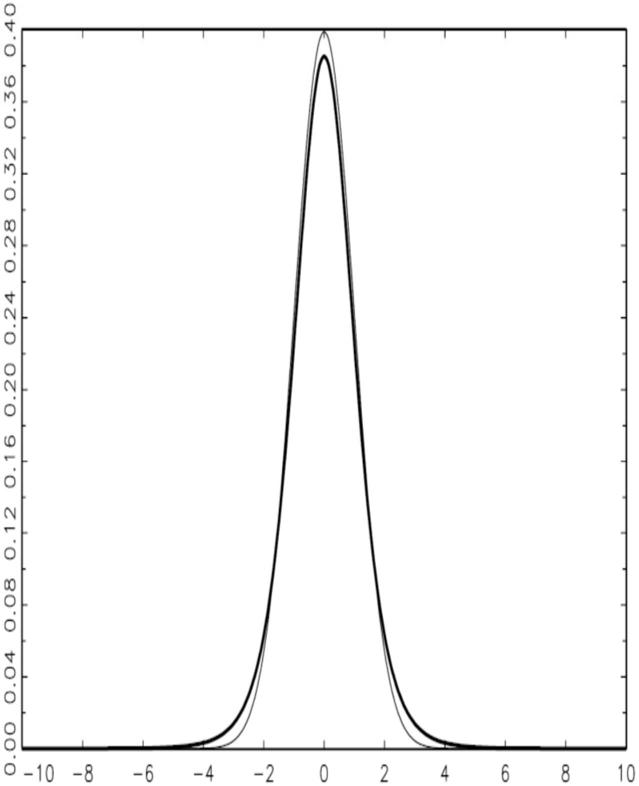
$$P = \begin{bmatrix} \Pr(s_t = 1 | s_{t-1} = 1) & \Pr(s_t = 2 | s_{t-1} = 1) \\ \Pr(s_t = 1 | s_{t-1} = 2) & \Pr(s_t = 2 | s_{t-1} = 2) \end{bmatrix} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$

This is the transition probability matrix of the regimes.

We assume that it is constant over time, and that it is determined by two parameters: $p$ and $q$.

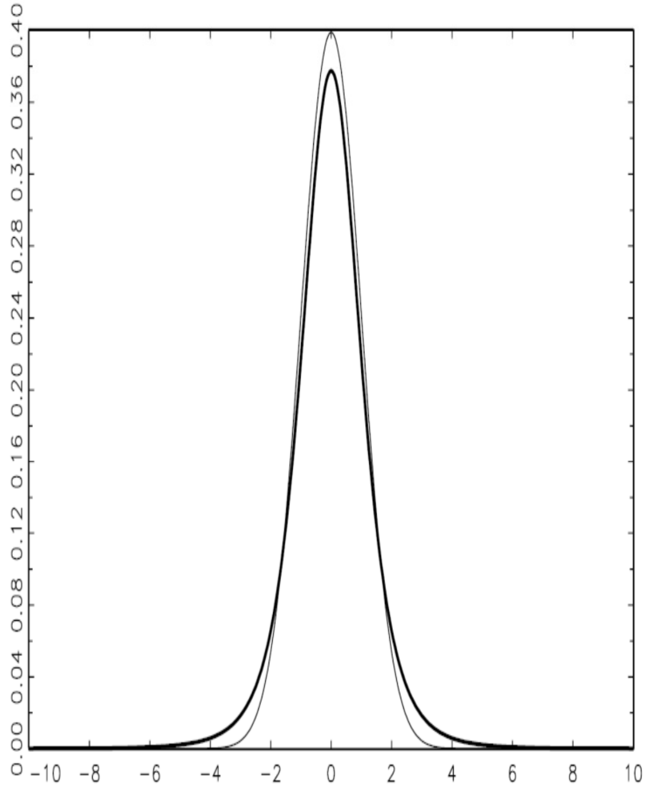
$t$-GARCH

$\delta_1 = 1.6313$



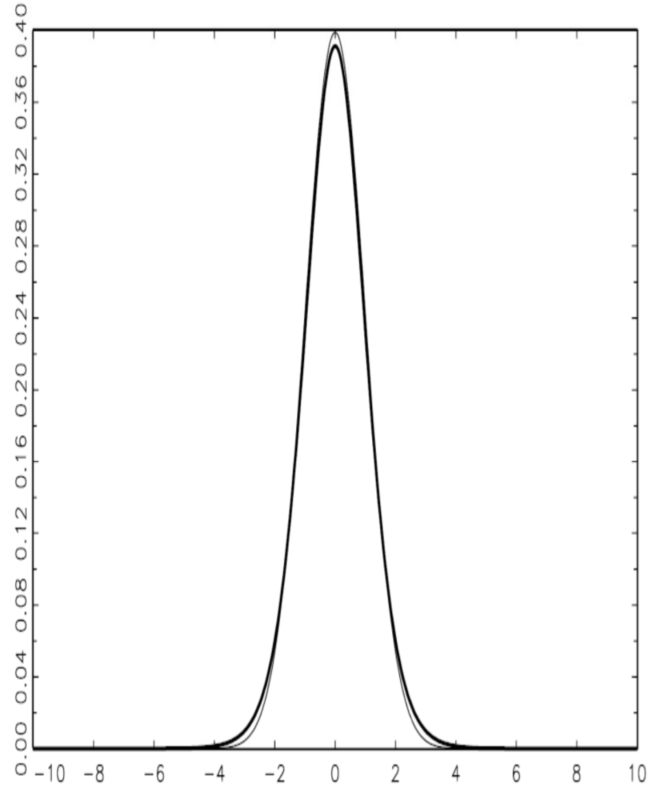

$t$-GARCH high-volatility $s_t = 1$

$\delta_{1,1} = 0.9038$



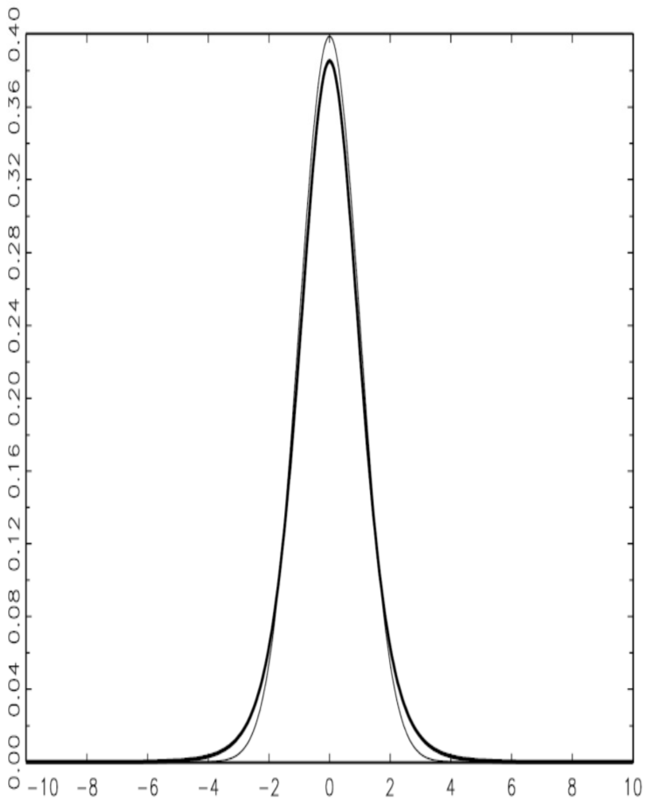

$t$-GARCH low-volatility $s_t = 2$

$\delta_{1,2} = 2.3860$



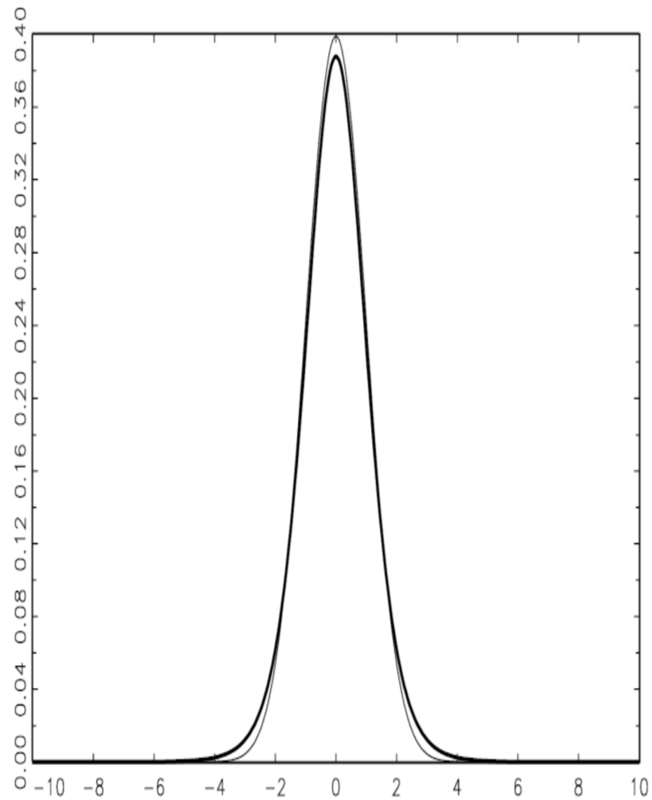
Beta-$t$-EGARCH

$\delta_1 = 1.6499$



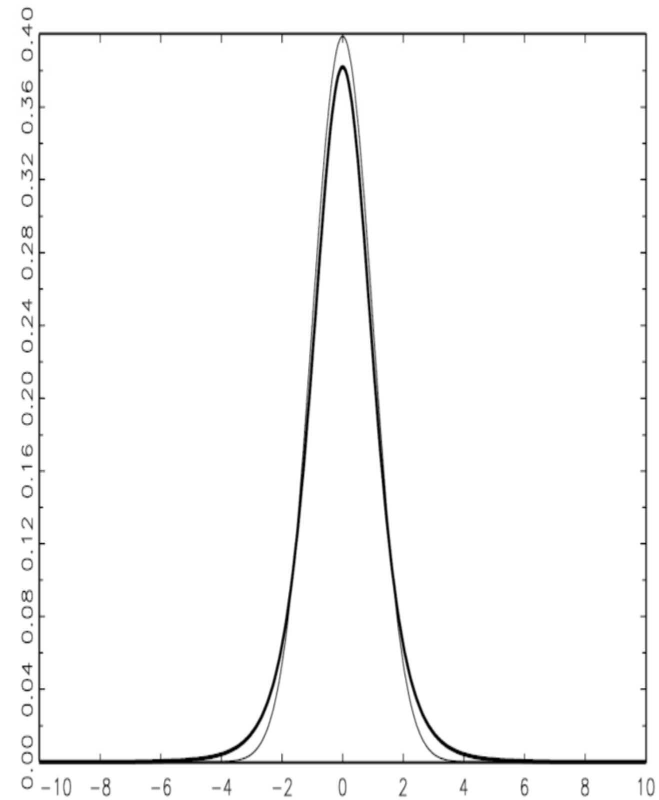

Beta-$t$-EGARCH high-volatility $s_t = 2$

$\delta_{1,2} = 1.9198$



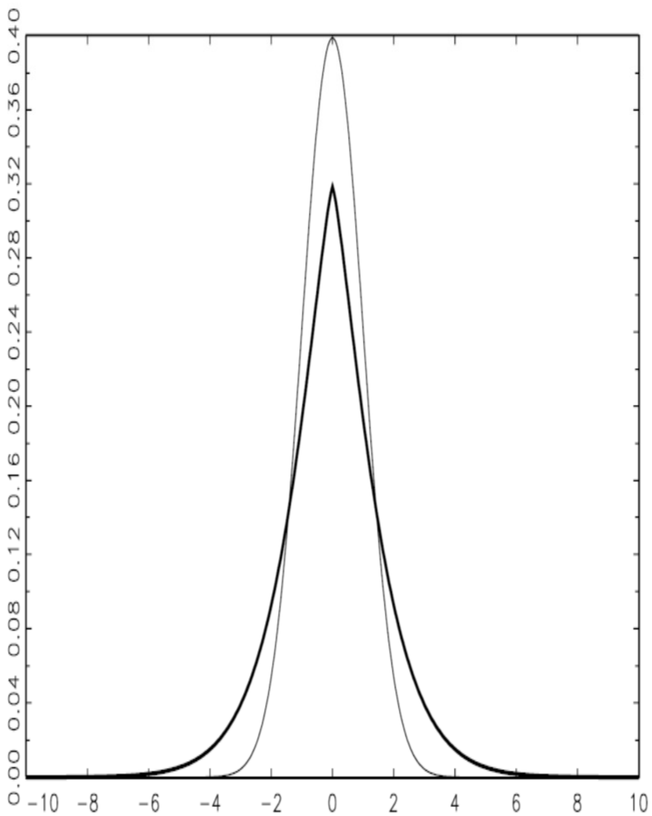

Beta-$t$-EGARCH low-volatility $s_t = 1$

$\delta_{1,1} = 1.3216$



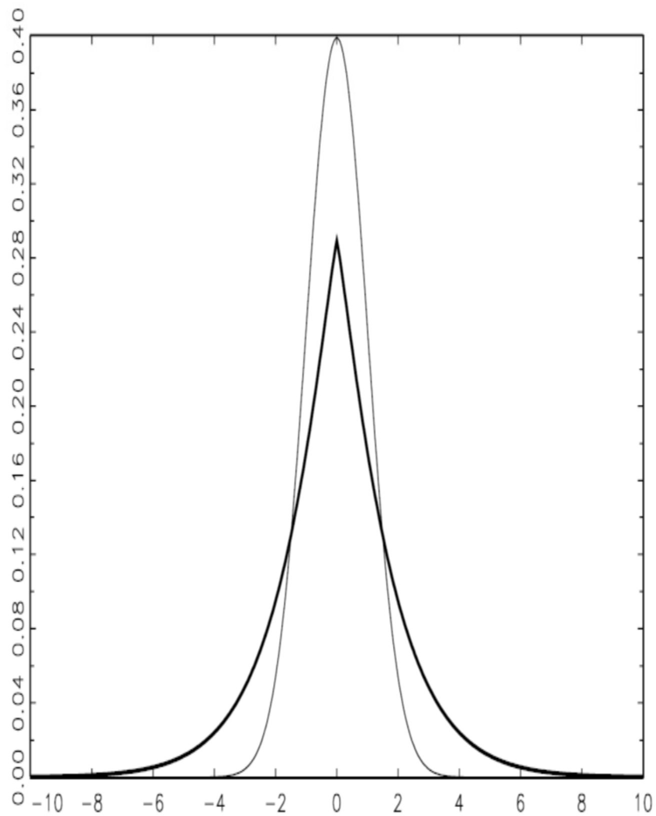
GED-EGARCH

$\delta_1 = 0.2657$



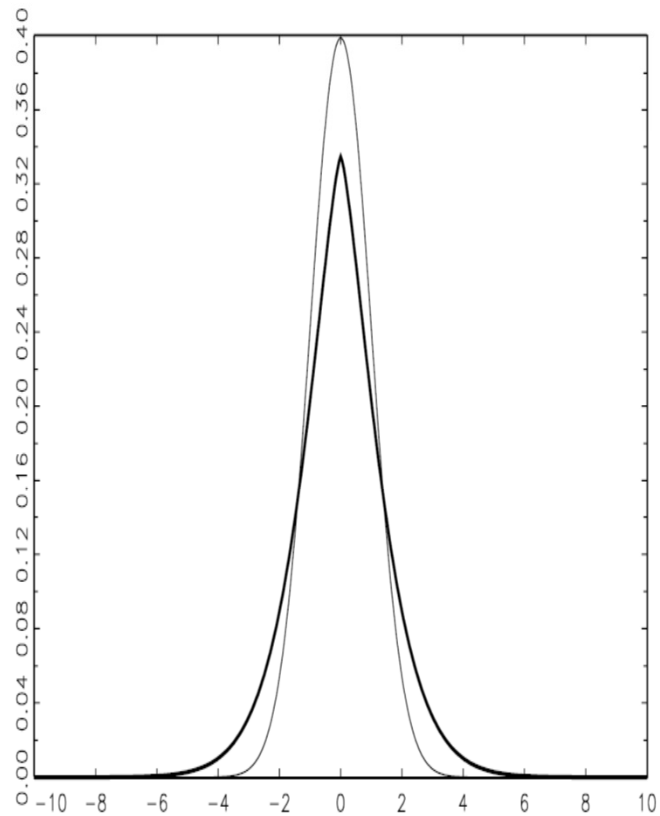

GED-EGARCH high-volatility $s_t = 2$

$\delta_{1,2} = 0.1463$



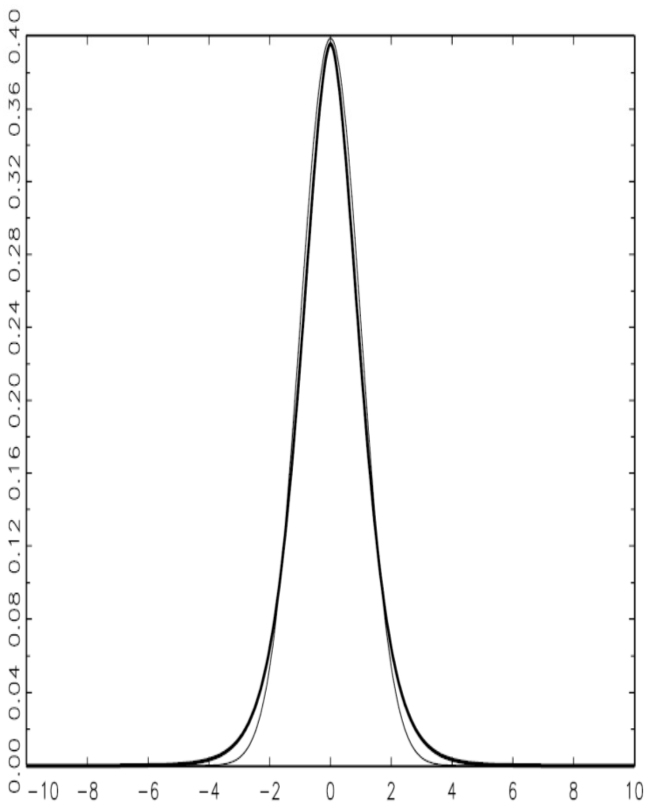

GED-EGARCH low-volatility $s_t = 1$

$\delta_{1,1} = 0.3777$



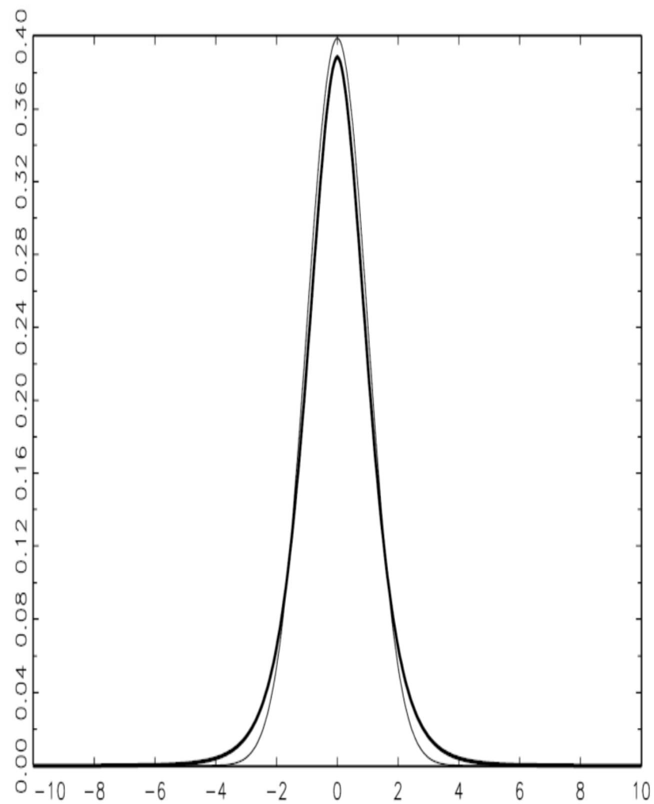
Gen-$t$-EGARCH

$\delta_1 = 1.9732$, $\delta_2 = 0.5886$



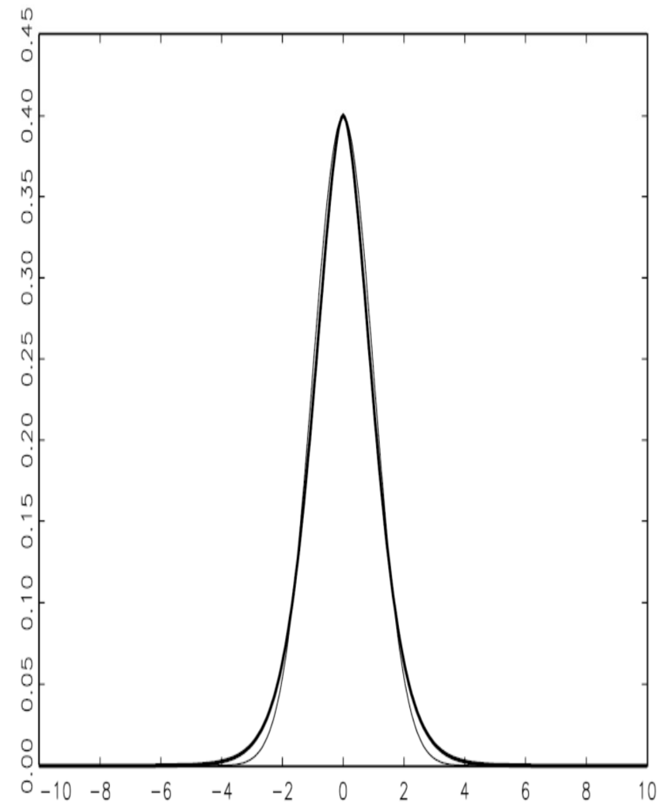

Gen-$t$-EGARCH high-volatility $s_t = 1$

$\delta_{1,1} = 1.5554$, $\delta_{2,1} = 0.6368$



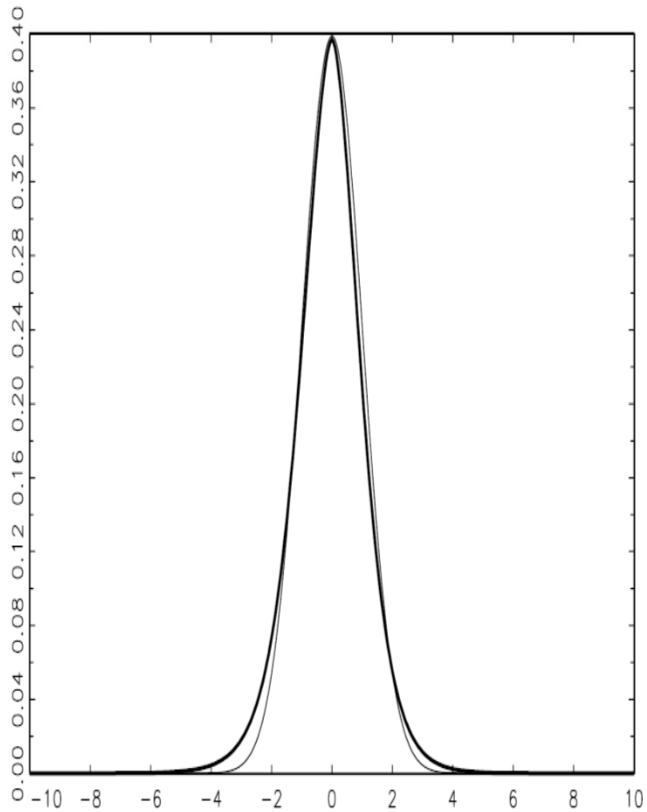

Gen-$t$-EGARCH low-volatility $s_t = 2$

$\delta_{1,2} = 2.3841$, $\delta_{2,2} = 0.5698$



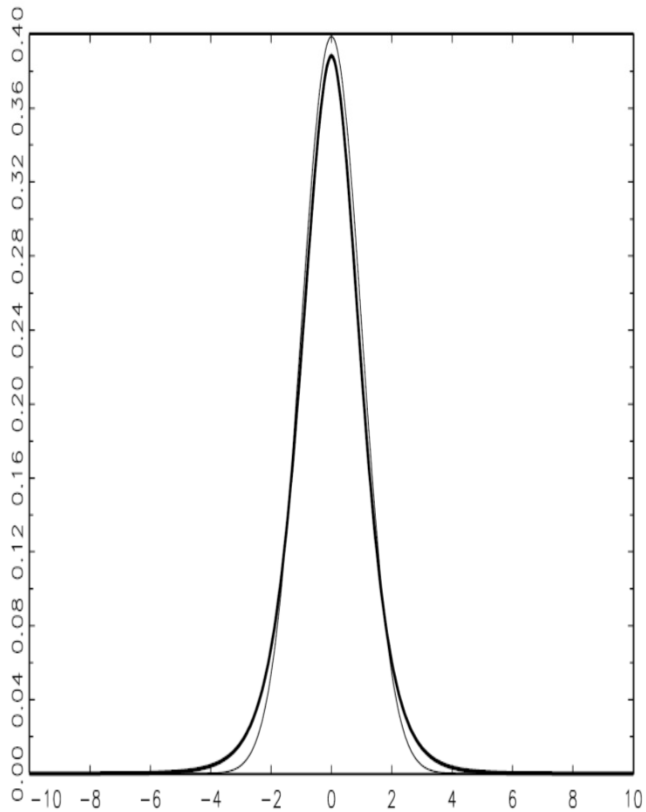
Skew-Gen-$t$-EGARCH

$\delta_1 = -0.0503,\ \delta_2 = 2.0343,\ \delta_3 = 0.5809$



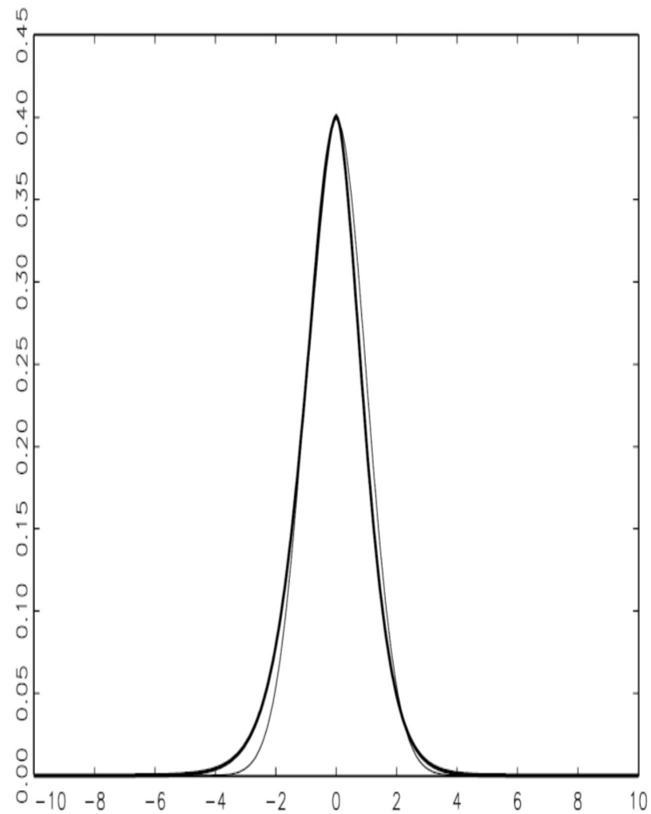

Skew-Gen-$t$-EGARCH high-volatility $s_t = 2$

$\delta_{1,2} = -0.0219,\ \delta_{2,2} = 1.6216,\ \delta_{3,2} = 0.6461$



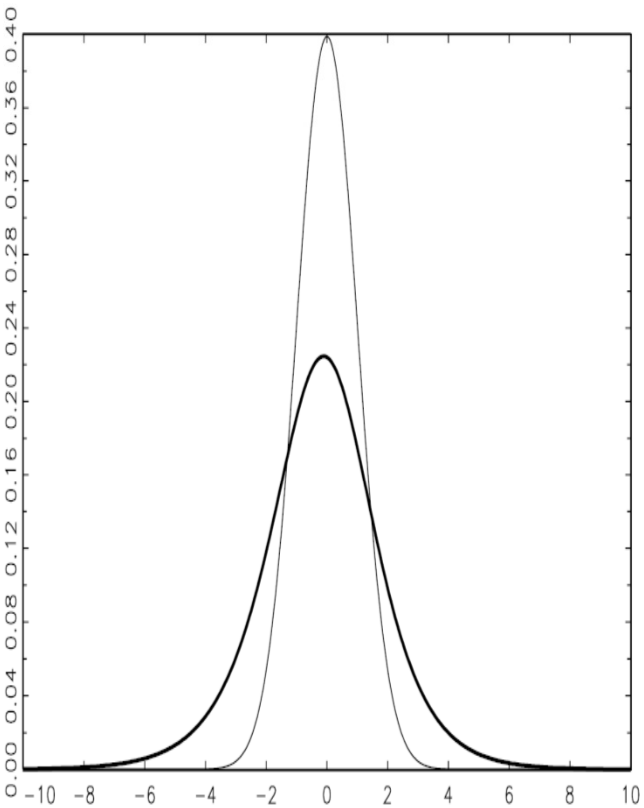

Skew-Gen-$t$-EGARCH low-volatility $s_t = 1$

$\delta_{1,1} = -0.0834,\ \delta_{2,1} = 2.4291,\ \delta_{3,1} = 0.5657$



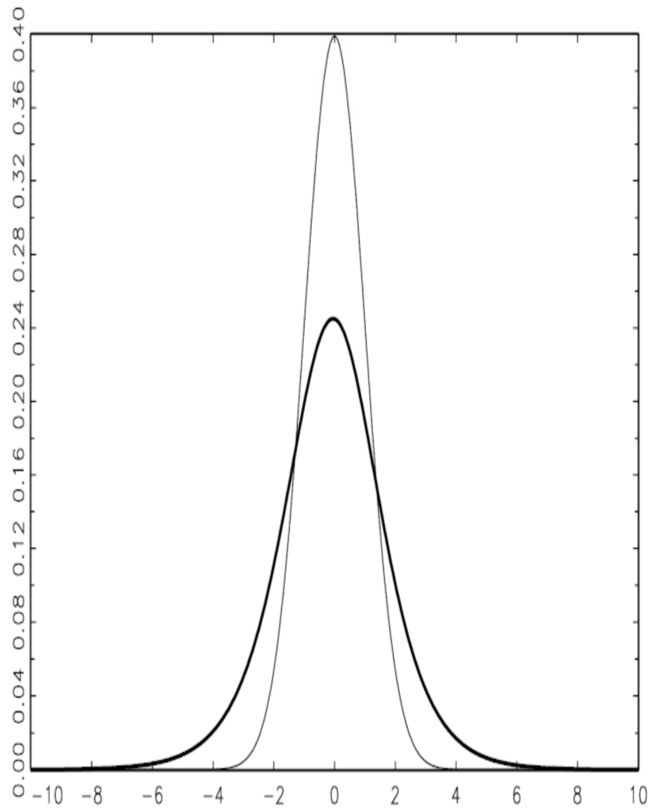
EGB2

$\delta_1 = -0.2253, \delta_2 = -0.1160$



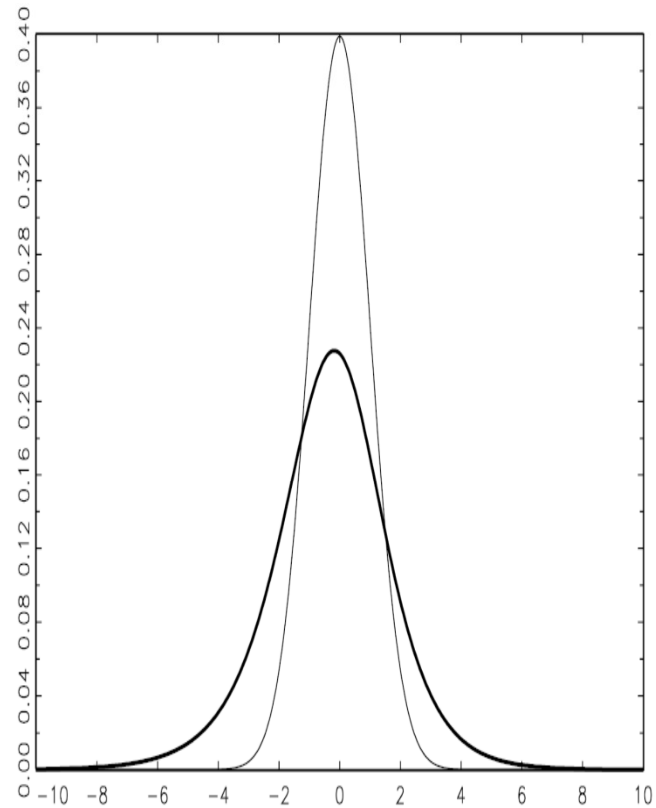

EGB2 high-volatility $s_t = 1$

$\delta_{1,1} = -0.0585, \delta_{2,1} = -0.0067$



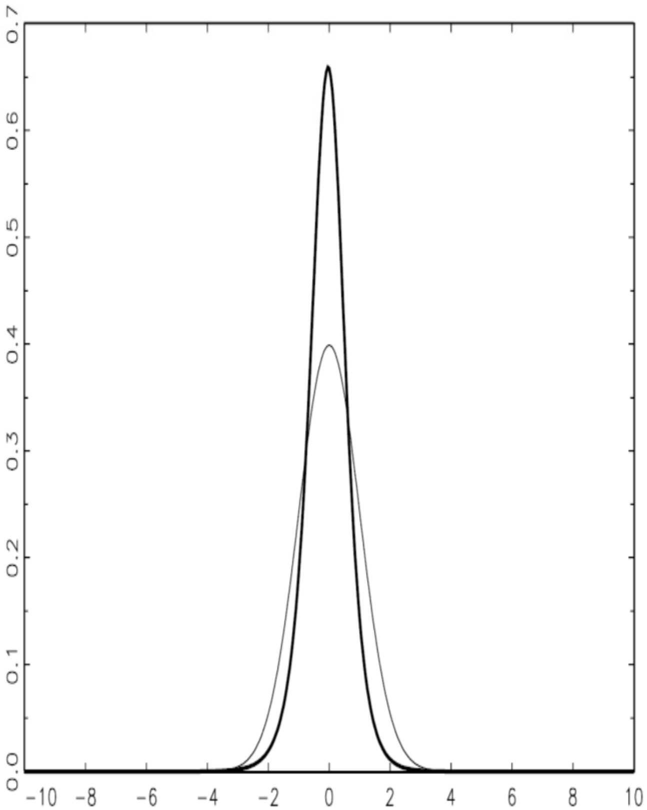

EGB2 low-volatility $s_t = 2$

$\delta_{1,2} = -0.2371, \delta_{2,2} = -0.0557$



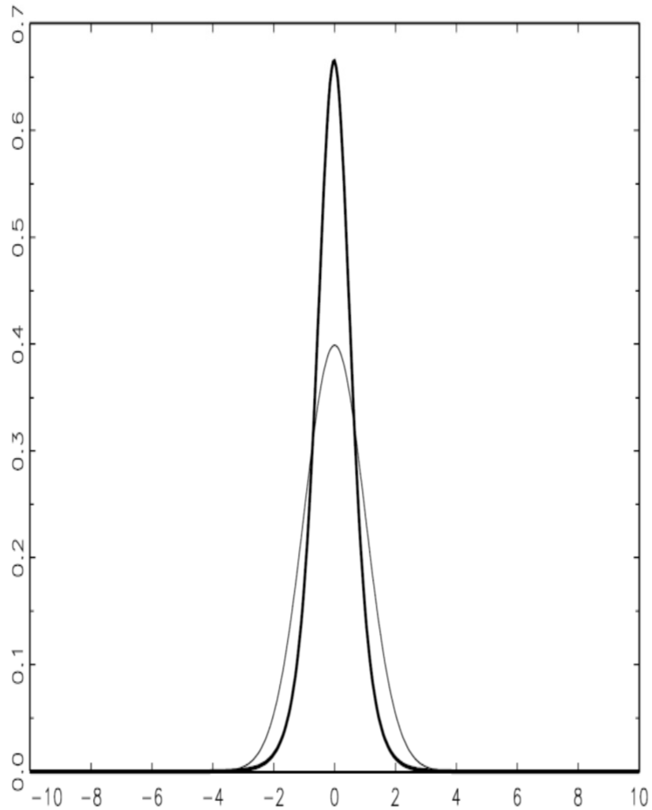
NIG

$\delta_1 = 0.7050, \delta_2 = -0.0659$



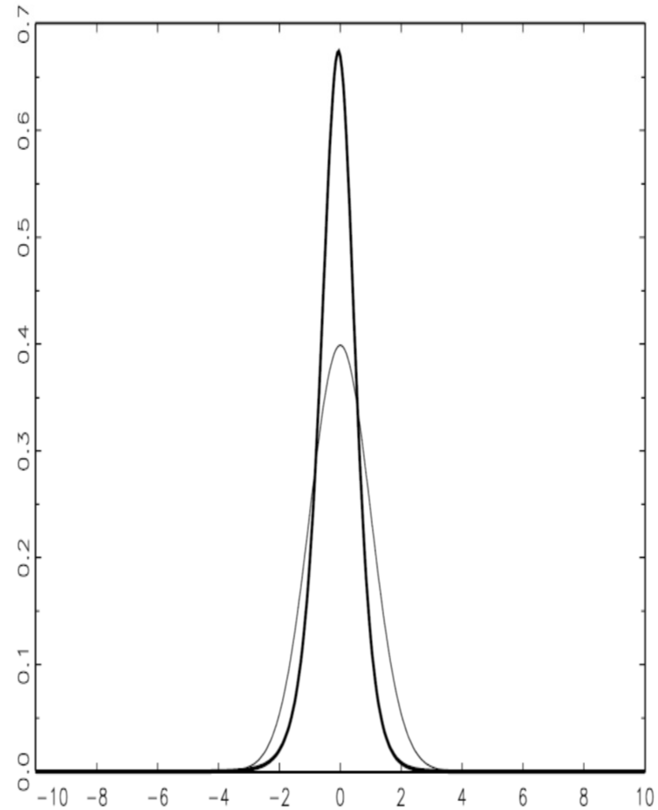

NIG high-volatility $s_t = 1$

$\delta_{1,1} = 0.7952, \delta_{2,1} = -0.0317$




NIG low-volatility $s_t = 2$

$\delta_{1,2} = 0.7721, \delta_{2,2} = -0.1068$



# Data

As an illustration, we use daily log-return data from the Standard & Poor's 500 (S&P 500) for period 1950 to 2016:

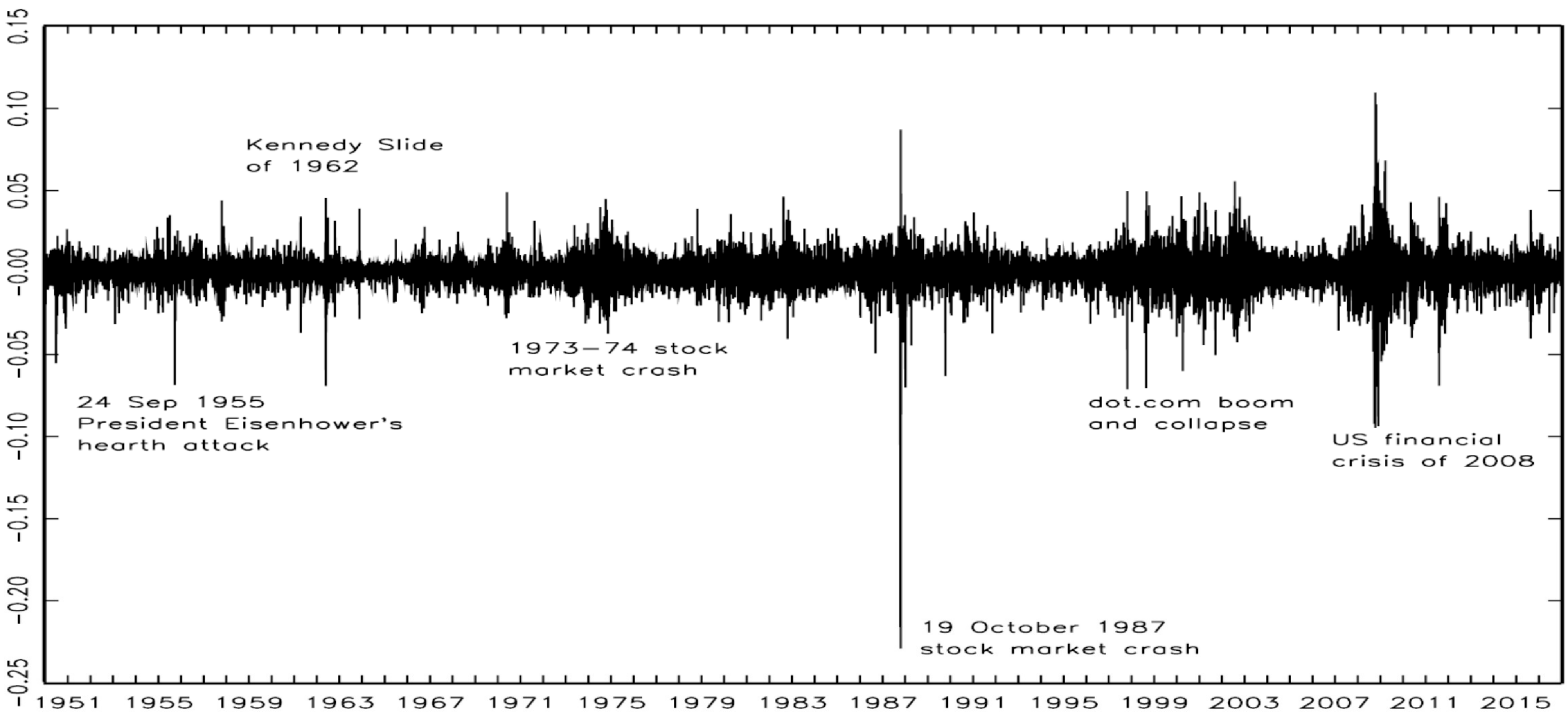



**Fig. 1. S&P 500 daily return for period 3rd January 1950 to 30th December 2016.**

Table 1. Descriptive statistics

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Start date | 4-Jan-1950 | PACF(1) | 0.0273*** | PACF(11) | −0.0142* | PACF(21) | −0.0181** |
| End date | 30-Dec-2016 | PACF(2) | −0.0422*** | PACF(12) | 0.0280*** | PACF(22) | −0.0028 |
| Sample size $T$ | 16,858 | PACF(3) | 0.0028 | PACF(13) | −0.0005 | PACF(23) | −0.0007 |
| Minimum | −0.2290 | PACF(4) | −0.0082 | PACF(14) | −0.0010 | PACF(24) | 0.0103 |
| Maximum | 0.1096 | PACF(5) | −0.0129* | PACF(15) | −0.0130* | PACF(25) | −0.0117 |
| Mean | 0.0003 | PACF(6) | −0.0055 | PACF(16) | 0.0325*** | PACF(26) | −0.0214*** |
| Standard deviation | 0.0097 | PACF(7) | −0.0190** | PACF(17) | −0.0045 | PACF(27) | 0.0192** |
| Skewness | −1.0110 | PACF(8) | 0.0105 | PACF(18) | −0.0205*** | PACF(28) | −0.0049 |
| Excess kurtosis | 27.1272 | PACF(9) | −0.0108 | PACF(19) | 0.0016 | PACF(29) | 0.0236*** |
| Corr($y_t^2$,$y_{t-1}$) | −0.0877 | PACF(10) | 0.0130* | PACF(20) | 0.0106 | PACF(30) | 0.0064 |

# Results

First, we estimate the *unrestricted versions* of all single-regime and MS models.

For all estimations, we use the Maximum Likelihood (ML) method for non-path-dependent MS models (Klaassen 2002, *Empirical Economics*).

We use the results of Abramson and Cohen (2007, *Econometric Theory*) to check the consistency and asymptotic normality of the ML estimates.

**Table 2. Parameter estimates and model diagnostics; unrestricted models**

| | $t$-GARCH | Beta-$t$-EGARCH | GED-EGARCH | Gen-$t$-EGARCH | Skew-Gen-$t$-EGARCH | EGB2-EGARCH | NIG-EGARCH |
|---|---|---|---|---|---|---|---|
| $c$ | 0.0005***(0.0001) | 0.0004***(0.0001) | 0.0004***(0.0000) | 0.0004***(0.0001) | 0.0008***(0.0001) | 0.0010***(0.0001) | 0.0010***(0.0001) |
| $\omega$ | 0.0000***(0.0000) | −0.0647***(0.0067) | −0.0761***(0.0076) | −0.0658***(0.0069) | −0.0578***(0.0071) | −0.0641***(0.0079) | −0.0516***(0.0064) |
| $\alpha$ | 0.0185***(0.0031) | 0.0375***(0.0020) | 0.0379***(0.0019) | 0.0383***(0.0020) | 0.0394***(0.0021) | 0.0386***(0.0020) | 0.0391***(0.0020) |
| $\alpha^*$ | 0.0718***(0.0045) | 0.0245***(0.0014) | 0.0204***(0.0013) | 0.0247***(0.0015) | 0.0252***(0.0015) | 0.0234***(0.0014) | 0.0241***(0.0015) |
| $\beta$ | 0.9141***(0.0042) | 0.9872***(0.0013) | 0.9859***(0.0014) | 0.9871***(0.0014) | 0.9887***(0.0014) | 0.9888***(0.0014) | 0.9889***(0.0014) |
| $\lambda_0$ | 0.0000**(0.0000) | −5.1642***(0.2196) | −5.4354***(0.2478) | −5.1436***(0.2250) | −5.1399***(0.2255) | −5.6943***(0.2376) | −4.6250***(0.2360) |
| $\delta_1$ | 1.6313***(0.0609) | 1.6499***(0.0573) | 0.2657***(0.0098) | 1.9732***(0.0943) | −0.0503***(0.0089) | −0.2253***(0.0613) | 0.7050***(0.0543) |
| $\delta_2$ | NA | NA | NA | 0.5886***(0.0278) | 2.0343***(0.0977) | −0.1160*(0.0650) | −0.0659***(0.0114) |
| $\delta_3$ | NA | NA | NA | NA | 0.5809***(0.0276) | NA | NA |
| $C_\lambda$ | 0.9685 | 0.8776 | 0.8802 | 0.8726 | 0.9094 | 0.8799 | 0.8794 |
| | $t$-GARCH | Beta-$t$-EGARCH | GED-EGARCH | Gen-$t$-EGARCH | Skew-Gen-$t$-EGARCH | EGB2-EGARCH | NIG-EGARCH |
| $c(1)$ | 0.0008***(0.0001) | 0.0005***(0.0001) | 0.0004***(0.0001) | 0.0010***(0.0001) | 0.0008***(0.0001) | 0.0010***(0.0002) | 0.0010***(0.0002) |
| $c(2)$ | 0.0003***(0.0001) | 0.0004***(0.0001) | 0.0011***(0.0002) | 0.0002***(0.0001) | 0.0011***(0.0002) | 0.0011***(0.0001) | 0.0011***(0.0001) |
| $\omega(1)$ | 0.0000***(0.0000) | −0.6688***(0.0711) | −0.0774***(0.0078) | −0.0393***(0.0110) | −0.0668***(0.0094) | −0.0373***(0.0088) | −0.0308***(0.0072) |
| $\omega(2)$ | 0.0000***(0.0000) | −0.0430***(0.0066) | −0.0598***(0.0116) | −0.0845***(0.0084) | −0.0418***(0.0107) | −0.0859***(0.0125) | −0.0694***(0.0101) |
| $\alpha(1)$ | 0.0073(0.0060) | 0.0291***(0.0066) | 0.0379***(0.0021) | 0.0200***(0.0038) | 0.0357***(0.0027) | 0.0189***(0.0024) | 0.0202***(0.0025) |
| $\alpha(2)$ | 0.0152***(0.0036) | 0.0329***(0.0022) | 0.0011(0.0025) | 0.0336***(0.0027) | 0.0195***(0.0036) | 0.0421***(0.0032) | 0.0424***(0.0033) |
| $\alpha^*(1)$ | 0.1433***(0.0169) | 0.0838***(0.0074) | 0.0298***(0.0017) | 0.0047*(0.0028) | 0.0415***(0.0025) | 0.0049***(0.0018) | 0.0057***(0.0019) |
| $\alpha^*(2)$ | 0.0678***(0.0055) | 0.0199***(0.0016) | −0.0095***(0.0029) | 0.0391***(0.0024) | 0.0059**(0.0026) | 0.0470***(0.0030) | 0.0476***(0.0029) |
| $\beta(1)$ | 0.7375***(0.0233) | 0.8742***(0.0135) | 0.9854***(0.0015) | 0.9921***(0.0022) | 0.9874***(0.0019) | 0.9932***(0.0016) | 0.9932***(0.0016) |
| $\beta(2)$ | 0.9410***(0.0045) | 0.9913***(0.0013) | 0.9891***(0.0021) | 0.9836***(0.0017) | 0.9916***(0.0021) | 0.9856***(0.0023) | 0.9858***(0.0023) |
| $\lambda_0(1)$ | 0.0000(0.0004) | −4.9444*(2.6465) | −5.1726***(0.5019) | −5.3476***(0.8035) | −5.0969***(0.5784) | −5.7423***(0.7056) | −4.7149***(0.7250) |
| $\lambda_0(2)$ | 0.0000(0.0001) | −5.1593***(0.8047) | −4.7632(9.2990) | −5.1350***(0.6294) | −5.3379***(0.8527) | −5.5871***(0.7669) | −4.5064***(0.8191) |
| $\delta_1(1)$ | 0.9038***(0.1242) | 1.3216***(0.1080) | 0.3777***(0.0129) | 1.5554***(0.1674) | −0.0834***(0.0118) | −0.0585(0.0810) | 0.7952***(0.0804) |
| $\delta_1(2)$ | 2.3860***(0.1393) | 1.9198***(0.0813) | 0.1463***(0.0192) | 2.3841***(0.2078) | −0.0219(0.0191) | −0.2371***(0.0793) | 0.7721***(0.0814) |
| $\delta_2(1)$ | NA | NA | NA | 0.6368***(0.0590) | 2.4291***(0.2114) | −0.0067(0.0875) | −0.0317(0.0205) |
| $\delta_2(2)$ | NA | NA | NA | 0.5698***(0.0402) | 1.6216***(0.1647) | −0.0557(0.0823) | −0.1068***(0.0154) |
| $\delta_3(1)$ | NA | NA | NA | NA | 0.5657***(0.0402 | NA | NA |
| $\delta_3(2)$ | NA | NA | NA | NA | 0.6461***(0.0555 | NA | NA |
| $p$ | 0.9917***(0.0022) | 0.9941***(0.0017) | 0.9990***(0.0003) | 0.9945***(0.0016) | 0.9968***(0.0010) | 0.9971***(0.0008) | 0.9977***(0.0007) |
| $q$ | 0.9961***(0.0010) | 0.9979***(0.0006) | 0.9933***(0.0016) | 0.9968***(0.0010) | 0.9950***(0.0014) | 0.9973***(0.0009) | 0.9977***(0.0008) |
| $E(\sigma_1)$ | **9.86**% | 9.65% | 11.19% | **13.11**% | 9.03% | **12.03**% | **11.75**% |
| $E(\sigma_2)$ | 8.44% | **13.13**% | **13.80**% | 10.39% | **13.05**% | 8.14% | 8.17% |
| $C_{\lambda,1}$ | 0.8095 | 0.7168 | 0.8686 | 0.9261 | 0.9107 | 0.9318 | 0.8725 |
| $C_{\lambda,2}$ | 0.9864 | 0.8898 | 0.9692 | 0.8723 | 0.9431 | 0.8654 | 0.7527 |

**Table 3. Model performance; unrestricted models**

| | SR $R^2$(rank) | MS $R^2$(rank) | SR LL(rank) | MS LL(rank) | | LR |
|---|---|---|---|---|---|---|
| $t$-GARCH | **7.84**%(6) | 7.64%(7) | 3.4381(6) | **3.4430**(6) | | 0.0049***(0.0010) |
| Beta-$t$-EGARCH | **9.55**%(2) | 9.15%(4) | 3.4402(4) | **3.4442**(2) | | 0.0041***(0.0009) |
| GED-EGARCH | 6.19%(7) | **7.94**%(6) | 3.4349(7) | **3.4407**(7) | | 0.0057***(0.0013) |
| Gen-$t$-EGARCH | 9.67%(1) | **10.18**%(1) | 3.4405(2) | **3.4435**(5) | | 0.0031***(0.0007) |
| Skew-Gen-$t$-EGARCH | 9.54%(3) | **10.05**%(2) | 3.4415(1) | **3.4452**(1) | | 0.0038***(0.0007) |
| EGB2-EGARCH | **9.02**%(5) | 8.82%(5) | 3.4398(5) | **3.4439**(4) | | 0.0040***(0.0007) |
| NIG-EGARCH | 9.26%(4) | **9.27**%(3) | 3.4402(3) | **3.4441**(3) | | 0.0039***(0.0007) |
| | SR AIC(rank) | MS AIC(rank) | SR BIC(rank) | MS BIC(rank) | SR HQC(rank) | MS HQC(rank) |
| $t$-GARCH | −6.8753(6) | **−6.8842**(6) | −6.8721(6) | **−6.8768**(5) | −6.8743(6) | **−6.8817**(6) |
| Beta-$t$-EGARCH | −6.8795(4) | **−6.8866**(2) | −6.8763(3) | **−6.8792**(1) | −6.8784(3) | **−6.8842**(2) |
| GED-EGARCH | −6.8690(7) | **−6.8794**(7) | −6.8658(7) | **−6.8721**(7) | −6.8679(7) | **−6.8770**(7) |
| Gen-$t$-EGARCH | −6.8800(2) | **−6.8850**(5) | −6.8763(2) | **−6.8767**(6) | −6.8788(2) | **−6.8822**(5) |
| Skew-Gen-$t$-EGARCH | −6.8819(1) | **−6.8881**(1) | −6.8777(1) | **−6.8789**(2) | −6.8805(1) | **−6.8851**(1) |
| EGB2-EGARCH | −6.8787(5) | **−6.8856**(4) | −6.8751(5) | **−6.8774**(4) | −6.8775(5) | **−6.8829**(4) |
| NIG-EGARCH | −6.8795(3) | **−6.8860**(3) | −6.8758(4) | **−6.8778**(3) | −6.8783(4) | **−6.8833**(3) |

# Diagnostic tests

For all parameters two alternatives are estimated (one for each regime).

We test whether those two parameters are significantly different from each other:

**Table 4. Test statistics of differences in parameters; unrestricted MS models**

| | $t$-GARCH | Beta-$t$-EGARCH | GED-EGARCH | Gen-$t$-EGARCH |
|---|---|---|---|---|
| $c(1) - c(2)$ | $4.1734^{***}(0.0000)$ | $1.3771(0.1685)$ | $-4.4232^{***}(0.0000)$ | $5.8339^{***}(0.0000)$ |
| $\omega(1) - \omega(2)$ | $7.4185^{***}(0.0000)$ | $-8.7580^{***}(0.0000)$ | $-1.2582(0.2083)$ | $3.2649^{***}(0.0011)$ |
| $\alpha(1) - \alpha(2)$ | $-1.1354(0.2562)$ | $-0.5519(0.5810)$ | $11.4419^{***}(0.0000)$ | $-2.9547^{***}(0.0031)$ |
| $\alpha^*(1) - \alpha^*(2)$ | $4.2503^{***}(0.0000)$ | $8.4638^{***}(0.0000)$ | $11.7054^{***}(0.0000)$ | $-9.3172^{***}(0.0000)$ |
| $\beta(1) - \beta(2)$ | $-8.5860^{***}(0.0000)$ | $-8.6433^{***}(0.0000)$ | $-1.4022(0.1609)$ | $3.0773^{***}(0.0021)$ |
| $\lambda_0(1) - \lambda_0(2)$ | $0.0058(0.9954)$ | $0.0777(0.9381)$ | $-0.0440(0.9649)$ | $-0.2082(0.8350)$ |
| $\delta_1(1) - \delta_1(2)$ | $-7.9418^{***}(0.0000)$ | $-4.4255^{***}(0.0000)$ | $10.0050^{***}(0.0000)$ | $-3.1059^{***}(0.0019)$ |
| $\delta_2(1) - \delta_2(2)$ | NA | NA | NA | $0.9386(0.3479)$ |
| | Skew-Gen-$t$-EGARCH | EGB2-EGARCH | NIG-EGARCH | |
| $c(1) - c(2)$ | $-1.3355(0.1817)$ | $-0.2426(0.8083)$ | $-0.4443(0.6569)$ | |
| $\omega(1) - \omega(2)$ | $-1.7592^{*}(0.0786)$ | $3.1862^{***}(0.0014)$ | $3.1191^{***}(0.0018)$ | |
| $\alpha(1) - \alpha(2)$ | $3.5967^{***}(0.0003)$ | $-5.7818^{***}(0.0000)$ | $-5.3943^{***}(0.0000)$ | |
| $\alpha^*(1) - \alpha^*(2)$ | $9.9166^{***}(0.0000)$ | $-12.1009^{***}(0.0000)$ | $-12.0841^{***}(0.0000)$ | |
| $\beta(1) - \beta(2)$ | $-1.4749(0.1402)$ | $2.7230^{***}(0.0065)$ | $2.6247^{***}(0.0087)$ | |
| $\lambda_0(1) - \lambda_0(2)$ | $0.2339(0.8151)$ | $-0.1489(0.8817)$ | $-0.1907(0.8488)$ | |
| $\delta_1(1) - \delta_1(2)$ | $-2.7450^{***}(0.0061)$ | $1.5765(0.1149)$ | $0.2016(0.8402)$ | |
| $\delta_2(1) - \delta_2(2)$ | $3.0132^{***}(0.0026)$ | $0.4085(0.6829)$ | $2.9258^{***}(0.0034)$ | |
| $\delta_3(1) - \delta_3(2)$ | $-1.1733(0.2407)$ | NA | NA | |

# Diagnostic tests

For several pairs of parameters we find that they are not different.

Second, for those cases, we assume that they are identical and we estimate *restricted MS models*:

**Table 5. Parameter estimates and model diagnostics; restricted MS models**

| | $t$-GARCH | Beta-$t$-EGARCH | GED-EGARCH | Gen-$t$-EGARCH | Skew-Gen-$t$-EGARCH | EGB2-EGARCH | NIG-EGARCH |
|---|---|---|---|---|---|---|---|
| $c(1)$ | 0.0008***(0.0001) | 0.0004***(0.0000) | 0.0004***(0.0001) | 0.0010***(0.0001) | 0.0009***(0.0001) | 0.0010***(0.0001) | 0.0011***(0.0001) |
| $c(2)$ | 0.0003***(0.0001) | $c(1)$ | 0.0012***(0.0002) | 0.0002***(0.0001) | $c(1)$ | $c(1)$ | $c(1)$ |
| $\omega(1)$ | 0.0000***(0.0000) | −0.6547***(0.0674) | −0.0706***(0.0064) | −0.0398***(0.0111) | −0.0553***(0.0066) | −0.0341***(0.0081) | −0.0312***(0.0071) |
| $\omega(2)$ | 0.0000***(0.0000) | −0.0416***(0.0064) | $\omega(1)$ | −0.0838***(0.0084) | −0.0513***(0.0066) | −0.1000***(0.0124) | −0.0689***(0.0094) |
| $\alpha(1)$ | 0.0138***(0.0030) | 0.0323***(0.0020) | 0.0369***(0.0019) | 0.0204***(0.0038) | 0.0355***(0.0027) | 0.0221***(0.0023) | 0.0203***(0.0025) |
| $\alpha(2)$ | $\alpha(1)$ | $\alpha(1)$ | 0.0019(0.0026) | 0.0336***(0.0027) | 0.0206***(0.0035) | 0.0440***(0.0034) | 0.0417***(0.0032) |
| $\alpha^*(1)$ | 0.1391***(0.0172) | 0.0835***(0.0073) | 0.0303***(0.0016) | 0.0047*(0.0028) | 0.0414***(0.0025) | 0.0079***(0.0017) | 0.0056***(0.0019) |
| $\alpha^*(2)$ | 0.0691***(0.0053) | 0.0195***(0.0015) | −0.0088***(0.0028) | 0.0391***(0.0024) | 0.0060**(0.0026) | 0.0485***(0.0030) | 0.0471***(0.0029) |
| $\beta(1)$ | 0.7303***(0.0238) | 0.8765***(0.0128) | 0.9868***(0.0012) | 0.9921***(0.0022) | 0.9897***(0.0013) | 0.9939***(0.0015) | 0.9931***(0.0016) |
| $\beta(2)$ | 0.9418***(0.0042) | 0.9916***(0.0013) | $\beta(1)$ | 0.9838***(0.0017) | $\beta(1)$ | 0.9831***(0.0022) | 0.9859***(0.0021) |
| $\lambda_0(1)$ | 0.0000*(0.0000) | −5.1184***(0.2270) | −5.2011***(0.2647) | −5.3265***(0.1928) | −5.3177***(0.1868) | −5.7372***(0.1949) | −4.7055***(0.1927) |
| $\lambda_0(2)$ | $\lambda_0(1)$ | $\lambda_0(1)$ | $\lambda_0(1)$ | $\lambda_0(1)$ | $\lambda_0(1)$ | $\lambda_0(1)$ | $\lambda_0(1)$ |
| $\delta_1(1)$ | 0.9050***(0.1240) | 1.3195***(0.1085) | 0.3734***(0.0123) | 1.6911***(0.1245) | −0.0922***(0.0101) | −0.1738***(0.0641) | 0.7827***(0.0595) |
| $\delta_1(2)$ | 2.3810***(0.1389) | 1.9234***(0.0814) | 0.1882***(0.0172) | 2.3115***(0.1673) | −0.0110(0.0122) | $\delta_1(1)$ | $\delta_1(1)$ |
| $\delta_2(1)$ | NA | NA | NA | 0.5897***(0.0308) | 2.3170***(0.1683) | −0.0629(0.0672) | −0.0379***(0.0126) |
| $\delta_2(2)$ | NA | NA | NA | $\delta_2(1)$ | 1.7932***(0.1246) | $\delta_2(1)$ | −0.1030***(0.0128) |
| $\delta_3(1)$ | NA | NA | NA | NA | 0.5947***(0.0305) | NA | NA |
| $\delta_3(2)$ | NA | NA | NA | NA | $\delta_3(1)$ | NA | NA |
| $p$ | 0.9915***(0.0022) | 0.9941***(0.0016) | 0.9987***(0.0004) | 0.9945***(0.0016) | 0.9968***(0.0010) | 0.9984***(0.0005) | 0.9977***(0.0007) |
| $q$ | 0.9960***(0.0010) | 0.9978***(0.0006) | 0.9921***(0.0017) | 0.9968***(0.0010) | 0.9954***(0.0013) | 0.9980***(0.0007) | 0.9977***(0.0008) |
| $E(\sigma_1)$ | **9.84%** | 9.80% | 10.62% | **12.97%** | 8.30% | **11.66%** | **11.76%** |
| $E(\sigma_2)$ | 8.47% | **12.83%** | **14.26%** | 10.42% | **12.86%** | 8.41% | 8.25% |
| $C_{\lambda,1}$ | 0.8066 | 0.7138 | 0.8739 | 0.9235 | 0.9145 | 0.9271 | 0.8726 |
| $C_{\lambda,2}$ | 0.9863 | 0.8919 | 0.9619 | 0.8732 | 0.9392 | 0.8573 | 0.7547 |

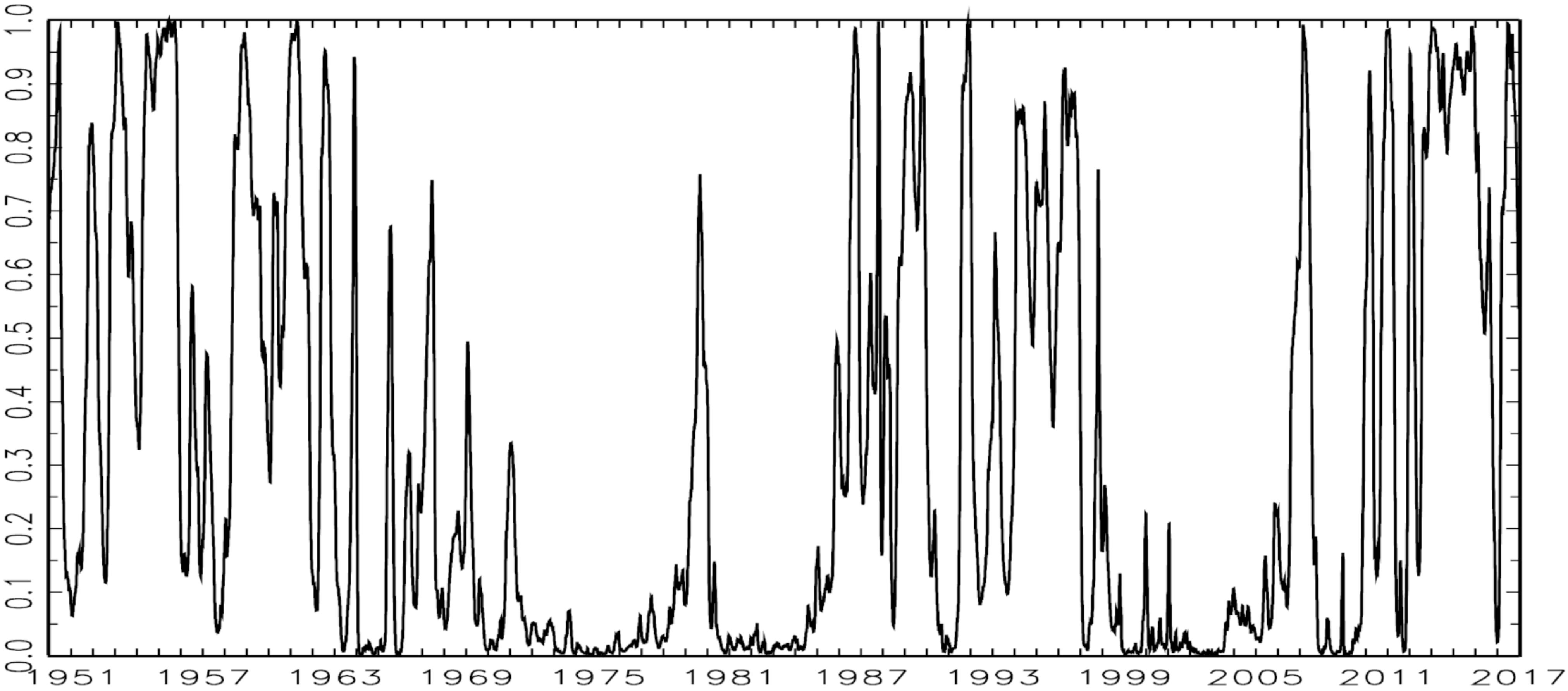
**Table 6. Model performance; restricted MS models**

| | SR $R^2$(rank) | MS $R^2$(rank) | SR LL(rank) | MS LL(rank) | | LR |
|---|---|---|---|---|---|---|
| $t$-GARCH | **7.84%**(6) | 7.60%(6) | 3.4381(6) | **3.4430**(6) | | 0.0049***(0.0010) |
| Beta-$t$-EGARCH | **9.55%**(2) | 9.09%(4) | 3.4402(4) | **3.4442**(2) | | 0.0040***(0.0009) |
| GED-EGARCH | **6.19%**(7) | 4.40%(7) | 3.4349(7) | **3.4405**(7) | | 0.0056***(0.0013) |
| Gen-$t$-EGARCH | 9.67%(1) | **10.18%**(1) | 3.4405(2) | **3.4435**(4) | | 0.0030***(0.0007) |
| Skew-Gen-$t$-EGARCH | 9.54%(3) | **10.04%**(2) | 3.4415(1) | **3.4451**(1) | | 0.0037***(0.0007) |
| EGB2-EGARCH | **9.02%**(5) | 8.08%(5) | 3.4398(5) | **3.4431**(5) | | 0.0032***(0.0007) |
| NIG-EGARCH | **9.26%**(4) | 9.23%(3) | 3.4402(3) | **3.4441**(3) | | 0.0039***(0.0007) |
| | SR AIC(rank) | MS AIC(rank) | SR BIC(rank) | MS BIC(rank) | SR HQC(rank) | MS HQC(rank) |
| $t$-GARCH | −6.8753(6) | **−6.8843**(6) | −6.8721(6) | **−6.8779**(5) | −6.8743(6) | **−6.8822**(6) |
| Beta-$t$-EGARCH | −6.8795(4) | **−6.8868**(2) | −6.8763(3) | **−6.8809**(2) | −6.8784(3) | **−6.8849**(2) |
| GED-EGARCH | −6.8690(7) | **−6.8795**(7) | −6.8658(7) | **−6.8735**(7) | −6.8679(7) | **−6.8775**(7) |
| Gen-$t$-EGARCH | −6.8800(2) | **−6.8852**(4) | −6.8763(2) | **−6.8778**(6) | −6.8788(2) | **−6.8827**(4) |
| Skew-Gen-$t$-EGARCH | −6.8819(1) | **−6.8884**(1) | −6.8777(1) | **−6.8810**(1) | −6.8805(1) | **−6.8859**(1) |
| EGB2-EGARCH | −6.8787(5) | **−6.8845**(5) | −6.8751(5) | **−6.8780**(4) | −6.8775(5) | **−6.8823**(5) |
| NIG-EGARCH | −6.8795(3) | **−6.8864**(3) | −6.8758(4) | **−6.8795**(3) | −6.8783(4) | **−6.8841**(3) |

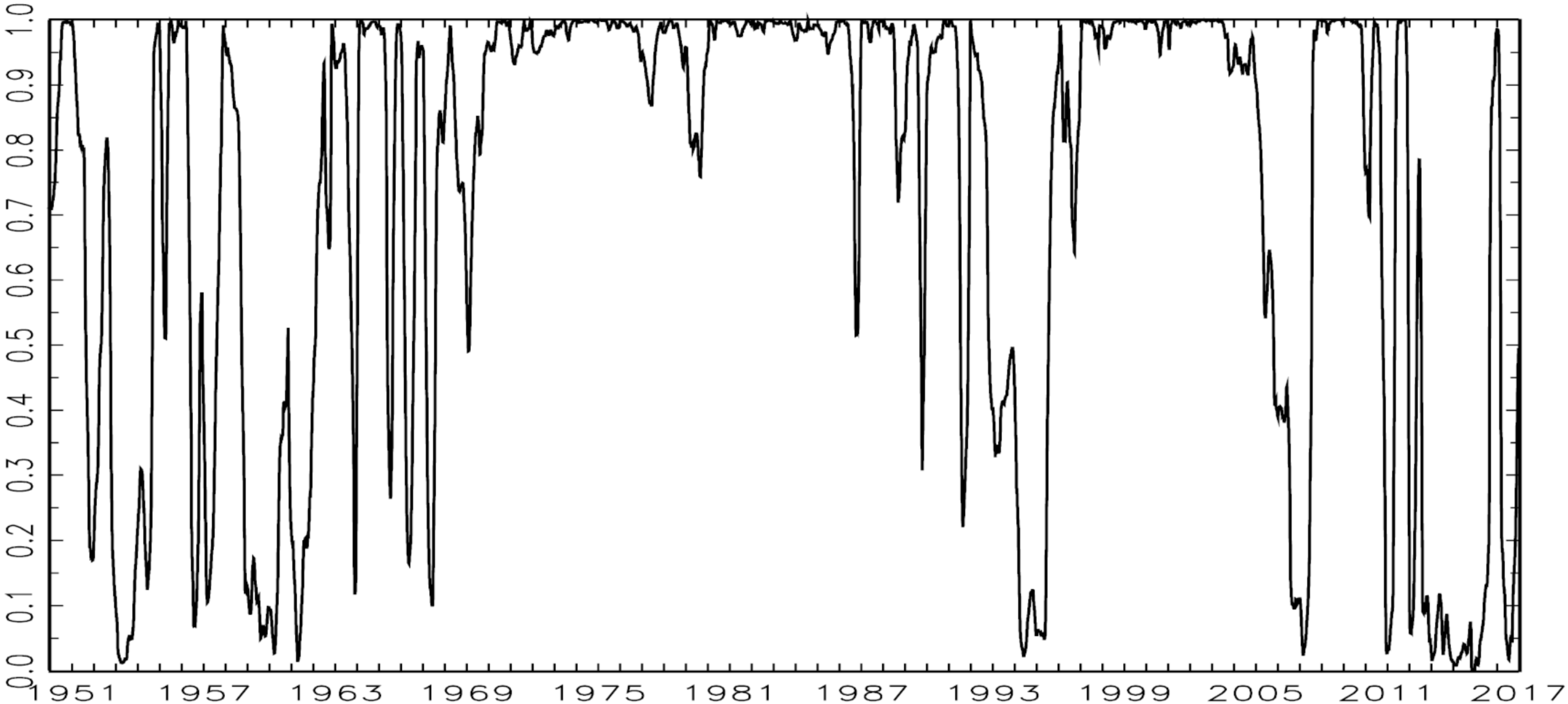
# Probability of high-volatility regime

In the following figures, we present the evolution of the smoothed probability of the high-volatility regime:

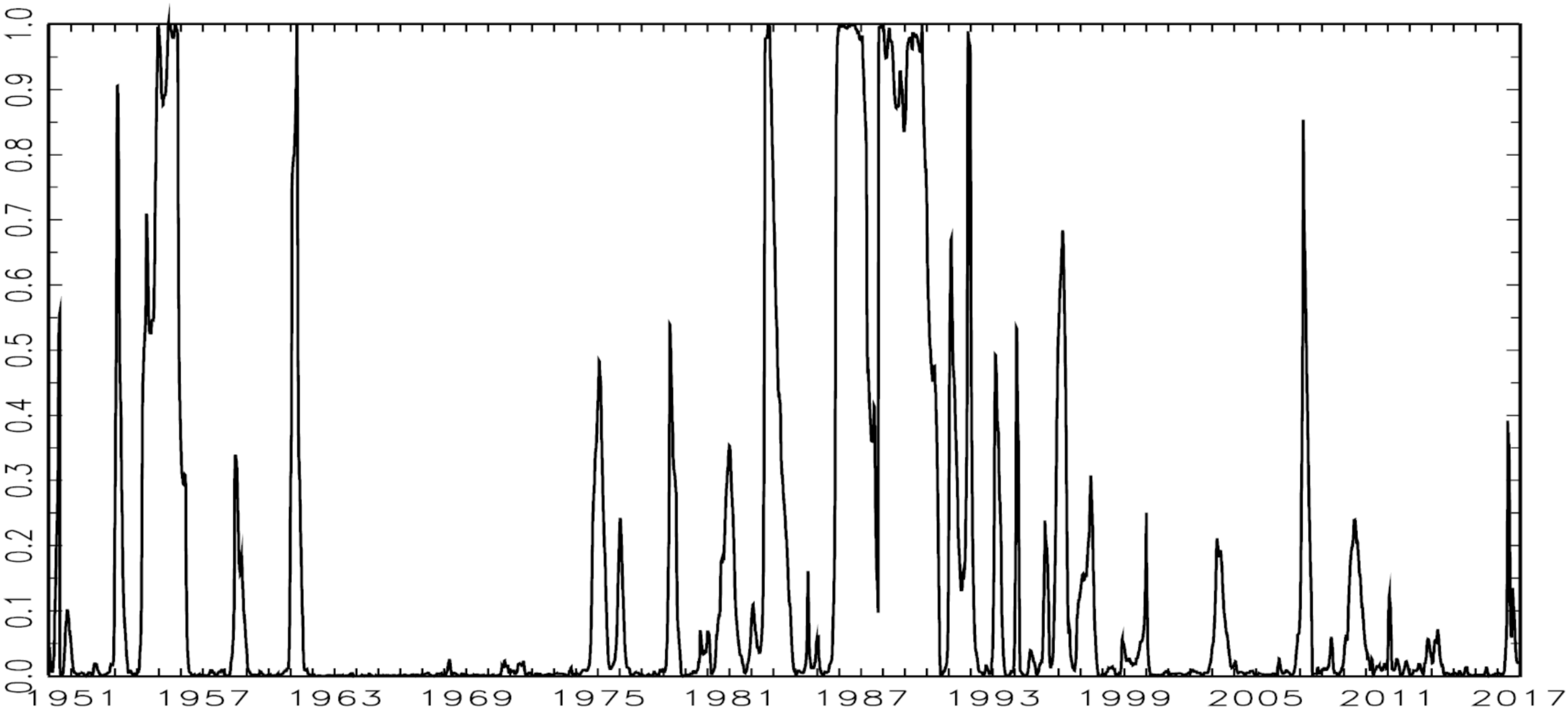
# MS $t$-GARCH smoothed probability $s_t = 1$



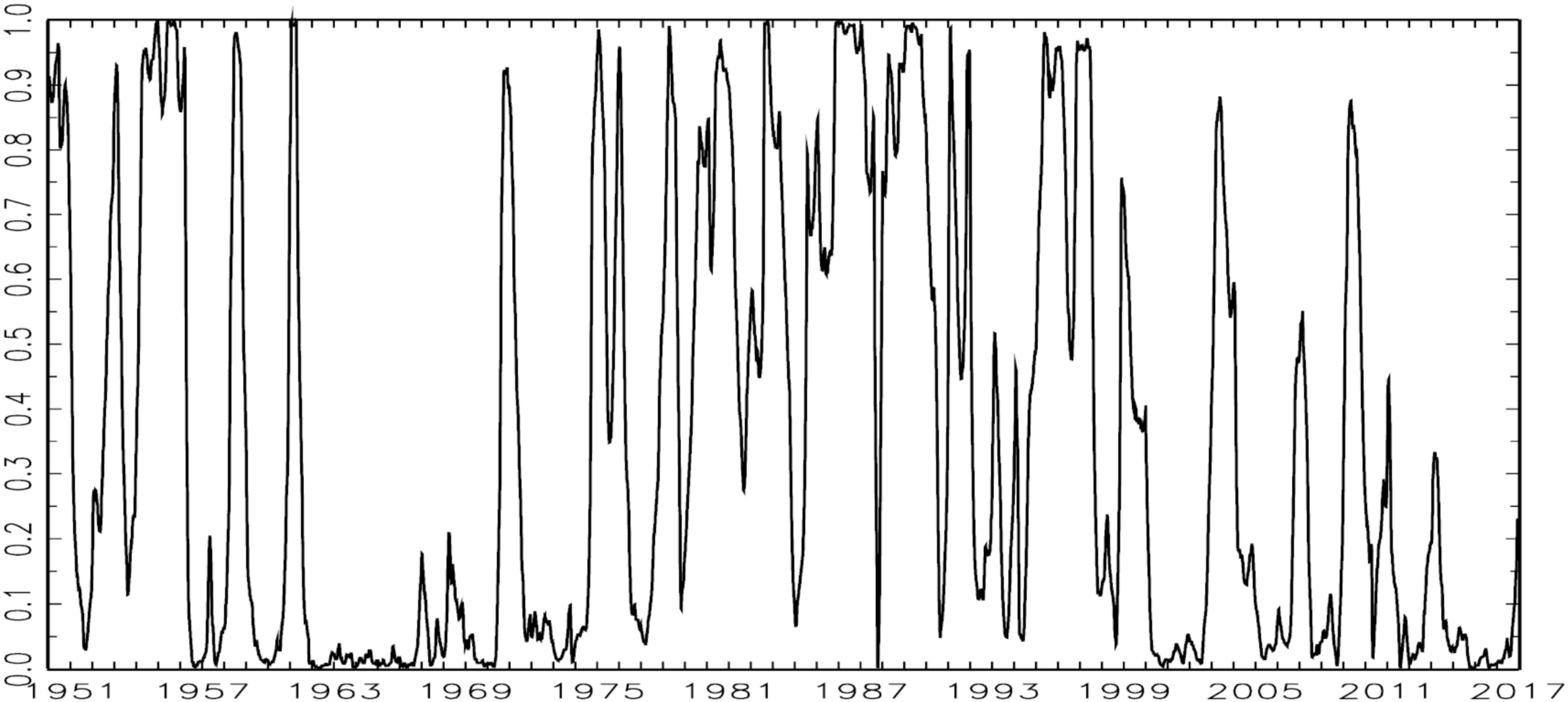
# MS Beta-$t$-EGARCH smoothed probability $s_t = 2$



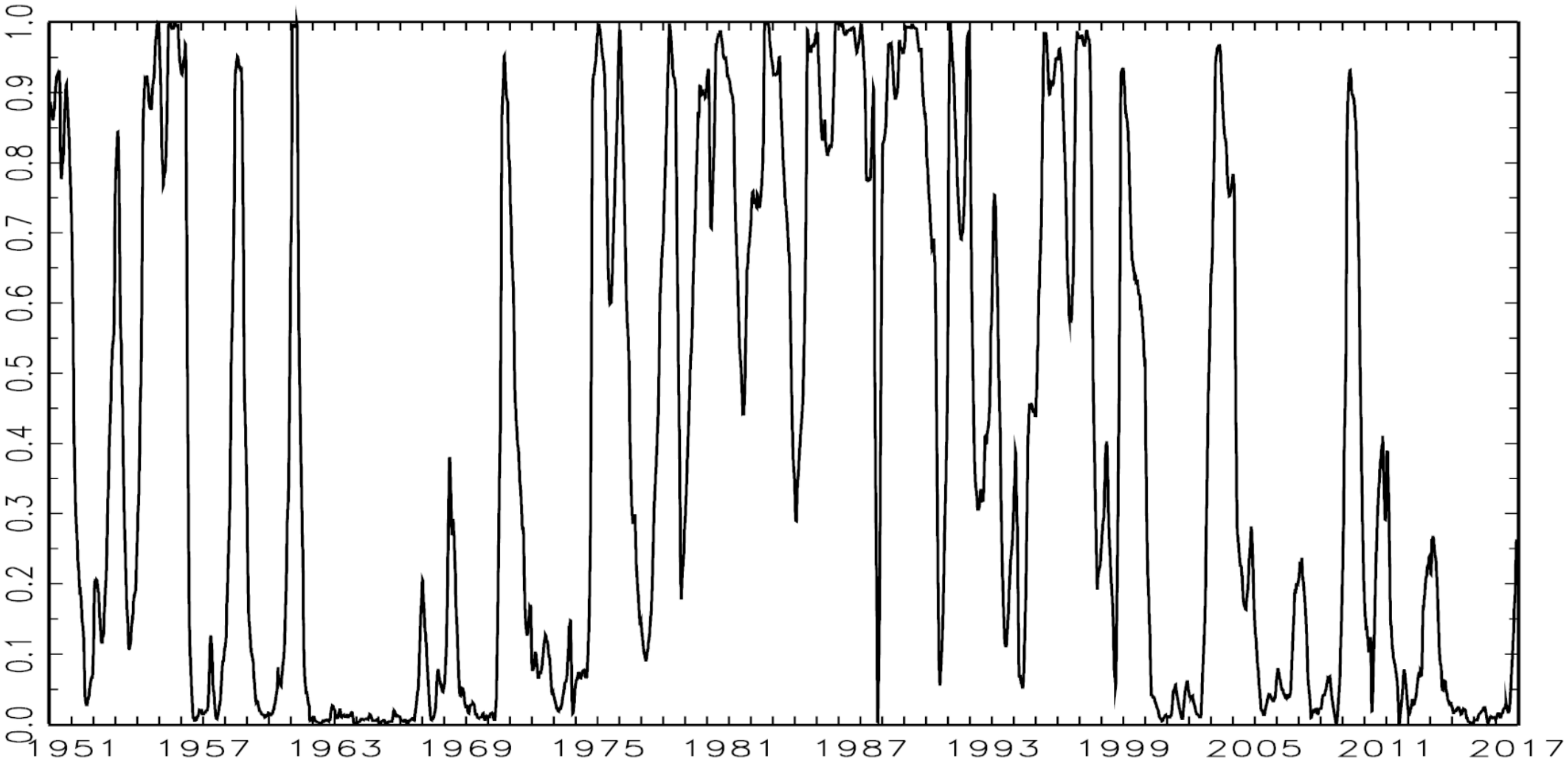
# MS GED-EGARCH smoothed probability $s_t = 2$



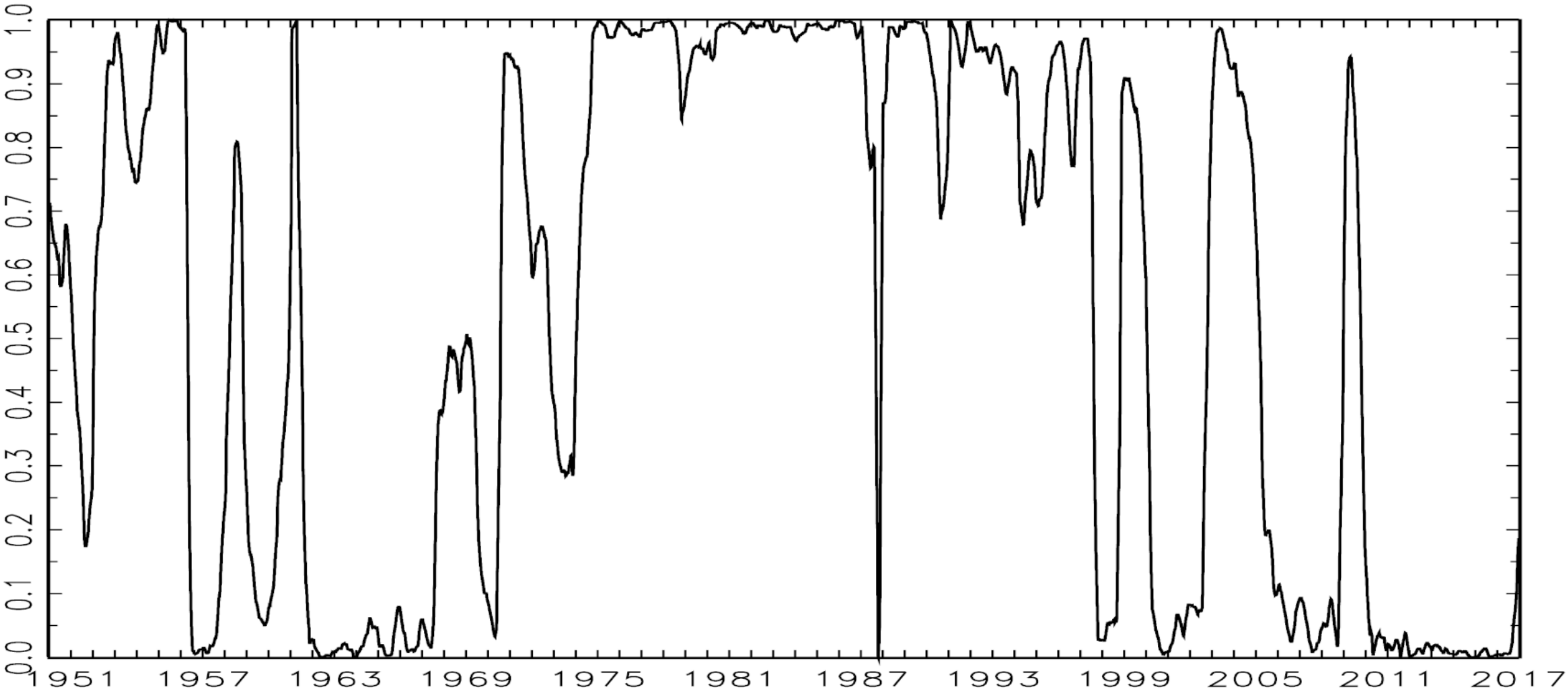
# MS Gen-$t$-EGARCH smoothed probability $s_t = 1$



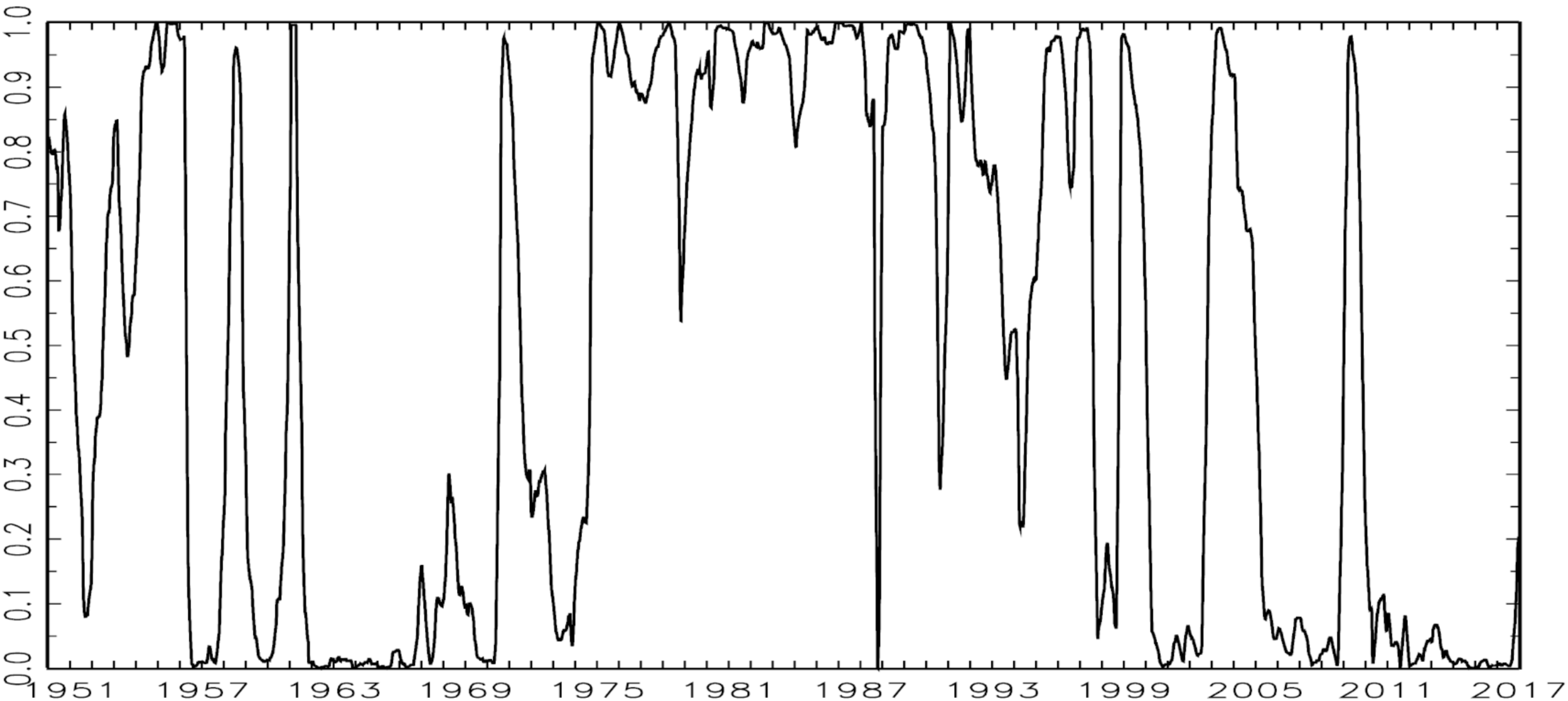
# MS Skew-Gen-$t$-EGARCH smoothed probability $s_t = 2$



# MS EGB2 smoothed probability $s_t = 1$



# MS NIG smoothed probability $s_t = 1$



# Thank you for your attention!

*SBLAZSEK@UFM.EDU*