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Score-driven models of local level, seasonality and volatility:

an application to the currency exchange rate of Indian rupee to USD

Astrid Ayala and Szabolcs Blazsek

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Astrid Ayala and Szabolcs Blazsek*

School of Business, Universidad Francisco Marroquín, Guatemala City 01010, Guatemala

Abstract: In this paper, we suggest the new Skew-Gen-t-DCS (skewed generalized t distribution, dynamic conditional score) and NIG-DCS (normal-inverse Gaussian distribution) models for the Indian rupee (INR) to USD exchange rate. Both time series models include stochastic local level and stochastic seasonality components, and they are alternatives to the t-DCS and EGB2-DCS (exponential generalized beta distribution of the second kind) models, respectively, from the body of literature. We also consider DCS-EGARCH (exponential generalized autoregressive conditional heteroscedasticity) volatility dynamics for the irregular component. We find that Skew-Gen-t-DCS is superior to t-DCS and NIG-DCS is superior to EGB2-DCS. For INR/USD, a significant annual stochastic seasonality with dynamic amplitude is identified for the period of 1st January 1982 to 7th July 2017. We relate the seasonality of INR/USD to the seasonality of Indian exports and imports. We provide an economic analysis of several product groups of exports and imports, and highlight the importance of pearls, precious and semi-precious stones, gold and silver imports that are concentrated during the wedding season of India. The results of this paper motivate the use of the new DCS models by central bankers, financial analysts and investors, and private firms that undertake export or import activities.

Keywords: Indian rupee (INR) to USD; dynamic conditional score (DCS) models; stochastic seasonality; trimming; Winsorizing; extreme observations

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*Corresponding author. Postal address: School of Business, Universidad Francisco Marroquín, Calle Manuel F. Ayau, Zona 10, Guatemala City 01010, Guatemala. E-mail and telephone: sblazsek@ufm.edu, +502 2338-7783.

I. Introduction

The real gross domestic product (GDP) per capita of India has increased significantly during the last twenty years (Fig. 1(a)). Alongside this important real GDP per capita growth, Indian exports and imports have also dramatically increased (Fig. 1(b)). Related to the evolution of Indian exports and imports, in this paper we demonstrate that the Indian rupee (INR) to the United States dollar (USD) exchange rate has an annual stochastic seasonality component with a dynamic amplitude that has significantly increased in recent decades. In our application, we use daily INR/USD data for the period of 1st January 1982 to 7th July 2017. We provide an economic analysis of INR/USD seasonality, with respect to Indian exports and imports.

We identify the stochastic seasonality component of INR/USD, by using two new score-driven models of stochastic local level, stochastic seasonality and dynamic volatility: Skew-Gen-t-DCS (skewed generalized t distribution, dynamic conditional score) model and NIG-DCS (normal-inverse Gaussian distribution) model. These models belong to the family of DCS models (Creal, Koopman and Lucas 2013; Harvey 2013), in which each dynamic equation is updated by the conditional score of the log-likelihood (LL) (hereinafter, score function). A general property of DCS models is that the score function discounts the effects of extreme values in the irregular component v_t when a time-varying parameter is updated.

In the body of literature, Harvey (2013, Chapter 3.6) and Harvey and Luati (2014) introduce the dynamic Student's t location model (t-DCS, hereinafter) that includes stochastic local level, stochastic seasonality and homoscedastic irregular components. Caivano et al. (2016) introduce the dynamic EGB2 (exponential generalized beta distribution of the second kind) location model (EGB2-DCS, hereinafter) that also includes stochastic local level, stochastic seasonality and homoscedastic irregular components. Extreme observations are discounted in different ways in the location equations of the t-DCS and EGB2-DCS models: For t-DCS, the score function converges to zero as $|v_t| \to \infty$. Thus, the score function in the location equation performs a soft form of trimming of extreme observations. On the other hand, for EGB2-DCS, the score function converges to positive and negative non-zero values as $v_t \to +\infty$ and $v_t \to -\infty$, respectively.

Hence, the score function in the location equation performs a soft form of Winsorizing of extreme observations. As to which type of discounting is more effective, it is an open question in the body of the relevant DCS literature (Harvey 2013; Harvey and Luati 2014; Caivano et al. 2016).

In this paper, we contribute to the body of literature on DCS models, as follows: (i) We use DCS-EGARCH (exponential generalized autoregressive conditional heteroscedasticity) (Harvey 2013) scale dynamics for the irregular component in all DCS models with stochastic local level and stochastic seasonality; extending the works of Harvey (2013), Harvey and Luati (2014) and Caivano et al. (2016). We find that INR/USD has significant volatility dynamics.

(ii) For the new Skew-Gen-t-DCS model, we show that a soft form of trimming is performed in the location equation; similar to the t-DCS location model (Harvey 2013; Harvey and Luati 2014). For the INR/USD data, we compare the statistical performances of t-DCS and Skew-Gen-t-DCS, where both models include the stochastic local level, stochastic seasonality and dynamic volatility components. We find that the more general Skew-Gen-t-DCS model fits better to the data and it is more parsimonious than the t-DCS model.

(iii) For the new NIG-DCS model, we show that a soft form of Winsorizing is performed in the location equation; similar to the EGB2-DCS location model (Caivano et al. 2016). For the INR/USD data, we compare the statistical performances of EGB2-DCS and NIG-DCS, where both models include the stochastic local level, stochastic seasonality and dynamic volatility components. We find that the NIG-DCS model fits better to the data and it is more parsimonious than the EGB2-DCS model.

(iv) We relate the seasonality of INR/USD to the seasonality of several important export and import product groups of India. In that analysis, we study the correlation between the seasonality component of each product group and the seasonality component of INR/USD. We find that the seasonality component of Indian exports has a negative correlation coefficient with the seasonality component of IRN/USD, which is related to the fact that the USD export income is exchanged to INR. We also find that the seasonality component of Indian imports has a positive correlation coefficient with the seasonality component of IRN/USD, which is related to the fact that INR is exchanged to USD to finance those imports. With respect to the seasonality of imports, we highlight the importance of pearls, precious and semi-precious stones, gold and silver. Those imports are particularly important during the wedding season in India, from March to June in every year. We also present the correlation coefficient of the seasonality component of IRN/USD and the seasonality component of several important product groups of exports and imports, which validate the significant amplitude of INR/USD seasonality.

The remainder of this work is organized as follows. Section II presents the econometric models. Section III reviews the statistical inferences. Section IV presents the empirical results. Section V concludes.

[APPROXIMATE LOCATION OF FIGURE 1]

II. Econometric models

In this paper, we use DCS models that decompose the daily INR/USD exchange rate time series, p_t for $t = 1, \ldots, T$, to the local level μ_t, seasonality s_t and irregular v_t components, as follows: $p_t = \mu_t + s_t + v_t = \mu_t + s_t + \exp(\lambda_t)\epsilon_t$, where we factorize the irregular component to the product of the dynamic scale parameter $\exp(\lambda_t)$ and the noise term ϵ_t. For ϵ_t, we use the Student's t, Skew-Gen-t, EGB2 and NIG distributions. We present mathematical details for those probability distributions in the Appendix. In the following, we present the dynamics of the local level, seasonality and irregular components.

The local level component is specified according to the dynamic equation $\mu_t = \mu_{t-1} + \delta u_{\mu,t-1}$. This equation is updated by the score function $u_{\mu,t}$ with respect to μ_t. We present $u_{\mu,t}$ for Student's t, Skew-Gen-t, EGB2 and NIG noise specifications in the Appendix. We initialize μ_t by using the first observation of the IRN/USD exchange rate p_1. It is noteworthy that, as an alternative, we also make use of parameter μ_0 to initialize μ_t. We obtain similar results for both alternatives, therefore, we only report results for $\mu_1 = p_1$ in this paper.

The annual seasonality component is specified as $s_t = D_t'\rho_t = (D_{\text{Jan},t}, D_{\text{Feb},t}, \ldots, D_{\text{Dec},t})'\rho_t$, where the monthly dummies $D_{j,t}$ with $j \in \{\text{Jan}, \ldots, \text{Dec}\}$ select an element from the 12×1 vector of dynamic variables ρ_t. The vector ρ_t is formulated according to the first-order $I(1)$ (Hamilton

1994) equation: $\rho_t = \rho_{t-1} + \gamma_t u_{\mu,t-1}$. The vector ρ_t is updated by the score function $u_{\mu,t}$ and $u_{\mu,t}$ is scaled by using the 12×1 vector of dynamic parameters γ_t. It is noteworthy that the same score function $u_{\mu,t}$ updates both μ_t and s_t, since the conditional scores with respect to μ_t and s_t are identical. Each element of the γ_t vector is given by $\gamma_{jt} = \gamma_j$ for $D_{jt} = 1$ and $\gamma_{jt} = -\gamma_j/(12-1)$ for $D_{jt} = 0$, where γ_j with $j \in \{\text{Jan}, \ldots, \text{Dec}\}$ are parameters to be estimated. This specification ensures that the sum of the seasonality parameters is zero, hence, s_t has mean zero and it is effectively separated from μ_t. The fact that s_t is $I(0)$ indicates that the parameterization of the seasonality component compensates the $I(1)$ specification of the dynamic parameter ρ_t. We initialize ρ_t by estimating the equation $p_t = a + bt + c_{\text{Jan}} D_{\text{Jan},t} + \ldots + c_{\text{Dec}} D_{\text{Dec},t} + \epsilon_t$, under the restriction $c_{\text{Jan}} + \ldots + c_{\text{Dec}} = 0$, by using the non-linear least squares (NLS) method (Harvey 2013, Chapter 3.6; Harvey and Luati 2014). For the initialization of ρ_t, we use data from the first year of the full data window (i.e. the first 261 observations of the sample, from 1982; see Section IV). The initial values of ρ_t are the NLS estimates of $c_{\text{Jan}}, \ldots, c_{\text{Dec}}$.

The time-varying scale of the irregular component v_t is specified by using the DCS-EGARCH model: $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1}$. This equation is updated by the score function $u_{\lambda,t}$ with respect to λ_t. We present $u_{\lambda,t}$ for the Student's t, Skew-Gen-t, EGB2 and NIG distributions in the Appendix. We initialize λ_t by using parameter λ_0. The DCS-EGARCH model for the Student's t and EGB2 distributions is named as Beta-t-EGARCH (Harvey and Chakravarty 2008) and EGB2-EGARCH (Caivano and Harvey 2014), respectively. We name the DCS-EGARCH model for the Skew-Gen-t and NIG distributions as Skew-Gen-t-EGARCH and NIG-EGARCH, respectively. For the DCS-EGARCH models, we also refer to the related works of Harvey and Sucarrat (2014) and Harvey and Lange (2017).

III. Statistical inference

All models of this paper are estimated by using the maximum likelihood (ML) method (Davidson and MacKinnon 2003). The ML estimates of parameters are given by

$$\hat{\Theta}_{\text{ML}} = \arg\max_{\Theta} \text{LL}(p_1, \ldots, p_T; \Theta) = \arg\max_{\Theta} \sum_{t=1}^{T} \ln f(p_t | p_1, \ldots, p_{t-1}; \Theta) \tag{1}$$

where Θ denotes the vector of time-constant parameters. For different noise term ϵ_t specifications, the formulation of the log of the conditional density $\ln f(\cdot)$ is presented in the Appendix. We estimate the standard errors of parameters by using the inverse information matrix (Harvey 2013). We estimate the transformed values of some parameters, for which we use the delta method (Davidson and MacKinnon 2003) to estimate standard errors.

For the local level and stochastic seasonality components, the asymptotic properties of the ML estimator hold, because the dynamic parameters of those equations are set to the value one, rather than an estimate value (Harvey 2013). For DCS-EGARCH, the conditions for the consistency and asymptotic normality of ML are $|\beta| < 1$ and $C_\lambda = \beta^2 + 2\beta\alpha E(\partial u_{\lambda,t}/\partial\lambda_t) + \alpha^2 E[(\partial u_{\lambda,t}/\partial\lambda_t)^2] < 1$ (Harvey 2013). We evaluate $\partial u_{\lambda,t}/\partial\lambda_t$ and $(\partial u_{\lambda,t}/\partial\lambda_t)^2$ numerically. For C_λ, the expectations are estimated by using the sample average. If $\partial u_{\lambda,t}/\partial\lambda_t$ and $(\partial u_{\lambda,t}/\partial\lambda_t)^2$ are covariance stationary, then the sample average will be a consistent estimator of their expected value (Hamilton 1994, Chapter 7.2). The use of the sample average estimator is validated by the augmented Dickey–Fuller (ADF) unit root test with constant (Dickey and Fuller 1979).

IV. Empirical results

Data

We use data from the daily closing INR/USD exchange rate for the period of 3rd January 1973 to 7th July 2017 (source: Bloomberg). For the period of 3rd January 1973 to 31st December 1981, the INR/USD time series includes several constant level periods with zero volatility and other periods with step-like evolution of the exchange rate (Fig. 2(a)). The DCS models used in this paper are not adequate for this 'pre-sample period'. In this study, we use the sample period of 1st January 1982 to 7th July 2017 (Fig. 2(b)). From 1982, a managed float regime was introduced in India, and the INR/USD exchange rate became more volatile. The foreign exchange rate policy of the Central Bank of India implies intervention in currency markets only in order to reduce the INR/USD volatility and without influencing the direction of the INR value in relation to other currencies (https://www.centralbank.net.in).

We present descriptive statistics for the INR/USD level p_t and the INR/USD log-return

$\ln(p_t/p_{t-1})$ in Table 1. We also present the results for the ADF test in Table 1, which suggest that p_t is a I(1) process (this motivates the use of the local level component with unit root) and $\ln(p_t/p_{t-1})$ is a $I(0)$ process. Significant jumps and falls in INR/USD are observed in Fig. 2(c), which motivate the use of alternative DCS specifications that discount differently the extreme observations. Furthermore, volatility clustering is also observed in Fig. 2(c), which motivates the use of DCS-EGARCH.

[APPROXIMATE LOCATION OF TABLE 1 AND FIGURE 2]

ML estimates

For all models, we present parameter estimates, ML conditions and likelihood-based statistical performance metrics in Table 2. For all specifications, the EGARCH estimates support the consistency and asymptotic normality of the ML estimator: $|\beta| < 1$ and $C_\lambda < 1$. The ADF tests of $\partial u_{\lambda,t}/\partial \lambda_t$ and $(\partial u_{\lambda,t}/\partial \lambda_t)^2$ show that the derivative of the score function is covariance stationary (Table 2), hence, the estimation of C_λ is validated.

We use the following metrics to compare statistical performances: LL, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan-Quinn Criterion (HQC) (Davidson and MacKinnon 2003). These metrics suggest that: (i) Skew-Gen-t-DCS is superior to t-DCS; (ii) NIG-DCS is superior to EGB2-DCS; (iii) all LL-based statistical performance metrics support the use of Skew-Gen-t-DCS. For INR/USD, this last point suggests that the discounting of extreme observations is more effective for Skew-Gen-t-DCS than for NIG-DCS.

[APPROXIMATE LOCATION OF TABLE 2]

Trimming and Winsorizing of extreme observations

Discounting of extreme observations is undertaken by the score function for all DCS models. We present the score functions $u_{\mu,t}$ and $u_{\lambda,t}$ for all DCS models of this paper in Fig. 3. All score functions are computed by using: (i) the ML estimates of the shape parameters; (ii) the ML estimates of the unconditional mean $E(\lambda_t) = \hat{\omega}/(1 - \hat{\beta})$.

According to Fig. 3(a), $u_{\mu,t}$ performs a smooth form of trimming of the extreme observations for t-DCS and Skew-Gen-t-DCS. This figure shows that observations are discounted more for

Skew-Gen-t-DCS in the central part of the distribution than for t-DCS. Fig. 3(a) also shows that observations are discounted in similar ways in the extreme parts of the distribution for t-DCS and Skew-Gen-t-DCS. According to Fig. 3(b), $u_{\lambda,t}$ performs a smooth form of Winsorizing of the extreme observations for t-DCS and Skew-Gen-t-DCS. This figure shows that observations are discounted in similar ways for t-DCS and Skew-Gen-t-DCS in the central part of the distribution. Fig. 3(b) shows that observations are discounted more for t-DCS than for Skew-Gen-t-DCS in the extreme parts of the distribution. The LL-based metrics from Table 2 suggest that the discounting of extreme observations of Skew-Gen-t-DCS is more effective than that of t-DCS.

According to Fig. 3(c), $u_{\mu,t}$ undertakes a smooth form of Winsorizing for EGB2-DCS and NIG-DCS. We also present in Fig. 3(d) that $u_{\lambda,t}$ increases linearly as $|\epsilon_t| \to \infty$. We present in Figs. 3(c-d) for $u_{\mu,t}$ and $u_{\lambda,t}$, respectively, that extreme observations are discounted more for EGB2-DCS than for NIG-DCS. In addition, observations are discounted in an asymmetric way in the left and right tails of the distribution for both EGB2-DCS and NIG-DCS (Figs. 3(c-d)). We find that observations are discounted more in the right tail than the extreme observations in the left tail for both $u_{\mu,t}$ and $u_{\lambda,t}$. The LL-based metrics from Table 2 suggest that the discounting of extreme observations of NIG-DCS is more effective than that of EGB2-DCS.

[APPROXIMATE LOCATION OF FIGURE 3]

Stochastic seasonality

Significant stochastic annual seasonality s_t estimates are shown in Figs. 4(a-d) for t-DCS, Skew-Gen-t-DCS, EGB2-DCS and NIG-DCS, respectively. Based on those figures, we identify three periods with different seasonality effects: (i) 1st January 1982 to 31st December 1993; (ii) 3rd January 1994 to 31st December 2003; (iii) 1st January 2004 to 7th July 2017. We study monthly average seasonality effects within each period by estimating the following linear regression model: $s_t = \kappa_{\text{Jan}} D_{\text{Jan},t} + \ldots + \kappa_{\text{Dec}} D_{\text{Dec},t} + \epsilon_t$ (Table 3). The OLS-HAC (ordinary least squares, heteroscedasticity and autocorrelation consistent) estimates (Newey and West 1987) suggest that the seasonality component weakens the INR with respect to the USD in the first half of the year (Table 3). On the other hand, the seasonality component strengthens the INR with respect to the USD in the second half of the year (Table 3).

From Fig. 4 and Table 3, we conclude that the annual seasonality of INR/USD is the most significant for the period of 1st January 2004 to 7th July 2017. This can be explained by the relatively high real GDP per capita growth rates for that period (Fig. 1(a)). For the preceding period, 1st January 1982 to 31st December 2003, the compounded real GDP per capita growth rate of India is 3.57% (source: The Conference Board Total Economy Database). On the other hand, for the period of 1st January 2004 to 7th July 2017, the same growth rate is 6.15% (source: The Conference Board Total Economy Database).

We analyze the significant annual seasonality component in the INR/USD exchange rate, with respect to the evolution of Indian exports and imports. Firstly, in Table 4(a) and Table 4(b), we present the relative importance of different product groups that were exported and imported, respectively, for the period of 2011 to 2013 (source: Open Government Data Platform India, https://data.gov.in). From Table 4(b), we highlight the importance of two imported product groups: Pearls, precious and semi-precious stones; Gold and silver. Those products are mostly imported to India during the wedding season, in every year from March to June.

There are more than ten million annual weddings in India (Kannan 2013). Wedding related expenses are important, as those expenses may be more than six times the annual income of an Indian family (Bloch et al. 2004). Gold has an important role in Indian weddings (Baur 2013), because of its symbolism of wealth, prosperity and security (Kannan 2013). Wedding expenses can be classified as dowries and the cost of the wedding celebration (Bloch et al. 2004; Shenk 2007). The effect of the end of the wedding season on the seasonality of INR/USD exchange is observed in Table 3. When the pearls, precious and semi-precious stones, gold and silver imports fall after the wedding period, the INR becomes stronger with respect to the USD.

Secondly, we use data on the total exports and total imports, and also the exports and imports of several important product groups (source: Bloomberg) (see the list of those product groups in Table 5). Due to data availability, we use data for the third period that are identified in Fig. 4 and presented in Table 3. We believe that the analysis of this period is relevant because it is the most recent time period of our sample and the amplitude of the seasonality component for this period is the most significant with respect to other periods of the sample.

For the exports and imports data, we estimate the t-DCS model with homoscedastic errors (i.e. $\alpha = \beta = 0$ in the DCS-EGARCH model of Section II). We estimate this restricted t-DCS specification instead of the more general DCS models of Section II, due to the limited number of monthly observations (i.e. the corresponding sample size is 163 monthly observations during the period of January 2004 to July 2017). This t-DCS specification provides the estimates of the annual seasonality component for each product group of exports and imports. We estimate the correlation coefficient of the Skew-Gen-t-DCS seasonality component of INR/USD and the t-DCS seasonality component of each of the product groups of Table 5, where we present the correlation coefficient estimates. According to those estimates, the correlation coefficient for exports from India (INMTEXIR Index) is -0.3531 (Table 5). This shows that the USD income from exports is exchanged to INR, which strenghtens the INR with respect to USD. The estimates in Table 5 also show that the correlation coefficient for imports to India (INMTINIR Index) is 0.0648. This shows that as INR is exchanged to USD to finance the imports to India, the INR becomes weaker with respect to the USD. Furthermore, from Table 5, we highlight two relevant product groups of imports, for which significant positive correlation coefficients are estimated: (i) petroleum, crude and products imports (INIMPETR Index) with the correlation coefficient 0.4618; (ii) gold imports (INIMGOLD Index) with the correlation coefficient 0.3463.

In Figs. 5(a-d), we present the significant seasonality components of the previously mentioned relevant variables: exports from India (INMTEXIR Index), imports to India (INMTINIR Index), petroleum, crude and products imports (INIMPETR Index) and gold imports (INIMGOLD Index), respectively. Those estimates support the annual stochastic seasonality of INR/USD.

[APPROXIMATE LOCATION OF TABLES 3 TO 5 AND FIGURES 4 AND 5]

V. Conclusions

We have proposed the application of new Skew-Gen-t-DCS and NIG-DCS models for the INR/USD exchange rate. Those models are alternatives to the t-DCS and the EGB2-DCS models, respectively. We have shown that the Skew-Gen-t location model performs a smooth form of trimming of extreme observations, similar to the Student's t location model. We have also shown that the NIG location model performs a smooth form of Winsorizing of extreme observations, similar to the EGB2 location model. We have extended the DCS models with stochastic local level and stochastic seasonality from the body of literature, by considering DCS-EGARCH scale dynamics of the irregular component.

We have used daily data from INR/USD for the period of 1st January 1982 to 7th July 2017, for which we have found a significant stochastic seasonality component. We have found that the Skew-Gen-t-DCS model is superior to the t-DCS model, and we have also found that the NIG-DCS model is superior to the EGB2-DCS model. Furthermore, with respect to the new DCS models, we have found that the Skew-Gen-t-DCS model discounts extreme observations more effectively than the NIG-DCS model. We have provided an analysis of several important product groups of Indian exports and imports, with respect to the annual seasonality component of INR/USD. We have highlighted the importance of pearls, precious and semi-precious stones, gold and silver imports that are concentrated during the wedding season of India in each year.

The results presented in this paper motivate the use of the new DCS models with stochastic local level, stochastic seasonality and dynamic volatility for the INR/USD exchange rate, for example, by central bankers, financial analysts and investors, and private firms that undertake export or import activities.

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Appendix

In this appendix, we present the mathematical formulae related to alternative probability distributions of the noise term ϵ_t: Student's t, Skew-Gen-t, EGB2 and NIG. For each of these distributions, we present the log of the conditional density function of p_t and we also present the score functions $u_{\mu,t}$ and $u_{\lambda,t}$ that update the location and the log-scale, respectively.

Firstly, $\epsilon_t \sim t[0, 1, \exp(\nu) + 2]$ is the Student's t distribution, where $\nu \in \mathbb{R}$ influences tail-thickness (this specification of degrees of freedom ensures a finite conditional variance of p_t). The log of the conditional density of p_t is

$$\ln f(p_t|p_1, \ldots, p_{t-1}) = \ln \Gamma\left[\frac{\exp(\nu)+3}{2}\right] - \ln \Gamma\left[\frac{\exp(\nu)+2}{2}\right] \tag{A.1}$$

$$-\frac{\ln(\pi) + \ln[\exp(\nu)+2]}{2} - \lambda_t - \frac{\exp(\nu)+3}{2} \ln\left\{1 + \frac{\epsilon_t^2}{\exp(\nu)+2}\right\}$$

where $\Gamma(x)$ is the gamma function. The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t|p_1, \ldots, p_{t-1})}{\partial \mu_t} = \frac{\exp(\lambda_t)\epsilon_t}{\epsilon_t^2 + \exp(\nu) + 2} \times \frac{\exp(\nu)+3}{\exp(2\lambda_t)} = u_{\mu,t} \times \frac{\exp(\nu)+3}{\exp(2\lambda_t)} \tag{A.2}$$

where $u_{\mu,t}$ is defined by the second equality. The score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1, \ldots, p_{t-1})}{\partial \lambda_t} = \frac{[\exp(\nu)+3]\epsilon_t^2}{\exp(\nu)+2+\epsilon_t^2} - 1 \tag{A.3}$$

Secondly, $\epsilon_t \sim$ Skew-Gen-$t[0, 1, \tanh(\tau), \exp(\nu)+2, \exp(\eta)]$ (McDonald and Michelfelder 2017), where $\tanh(x)$ is the hyperbolic tangent function, and $\tau \in \mathbb{R}$, $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ influence the asymmetry, tail-thickness and peakedness, respectively (the degrees of freedom specification ensures that the conditional variance of p_t is finite). The log of the conditional density of p_t is

$$\ln f(p_t|p_1, \ldots, p_{t-1}) = \eta - \lambda_t - \ln(2) - \frac{\ln[\exp(\nu)+2]}{\exp(\eta)} - \ln \Gamma\left[\frac{\exp(\nu)+2}{\exp(\eta)}\right] \tag{A.4}$$

$$-\ln \Gamma[\exp(-\eta)] + \ln \Gamma\left[\frac{\exp(\nu)+3}{\exp(\eta)}\right]$$

$$-\frac{\exp(\nu)+3}{\exp(\eta)} \ln\left\{1 + \frac{|\epsilon_t|^{\exp(\eta)}}{[1+\tanh(\tau)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta)} \times [\exp(\nu)+2]}\right\}$$

where $\text{sgn}(x)$ is the signum function. The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = \frac{\exp(\lambda_t)\epsilon_t|\epsilon_t|^{\exp(\eta)-2}}{|\epsilon_t|^{\exp(\eta)} + [1+\tanh(\tau)\text{sgn}(\epsilon_t)]^{\exp(\eta)}[\exp(\nu)+2]} \times \frac{\exp(\nu)+3}{\exp(2\lambda_t)} \quad \text{(A.5)}$$

$$= u_{\mu,t} \times \frac{\exp(\nu)+3}{\exp(2\lambda_t)}$$

where $u_{\mu,t}$ is defined by the second equality. The score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \lambda_t} = \frac{|\epsilon_t|^{\exp(\eta)}[\exp(\nu)+3]}{|\epsilon_t|^{\exp(\eta)} + [1+\tanh(\tau)\text{sgn}(\epsilon_t)]^{\exp(\eta)}[\exp(\nu)+2]} - 1 \quad \text{(A.6)}$$

Thirdly, $\epsilon_t \sim \text{EGB2}[0,1,\exp(\xi),\exp(\zeta)]$, where $\xi \in \mathbb{R}$ and $\zeta \in \mathbb{R}$ influence both asymmetry and tail-thickness. The log of the conditional density of p_t is

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = \exp(\xi)\epsilon_t - \lambda_t - \ln\Gamma[\exp(\xi)] - \ln\Gamma[\exp(\zeta)] \quad \text{(A.7)}$$

$$+\ln\Gamma[\exp(\xi)+\exp(\zeta)] - [\exp(\xi)+\exp(\zeta)]\ln[1+\exp(\epsilon_t)]$$

The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = u_{\mu,t} \times \{\Psi^{(1)}[\exp(\xi)] + \Psi^{(1)}[\exp(\zeta)]\}\exp(2\lambda_t) \quad \text{(A.8)}$$

$$u_{\mu,t} = \{\Psi^{(1)}[\exp(\xi)] + \Psi^{(1)}[\exp(\zeta)]\}\exp(\lambda_t)\left\{[\exp(\xi)+\exp(\zeta)]\frac{\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\xi)\right\} \quad \text{(A.9)}$$

where $\Psi^{(1)}(x)$ is the trigamma function. Furthermore, the score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \lambda_t} = [\exp(\xi)+\exp(\zeta)]\frac{\epsilon_t\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\xi)\epsilon_t - 1 \quad \text{(A.10)}$$

Fourthly, $\epsilon_t \sim \text{NIG}[0,1,\exp(\nu),\exp(\nu)\tanh(\eta)]$ (Barndorff-Nielsen and Halgreen 1977), where $\nu \in \mathbb{R}$ and $\eta \in \mathbb{R}$ influence tail-thickness and asymmetry, respectively. The log of the conditional density of p_t is

$$\ln f(p_t|p_1,\ldots,p_{t-1}) = \nu - \lambda_t - \ln(\pi) + \exp(\nu)[1-\tanh^2(\eta)]^{1/2} \tag{A.11}$$

$$+\exp(\nu)\tanh(\eta)\epsilon_t + \ln K^{(1)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right] - \frac{1}{2}\ln(1+\epsilon_t^2)$$

where $K^{(1)}(x)$ is the modified Bessel function of the second kind of order 1. The score function $u_{\mu,t}$ is given by

$$\frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} = -\exp(\nu-\lambda_t)\tanh(\eta) + \frac{\epsilon_t}{\exp(\lambda_t)(1+\epsilon_t^2)} \tag{A.12}$$

$$+\frac{\exp(\nu-\lambda_t)\epsilon_t}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}$$

$$u_{\mu,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \mu_t} \times \exp(2\lambda_t) \tag{A.13}$$

where $K^{(0)}(x)$ and $K^{(2)}(x)$ are the modified Bessel functions of the second kind of orders 0 and 2, respectively. The score function $u_{\lambda,t}$ is

$$u_{\lambda,t} = \frac{\partial \ln f(p_t|p_1,\ldots,p_{t-1})}{\partial \lambda_t} = -1 - \exp(\nu)\tanh(\eta)\epsilon_t + \frac{\epsilon_t^2}{1+\epsilon_t^2} \tag{A.14}$$

$$+\frac{\exp(\nu)\epsilon_t^2}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu)\sqrt{1+\epsilon_t^2}\right]}$$

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Table 1. Descriptive statistics.

Statistics	INR/USD p_t	INR/USD $\ln(p_t/p_{t-1})$
Start date	1st January 1982	1st January 1982
End date	7th July 2017	7th July 2017
Sample size	9078	9078
Minimum	9.0700	−0.0463
Maximum	68.8450	0.1281
Average	36.3468	0.0002
Standard deviation	16.7756	0.0043
Skewness	−0.1349	4.4755
Excess kurtosis	−0.9935	120.5426
ADF statistic, constant	−0.3777(0.9106)	$-65.4075^{***}(0.0001)$
ADF statistic, constant plus linear trend	−1.7007(0.7512)	NA
ADF statistic, constant plus quadratic trend	−1.7926(0.8854)	NA

Notes: Indian rupee (INR); United States dollar (USD); augmented Dickey–Fuller (ADF); not available (NA). p-values of the ADF test are reported in parentheses. *** indicates significance at the 1% level. Source of data: Bloomberg

Table 2. Parameter estimates and model diagnostics.

t-DCS		Skew-Gen-t-DCS		EGB2-DCS		NIG-DCS	
δ	10.1284***(0.2301)	δ	10.4577***(0.2851)	δ	0.8051***(0.0104)	δ	0.2922***(0.0094)
γ_{Jan}	0.0079(0.0505)	γ_{Jan}	0.0030(0.0519)	γ_{Jan}	0.0029(0.0042)	γ_{Jan}	0.0012(0.0015)
γ_{Feb}	0.0940**(0.0428)	γ_{Feb}	0.0949**(0.0437)	γ_{Feb}	0.0092**(0.0039)	γ_{Feb}	0.0033**(0.0014)
γ_{Mar}	0.0398*(0.0207)	γ_{Mar}	0.0376*(0.0210)	γ_{Mar}	0.0044***(0.0017)	γ_{Mar}	0.0015**(0.0006)
γ_{Apr}	−0.0869(0.0748)	γ_{Apr}	−0.0807(0.0757)	γ_{Apr}	−0.0037(0.0051)	γ_{Apr}	−0.0016(0.0019)
γ_{May}	0.2064***(0.0128)	γ_{May}	0.2148***(0.0134)	γ_{May}	0.0151***(0.0012)	γ_{May}	0.0055***(0.0004)
γ_{Jun}	0.0982***(0.0316)	γ_{Jun}	0.0855***(0.0308)	γ_{Jun}	0.0097***(0.0027)	γ_{Jun}	0.0034***(0.0009)
γ_{Jul}	−0.0164(0.0118)	γ_{Jul}	−0.0214*(0.0125)	γ_{Jul}	−0.0020*(0.0010)	γ_{Jul}	−0.0007**(0.0004)
γ_{Aug}	0.0971***(0.0181)	γ_{Aug}	0.0930***(0.0187)	γ_{Aug}	0.0075***(0.0014)	γ_{Aug}	0.0027***(0.0005)
γ_{Sep}	0.1207***(0.0273)	γ_{Sep}	0.1232***(0.0293)	γ_{Sep}	0.0105***(0.0023)	γ_{Sep}	0.0038***(0.0008)
γ_{Oct}	0.1389***(0.0468)	γ_{Oct}	0.1399***(0.0478)	γ_{Oct}	0.0116***(0.0035)	γ_{Oct}	0.0042***(0.0013)
γ_{Nov}	0.0751***(0.0247)	γ_{Nov}	0.0767***(0.0248)	γ_{Nov}	0.0072***(0.0020)	γ_{Nov}	0.0026***(0.0007)
γ_{Dec}	1.0327***(0.0558)	γ_{Dec}	1.0578***(0.0590)	γ_{Dec}	0.0878***(0.0046)	γ_{Dec}	0.0317***(0.0019)
ω	−0.0428***(0.0041)	ω	−0.0424***(0.0041)	ω	−0.0322***(0.0029)	ω	−0.0213***(0.0023)
β	0.9797***(0.0015)	β	0.9801***(0.0015)	β	0.9877***(0.0010)	β	0.9870***(0.0011)
α	0.1291***(0.0031)	α	0.1274***(0.0031)	α	0.0777***(0.0021)	α	0.0871***(0.0024)
λ_0	−3.7090***(0.7423)	λ_0	−3.7280***(0.6634)	λ_0	−4.0962***(0.4484)	λ_0	−3.1769***(0.4803)
ν	1.9243***(0.0299)	τ	0.0530***(0.0084)	ξ	0.4143***(0.0415)	ν	1.0217***(0.0331)
		ν	1.9818***(0.0356)	ζ	0.0998***(0.0359)	η	0.1611***(0.0108)
		η	0.6823***(0.0123)				
C_λ	0.6955	C_λ	0.6977	C_λ	0.7893	C_λ	0.7746
ADF$_1$	−89.5513***(0.0001)	ADF$_1$	−88.9554***(0.0001)	ADF$_1$	−85.1211***(0.0001)	ADF$_1$	−84.8098***(0.0001)
ADF$_2$	−91.1192***(0.0001)	ADF$_2$	−90.6116***(0.0001)	ADF$_2$	−94.6216***(0.0001)	ADF$_2$	−94.3849***(0.0001)
LL	1.0249	LL	**1.0268**	LL	1.0214	LL	**1.0232**
AIC	−2.0459	AIC	**−2.0493**	AIC	−2.0387	AIC	**−2.0421**
BIC	−2.0318	BIC	**−2.0336**	BIC	−2.0238	BIC	**−2.0272**
HQC	−2.0411	HQC	**−2.0439**	HQC	−2.0336	HQC	**−2.0371**

Notes: Dynamic conditional score (DCS); exponential generalized beta distribution of the second kind (EGB2); normal-inverse Gaussian distribution (NIG); augmented Dickey–Fuller (ADF); log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan-Quinn criterion (HQC). ADF$_1$ indicates the ADF test with constant results for $\partial u_{\lambda,t}/\partial\lambda_t$. ADF$_2$ indicates the ADF test with constant results for $(\partial u_{\lambda,t}/\partial\lambda_t)^2$ For EGARCH, $|\beta| < 1$ and $C_\lambda < 1$ are required for the consistency and asymptotic normality of the ML estimates. For the ADF test, p-values are reported in parentheses. Standard errors of parameters are reported in parentheses. We highlight with bold numbers the superior DCS specifications. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table 3. Linear regression of stochastic seasonality s_t on monthly dummies (OLS-HAC estimates).

	1st January 1982–31st December 1993	3rd January 1994–31st December 2003	1st January 2004–7th July 2017
κ_{Jan}	0.0246***(0.0015)	−0.0024(0.0016)	−0.0104***(0.0035)
κ_{Feb}	0.0206***(0.0007)	0.0062***(0.0020)	0.0118***(0.0034)
κ_{Mar}	0.0389***(0.0013)	0.0262***(0.0020)	0.0134***(0.0034)
κ_{Apr}	0.0670***(0.0020)	0.0302***(0.0023)	0.0350***(0.0034)
κ_{May}	−0.0566***(0.0028)	0.0015(0.0051)	0.1277***(0.0094)
κ_{Jun}	0.0049***(0.0012)	−0.0133***(0.0006)	0.0192***(0.0035)
κ_{Jul}	0.0338***(0.0020)	−0.0009(0.0017)	−0.0033(0.0026)
κ_{Aug}	−0.0159***(0.0008)	−0.0211***(0.0007)	0.0301***(0.0069)
κ_{Sep}	−0.0125***(0.0008)	−0.0100***(0.0015)	−0.0282***(0.0074)
κ_{Oct}	−0.0191***(0.0005)	−0.0279***(0.0012)	−0.0186***(0.0039)
κ_{Nov}	0.0000(0.0013)	−0.0130***(0.0008)	0.0166***(0.0027)
κ_{Dec}	−0.0732***(0.0078)	0.0241***(0.0056)	−0.2069***(0.0142)

Notes: Ordinary least squares (OLS); heteroscedasticity and autocorrelation consistent (HAC). Robust standard errors are reported in parentheses. *** indicates significance at the 1% level.

Table 4a. Commodity-wise exports of India (relative importance).

Exports (% points)	2010-11	2011-12	2011-12	2012-13
Commodity group	Jan to Dec	Jan to Dec	Apr to Nov	Apr to Nov
Agriculture and allied	**9.7355**	**12.3502**	**10.7279**	**13.9772**
Tea	0.2932	0.2794	0.2973	0.2719
Coffee	0.2631	0.3107	0.3023	0.3002
Cereals	1.3315	2.0980	1.6395	2.9641
Unmanufactured tobacco	0.2754	0.1986	0.1826	0.2380
Spices	0.7030	0.9057	0.9057	1.0841
Cashew nuts	0.2494	0.3049	0.3079	0.2647
Oil meals	0.9675	0.8081	0.6720	0.7502
Fruits, vegetables and pulses	0.5550	0.5198	0.5078	0.4945
Marine products	1.0415	1.1362	1.2325	1.2625
Raw cotton	1.1502	1.4814	0.9688	0.7270
Ores and minerals (excluding coal)	**3.4171**	**2.8055**	**2.6159**	**1.9794**
Iron ore	1.8717	1.5198	1.4048	0.6216
Processed minerals	0.8670	0.5983	0.5589	0.7156
Other ores and minerals	0.5833	0.6052	0.5757	0.5579
Manufactured goods	**68.9945**	**66.0664**	**66.9191**	**64.4770**
Leather and manufactures	0.9715	1.0100	1.0314	1.1162
Leather footwear	0.5857	0.5638	0.5682	0.5553
Gems and jewellerly	16.1178	14.7212	14.8776	15.3592
Drugs, pharmaceuticals and fine chemicals	4.2659	4.3538	4.1619	5.0282
Dyes and co-altar chemicals	1.2019	1.2742	1.2421	1.4500
Manufactures of metals	3.3672	3.1458	2.9361	3.5976
Machinery and instruments	4.7144	4.6984	4.5827	5.1883
Transport equipment	6.3906	6.9405	7.2832	6.2544
Primary and semi-finished iron and steel	1.5951	1.6979	1.6846	1.5824
Electronic goods	3.2947	3.0813	3.0608	2.9949
Cotton yarn, fabricates, made-ups etc.	2.3039	2.2342	2.2306	2.4510
Ready-made garments	4.6199	4.4949	4.3209	4.2869
Handicrafts	0.1023	0.0912	0.0902	0.0861
Crude and petroleum products (including coal)	**16.8362**	**18.7180**	**19.1733**	**18.8677**
Other and unclassified items	**0.9054**	**0.4869**	**0.5402**	**0.6940**
Total exports	**100.0000**	**100.0000**	**100.0000**	**100.0000**

Source: Open Government Data Platform India, https://data.gov.in

Table 4b. Commodity-wise imports of India (relative importance).

Imports (% points) Commodity group	2010-11 Jan to Dec	2011-12 Jan to Dec	2011-12 Apr to Nov	2012-13 Apr to Nov
Food and allied products	**2.9015**	**2.9562**	**3.1156**	**3.5014**
Cereals	0.0324	0.0150	0.0156	0.0168
Pulses	0.4244	0.3810	0.3883	0.4275
Cashew nuts	0.1573	0.2277	0.2954	0.2362
Edible oils	1.7724	1.9733	2.0600	2.4500
Fuel	**31.2787**	**35.2659**	**34.3407**	**38.0034**
Coal	2.6515	3.5662	3.7122	3.4985
Petroleum, oil and lubricants	28.6569	31.6663	30.6913	34.5554
Fertilizers	**1.8717**	**2.2743**	**2.3236**	**2.1822**
Paper board manufactures and newsprint	**0.5706**	**0.5249**	**0.5531**	**0.4988**
Capital goods	**13.7539**	**13.2941**	**12.6381**	**11.9069**
Machinery except electrical and machine tools	6.4494	6.1559	6.0676	5.7317
Electrical machinery	1.0393	0.9769	0.9753	0.9288
Transport equipment	3.0933	2.8784	2.5042	2.3138
Project goods	1.6618	1.8012	1.7038	1.5081
Others	**49.5505**	**45.7425**	**47.0674**	**43.8544**
Chemicals	5.2232	4.8587	4.9959	5.0913
Pearls, precious and semi-precious stones	9.3546	5.7278	6.0757	4.1421
Iron and steel	2.8062	2.4552	2.4485	2.2772
Non-ferrous metals	1.1035	0.9995	1.0419	1.0002
Gold and silver	11.5198	12.5833	13.0370	10.4552
Professional instruments, optical goods, etc.	1.1397	1.0727	1.0424	1.1068
Electronic goods	7.1833	6.6765	6.9708	6.4838
Total imports	**100.0000**	**100.0000**	**100.0000**	**100.0000**

Source: Open Government Data Platform India, https://data.gov.in

Table 5. Correlation coefficient of the seasonality component of INR/USD and the seasonality component of different export and import product groups for the period of 1st January 2004 to 7th July 2017 (source of data: Bloomberg).

Bloomberg ticker	Product group	Correlation coefficient
INMTEXIR Index	**Exports of India, merchandise exports including reexports**	**−0.3531**
INEXCOTT Index	Cotton yarn fabrics made up	−0.5825
INEXCARP Index	Carpets, ex-silk handmade	−0.4309
INEXWHEA Index	Wheat	−0.2815
INEXPRIM Index	Primary and semi-finished iron and steel	−0.2775
INEXBCHE Index	Cosmetics, toiletries	−0.2138
INEXCASH Index	Cashew	−0.1737
INEXIRST Index	Iron and steel bar rod etc.	−0.1338
INEXENGG Index	Machine tools	−0.0601
INEXELEC Index	Electronic goods	−0.0141
INEXIROR Index	Iron ore	0.1468
INEXPETR Index	Petroleum crude and products	0.2719
INEXHAND Index	Commodities handicrafts	0.3047
INEXGEMJ Index	Gems and jewellery	0.3143
INEXCOFF Index	Coffee	0.4519
INEXAGRI Index	Floriculture products	0.4565

Bloomberg ticker	Product group	Correlation coefficient
INMTIMIR Index	**Imports of India**	**0.0648**
INIMPETR Index	Petroleum, crude and products	0.4618
INIMCOAL Index	Coal coke briquettes	0.3817
INIMGOLD Index	Gold	0.3463
INIMPAPE Index	Paper boards and manufacturers	0.3272
INIMCHEP Index	Chemical materials and products	0.3081
INIMCHEM Index	Organic chemicals	0.3050
INIMMACH Index	Machinery, ex electrical and electronic	0.2972
INIMGOSI Index	Silver	0.2196
INIMRUBB Index	Synthetic and reclaimed rubber	0.1394
INIMPULP Index	Pulp and waste paper	0.0324
INIMELGO Index	Electronic goods	−0.0377
INIMCASH Index	Cashew nuts	−0.0413
INIMEDBL Index	Essential oil and cosmetic preparations	−0.0884
INIMBULK Index	Newsprint	−0.1161
INIMCERE Index	Cereals and cereal preparations	−0.1776
INIMFERT Index	Fertilizer manufactured	−0.1816
INIMIRST Index	Iron and steel	−0.2424
INIMCONS Index	Computer software in physical form	−0.2533
INIMCAPG Index	Cotton yarn and fabrics	−0.3414
INIMPULS Index	Pulses	−0.4463

Fig. 1(a). Real GDP per capita from 1981 to 2017 (source of data: The Conference Board Total Economy Database).

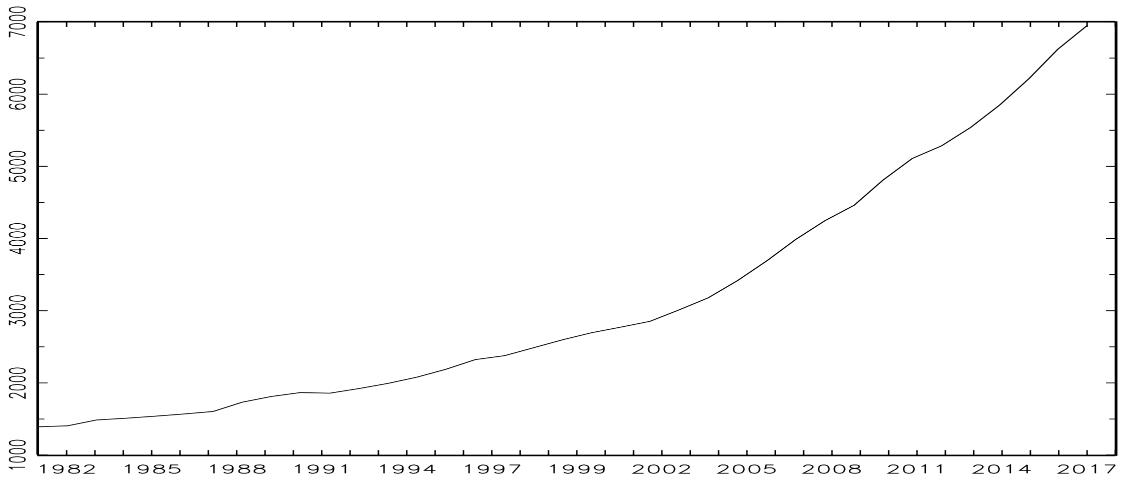

Fig. 1(b). Exports from India (thick) and imports to India (thin) from 1995 to 2017 (source of data: Bloomberg).

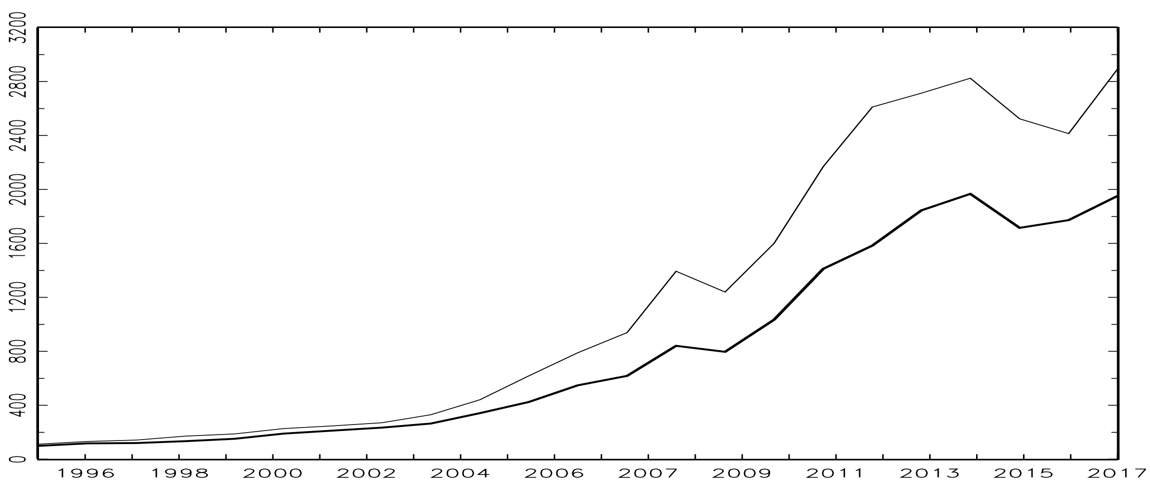

Fig. 1. Real GDP per capita, exports and imports of India. *Notes*: In Fig. 1(a), the real GDP per capita is in 2016 USD. In Fig. 1(b), we present the annual aggregates of the variables INMTEXIR Index INMTIMIR Index.

Fig. 2(a). p_t from 3rd January 1973 to 31st December 1981 (pre-sample period).

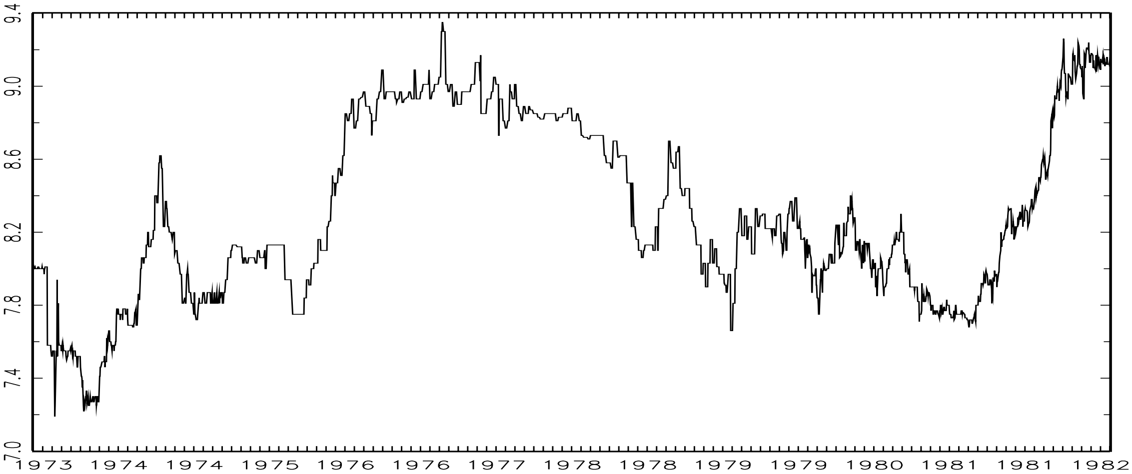

Fig. 2(b). p_t from 1st January 1982 to 7th July 2017 (sample period).

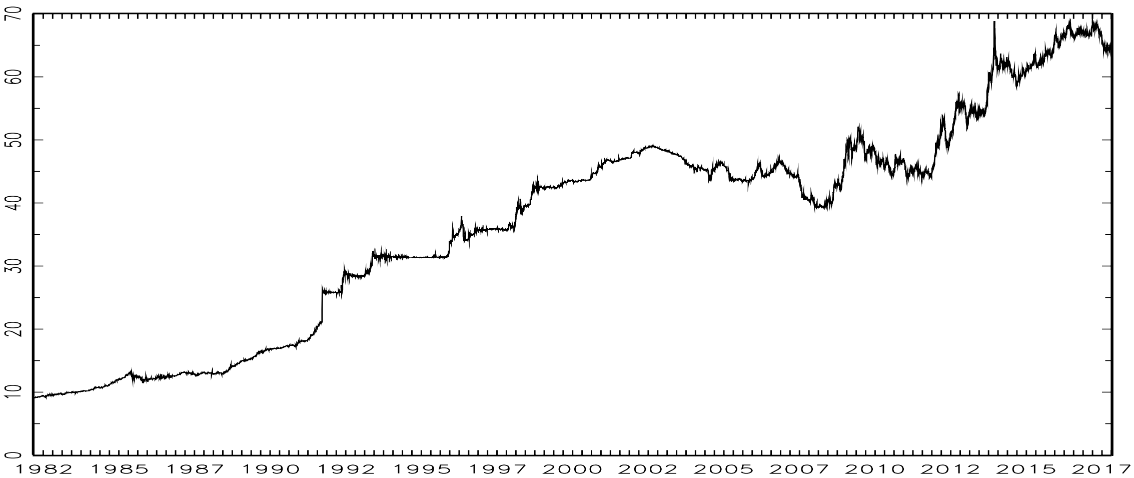

Fig. 2(c). $\ln(p_t/p_{t-1})$ from 1st January 1982 to 7th July 2017 (sample period).

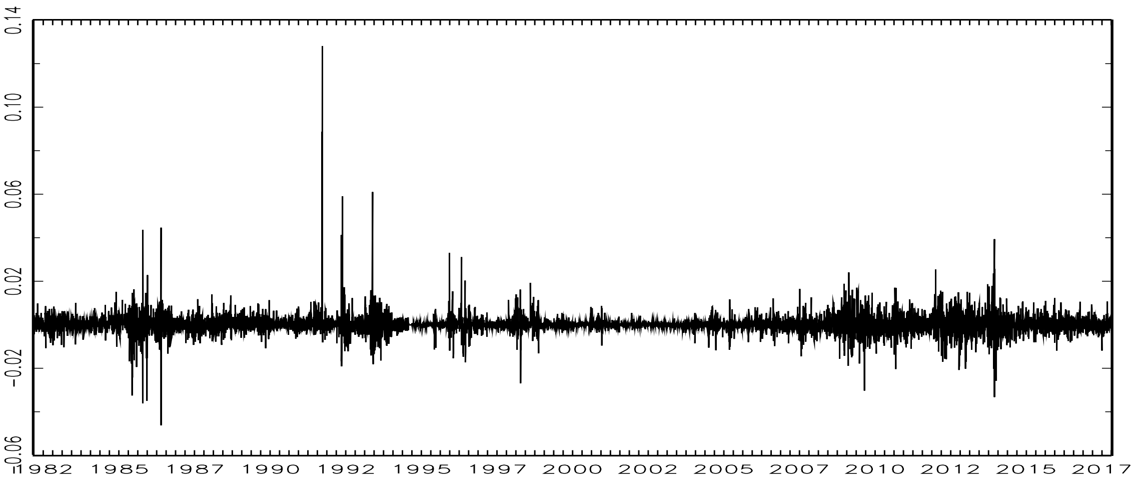

Fig. 2. Evolution of the INR/USD exchange rate for the period of 3rd January 1973 to 7th July 2017 (source: Bloomberg).

Fig. 3(a). $u_{\mu,t}$ for t-DCS (thin) and Skew-Gen-t-DCS (thick).

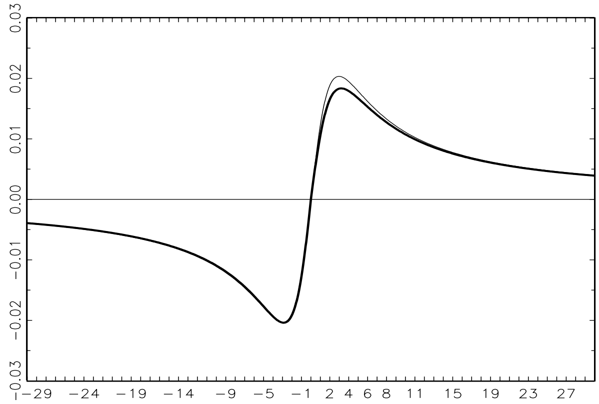

Fig. 3(b). $u_{\lambda,t}$ for t-DCS (thin) and Skew-Gen-t-DCS (thick).

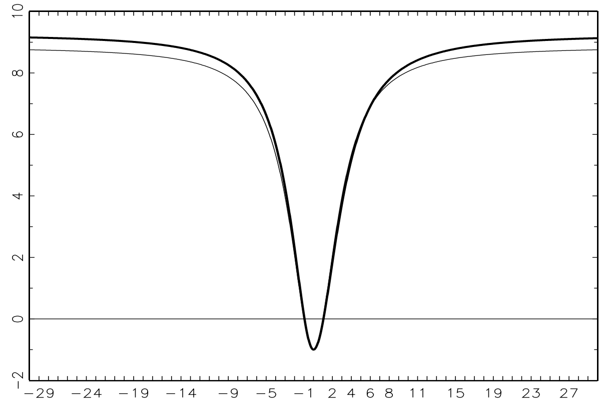

Fig. 3(c). $u_{\mu,t}$ for EGB2-DCS (thin) and NIG-DCS (thick).

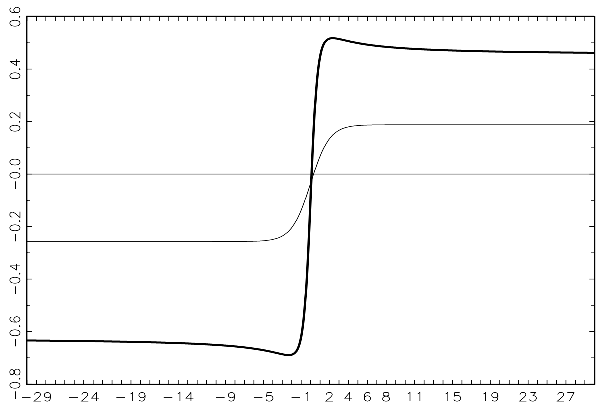

Fig. 3(d). $u_{\lambda,t}$ for EGB2-DCS (thin) and NIG-DCS (thick).

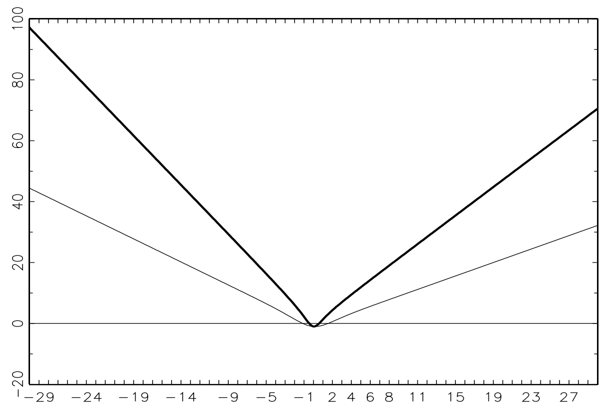

Fig. 3. Score functions of t-DCS, Skew-Gen-t-DCS, EGB2-DCS and NIG-DCS; estimated for the INR/USD time series
Notes: Score functions are presented as a function of the standardized error term ϵ_t.

Fig. 4(a). Annual seasonality s_t for t-DCS.

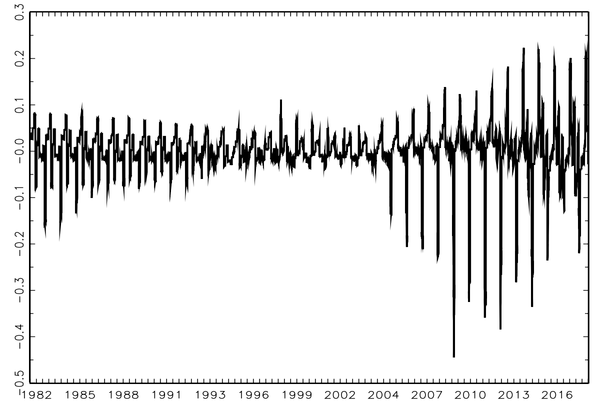

Fig. 4(b). Annual seasonality s_t for Skew-Gen-t-DCS.

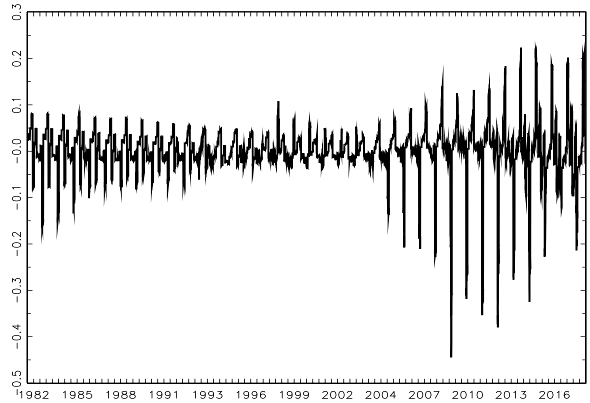

Fig. 4(c). Annual seasonality s_t for EGB2-DCS.

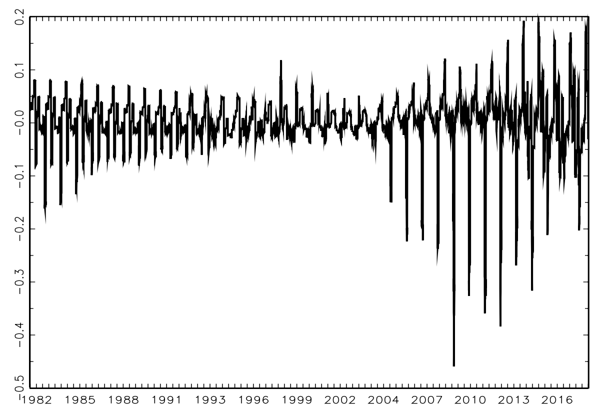

Fig. 4(d). Annual seasonality s_t for NIG-DCS.

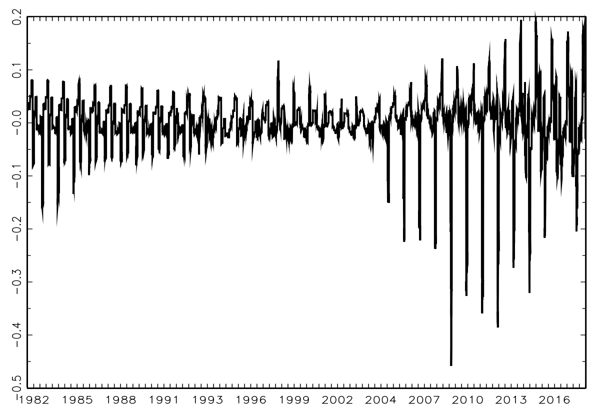

Fig. 4. INR/USD stochastic annual seasonality component s_t for the period of 1 January 1982 to 7 July 2017.

Fig. 5(a). Exports from India (INMTEXIR).

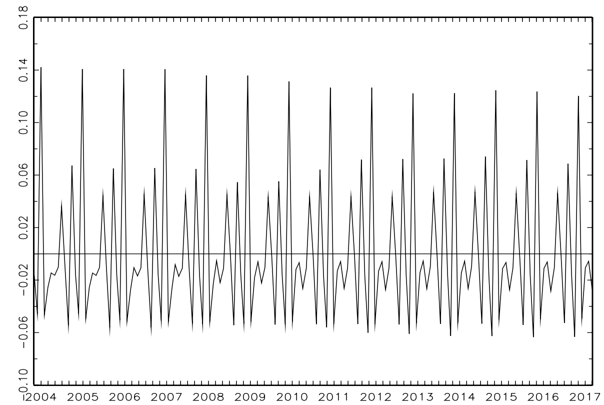

Fig. 5(b). Imports to India (INMTINIR).

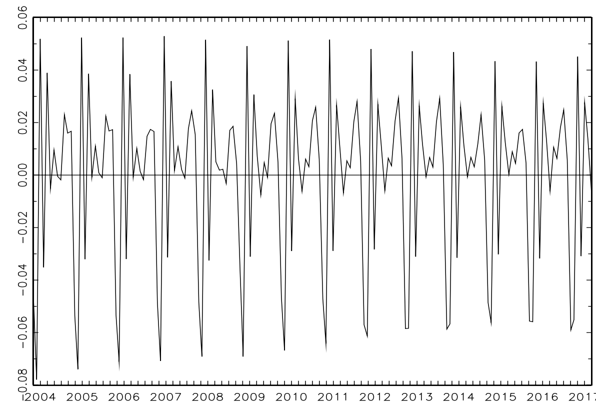

Fig. 5(c). Petroleum, crude and products imports (INIMPETR).

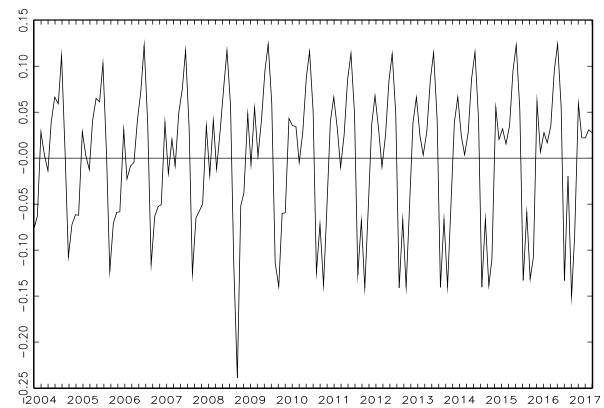

Fig. 5(d). Gold imports (INIMGOLD).

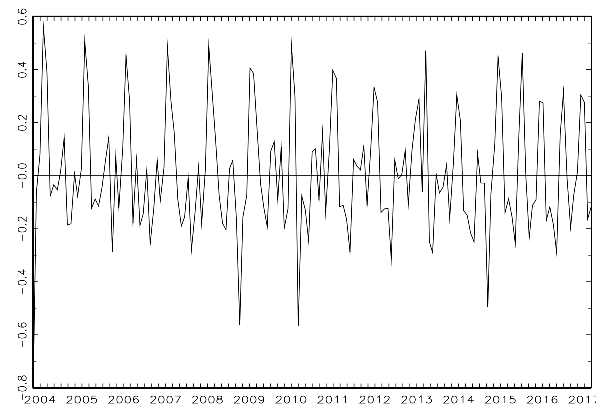

Fig. 5. Seasonality component s_t estimates of t-DCS with homoscedastic error for the period of January 2004 to July 2017.