Dynamic conditional score volatility models

Szabolcs Blazsek

GESG seminar – 30 January 2015

Universidad Francisco Marroquín, Guatemala

From GARCH(1,1) to Dynamic Conditional Score volatility models

GARCH(1,1)-normal (Bollerslev 1986; Taylor 1986)

- $y_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$ i.i.d.
- ${\sigma_t}^2 = \gamma + \beta(\sigma_{t-1})^2 + \alpha(y_{t-1})^2$

- An equivalent way to write this model is
- $y_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$ i.i.d.
- ${\sigma_t}^2 = \gamma + \varphi(\sigma_{t-1})^2 + \alpha(\sigma_{t-1})^2 u_{t-1}$ where $\varphi = \alpha + \beta$ and
- $u_t = ({y_t}^2/{\sigma_t}^2) - 1$
- $\{u_t\}$ is a martingale difference sequence

Martingale difference sequence
(Harvey 2013)

Definition:

- $\{u_t\}$ is a martingale difference sequence if for all t

a) $E(|u_t|) < \infty$

b) $E(u_t|u_1, \dots, u_{t-1}) = 0$

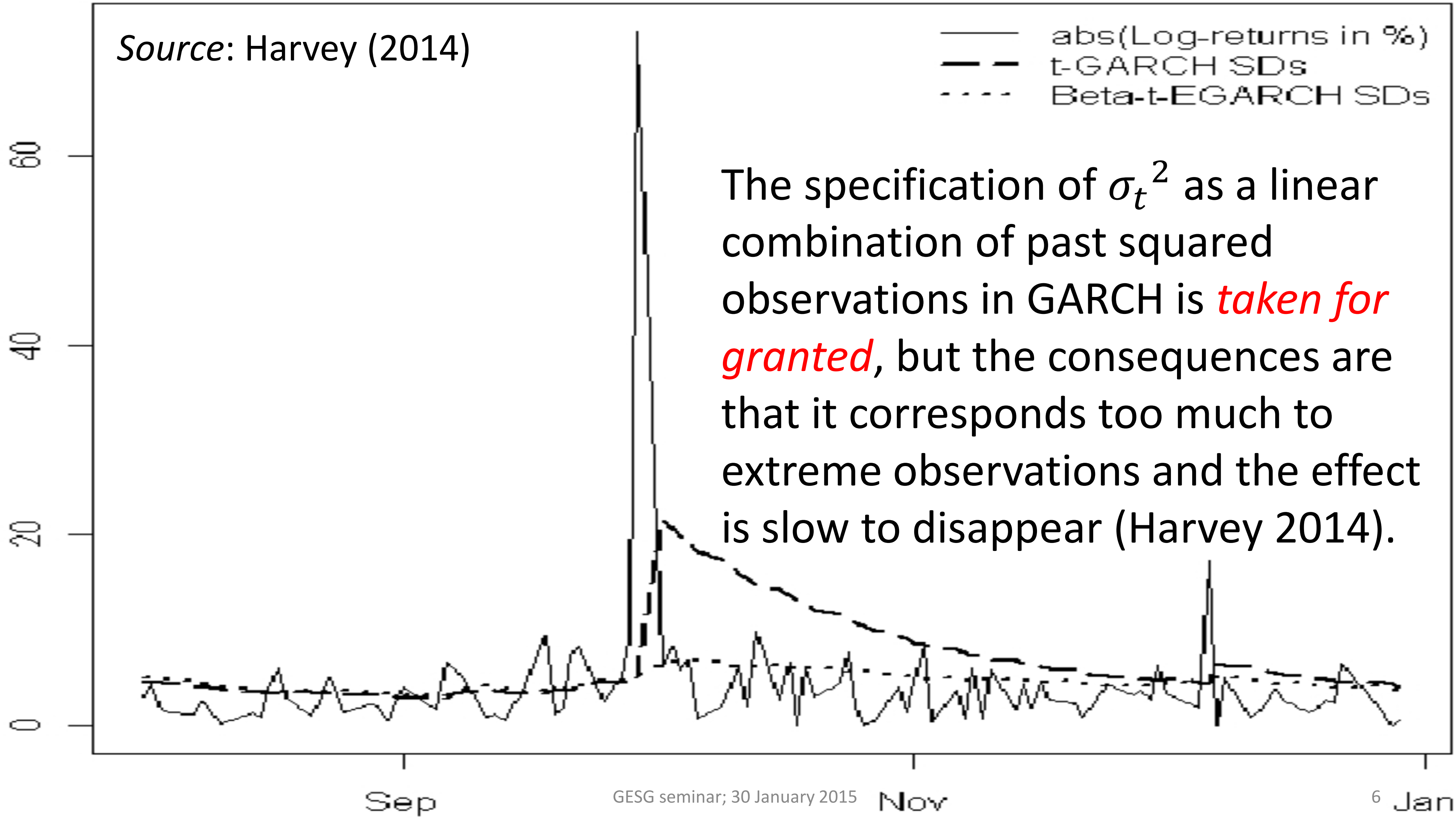
Consequences:

- $E(u_t) = 0$
- $E[u_t u_{t-j}] = 0$ for any j

GARCH(1,1)-t (Bollerslev 1987)

Motivation: Stock returns are known to be non-normal.

- Assume that ε_t has a Student $t(\nu)$ distribution.
- $y_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim t(\nu)$ i.i.d.
- ${\sigma_t}^2 = \gamma + \varphi(\sigma_{t-1})^2 + \alpha(\sigma_{t-1})^2 u_{t-1}$
- $u_t = ({y_t}^2 / {\sigma_t}^2) - 1$
- $\{u_t\}$ is a martingale difference sequence



Source: Harvey (2014)

The specification of ${\sigma_t}^2$ as a linear combination of past squared observations in GARCH is *taken for granted*, but the consequences are that it corresponds too much to extreme observations and the effect is slow to disappear (Harvey 2014).

Problem with GARCH(1,1)-t

- "The t distribution is employed in the predictive distribution of returns, however, it is not acknowledged in the design of the equation for ${\sigma_t}^2$" (Harvey 2014).
- Andrew Harvey's idea: Update ${\sigma_t}^2$ based on the conditional score of the time-varying scale parameter, σ_t.
- *Conditional score with respect to* ${\sigma_t}^2$: partial derivative of the log-density of y_t with respect to ${\sigma_t}^2$.
- The conditional score series, under some conditions, forms a martingale difference sequence (Harvey 2013, 2014).

Beta-t-GARCH(1,1) (Harvey 2013)

- Replace u_t in the ${\sigma_t}^2$ equation of GARCH(1,1)-t by another martingale difference sequence, as follows:
- $y_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim t(\nu)$ i.i.d.
- ${\sigma_t}^2 = \gamma + \varphi(\sigma_{t-1})^2 + \alpha(\sigma_{t-1})^2 u_{t-1}$
- $u_t = \frac{(\nu+1){y_t}^2}{(\nu-2){\sigma_t}^2 + {y_t}^2} - 1$
- u_t is proportional to the score of ${\sigma_t}^2$
- $\{u_t\}$ is a *martingale difference sequence*

Problem with Beta-*t*-GARCH(1,1)

Harvey (2013):

- The Beta-t-GARCH still suffers from some of the drawbacks of GARCH.
- Furthermore, the asymptotic distribution of the maximum likelihood estimates is not easy to derive.
- Harvey (2013) suggests an extension of the EGARCH model of Nelson (1991), the Beta-*t*-EGARCH, for which the asymptotic distribution of maximum likelihood estimates is known (Harvey 2013).
- Beta-*t*-EGARCH is an example of exponential DCS volatility models.

Exponential DCS volatility models

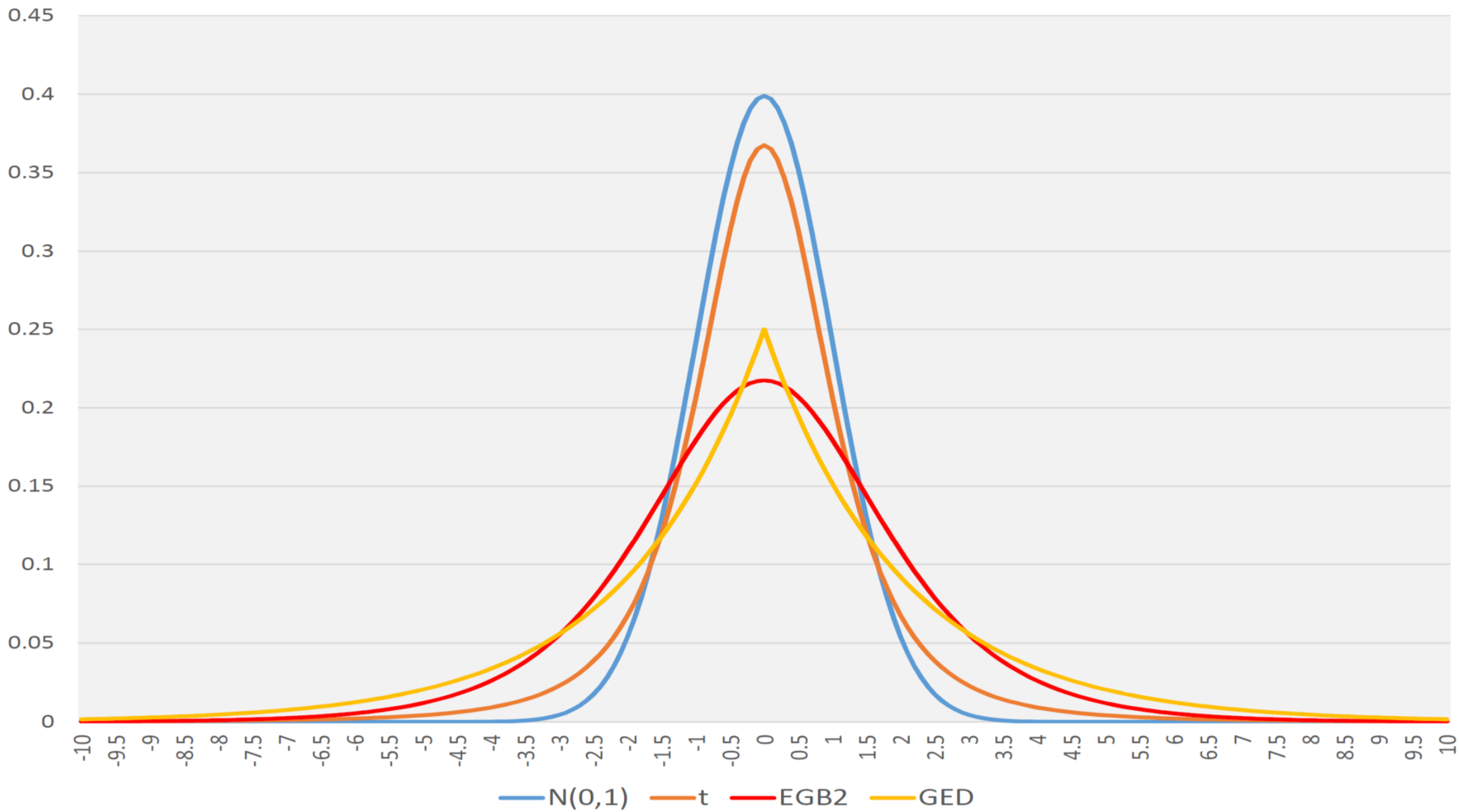
Exponential Dynamic Conditional Score (DCS) volatility models (Harvey 2013)

- Specification close to stochastic volatility models, for example,
- Harvey, Ruiz & Shephard (1994) and Harvey & Shephard (1996):
- $y_t = \exp(\lambda_t)\varepsilon_t$ with ε_t i.i.d.
- $\lambda_t = \gamma + \varphi\lambda_{t-1} + \kappa u_{t-1}$
- u_t is proportional to the score of λ_t
- $\{u_t\}$ is a *martingale difference sequence*
- Three choices for ε_t: a) $\varepsilon_t \sim t(\nu)$; b) $\varepsilon_t \sim$General Error Distribution(ν) (GED); c) $\varepsilon_t \sim$Exponential Generalized Beta distribution of the second kind(ξ, ζ) (EGB2)

Exponential Dynamic Conditional Score (DCS) volatility models (Harvey 2013)

Why are these alternatives?

- The choice is distribution for ε_t is a choice about how to weight extreme observations (outliers).
- If the probability mass around the tails of ε_t is higher then we will assign higher weight to outliers.
- Classification of distributions with respect to tails:
- Fat tailed → Light tailed → Heavy tailed
- (t) (EGB2) (GED)
- *See the density functions:*


0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-10
-9.5
-9
-8.5
-8
-7.5
-7
-6.5
-6
-5.5
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
N(0,1)
t
EGB2
GED

a) Beta-t-EGARCH(1,1) model (Harvey & Chakravarty 2008)

- $y_t = \exp(\lambda_t)\varepsilon_t$ with $\varepsilon_t \sim t(\nu)$ i.i.d.
- $\lambda_t = \gamma + \varphi\lambda_{t-1} + \kappa u_{t-1}$
- $b_t = \frac{{y_t}^2/\nu\exp(2\lambda_t)}{1+{y_t}^2/\nu\exp(2\lambda_t)}$
- $u_t = (\nu + 1)b_t - 1$
- u_t is proportional to the score of λ_t
- Impact of return on the score is symmetric.

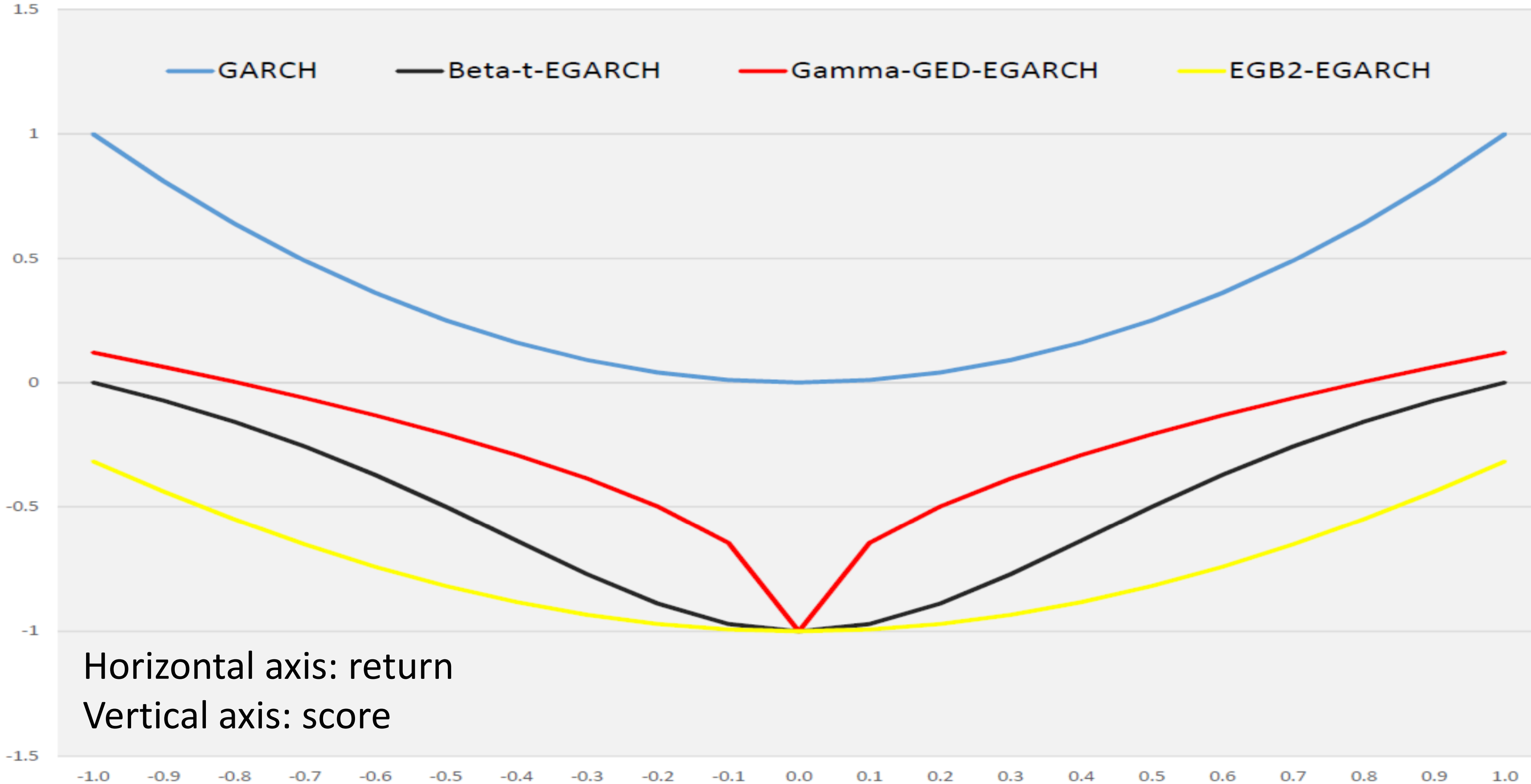
b) Gamma-GED-EGARCH(1,1) (Harvey 2013)

- $y_t = \exp(\lambda_t)\varepsilon_t$ with $\varepsilon_t \sim GED(\nu)$ i.i.d.
- $\lambda_t = \gamma + \varphi\lambda_{t-1} + \kappa u_{t-1}$
- $u_t = \frac{\nu}{2}|y_t \exp(-\lambda_t)|^{\nu} - 1$
- u_t is proportional to the score of λ_t
- Impact of return on the score is symmetric.

c) EGB2-EGARCH(1,1) (Caivano & Harvey 2014)

- $y_t = \exp(\lambda_t)\varepsilon_t$ with $\varepsilon_t \sim EGB2(\xi, \zeta)$ i.i.d.
- $\lambda_t = \gamma + \varphi\lambda_{t-1} + \kappa u_{t-1}$
- $b_t = \frac{\exp(y_t)}{1+\exp(y_t)}$
- $u_t = (\xi + \zeta) y_t \exp(-\lambda_t)\, b_t - \xi y_t \exp(-\lambda_t) - 1$
- u_t is proportional to the score of λ_t
- Impact of return on the score will be asymmetric if $\xi \neq \zeta$.

Impact of return on score



Dynamic conditional score models for location & scale

Quasi-ARMA(p, q) and Beta-t-EGARCH(1,1) (Harvey 2013)

- $y_t = \mu_t + \varepsilon_t$
- $\varepsilon_t = \exp(\lambda_t) e_t$ with $e_t \sim t(\nu)$ i.i.d.
- $\mu_t = \omega + \varphi_1 \mu_{t-1} + \cdots + \varphi_p \mu_{t-p} + \kappa_1 u_{\mu t-1} + \cdots + \kappa_q u_{\mu t-q}$
- $\lambda_t = \gamma + \varphi \lambda_{t-1} + \kappa u_{\lambda t-1}$
- $u_{\mu t} = \left[1 + \frac{{\varepsilon_t}^2}{\nu \exp(2\lambda_t)}\right]^{-1} \varepsilon_t$ (score of μ_t)
- $u_{\lambda t} = \frac{(\nu+1){\varepsilon_t}^2}{\nu \exp(2\lambda_t) + {\varepsilon_t}^2} - 1$ (score of λ_t)

Is Beta-*t*-EGARCH(1,1) superior to GARCH(1,1)?

Applied Economics, 2015

Szabolcs Blazsek & Marco Villatoro

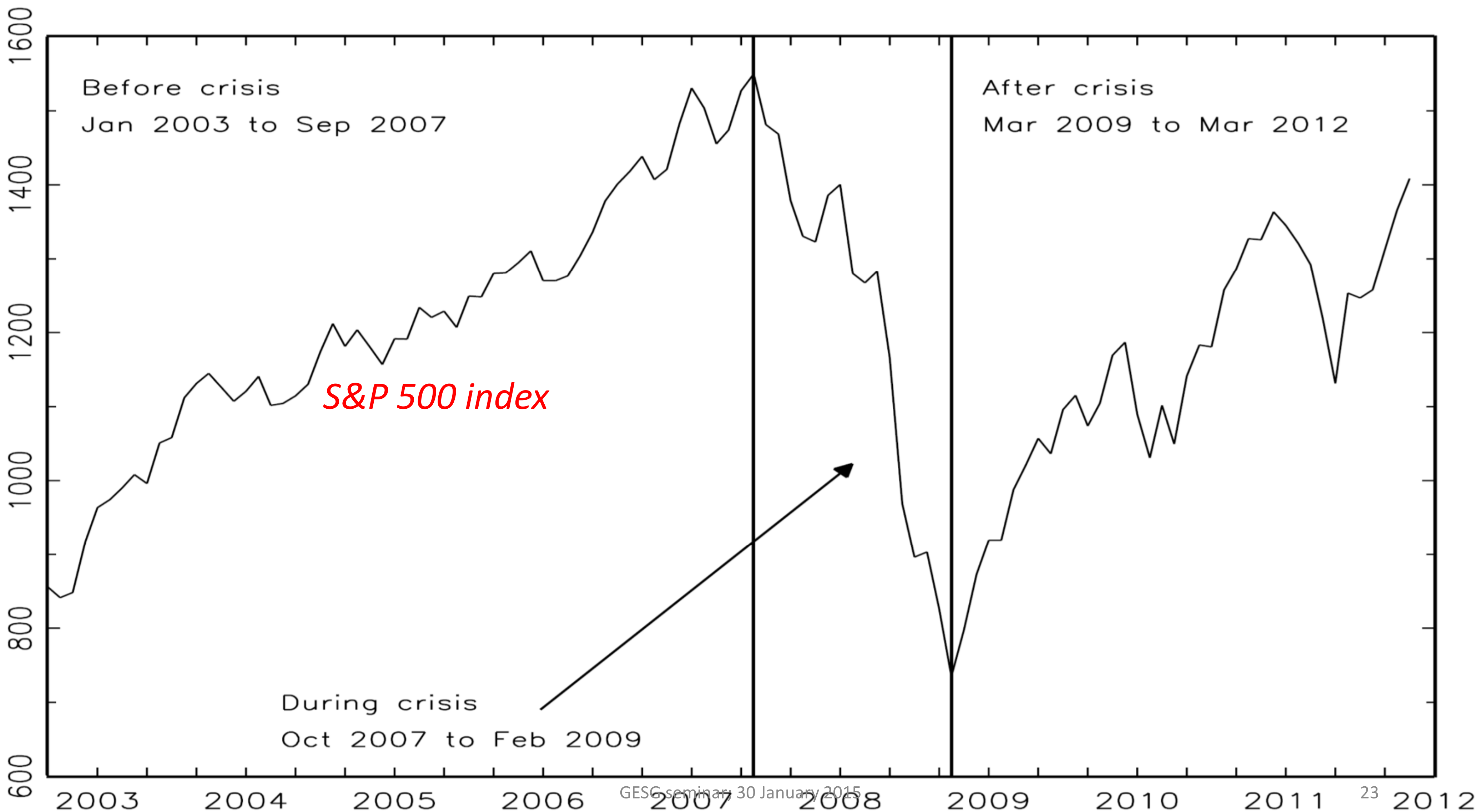
Universidad Francisco Marroquín, Guatemala

Motivation

- Hansen and Lunde (2005) compare the out-of-sample predictive performance of many ARCH-type models;
- GARCH(1,1)-normal, in many cases, is not outperformed by more sophisticated dynamic volatility models.
- This motivates our question:
- "Is Beta-*t*-EGARCH(1,1) superior to GARCH(1,1)?"

Summary of paper

- We compare statistical performance, in-sample point forecast precision and out-of-sample density forecast precision of *GARCH(1,1)-normal* and *Beta-t-EGARCH(1,1)*.
- We study the volatility of *nine global industry indices* for period April 2006 to July 2010; daily log returns.
- *Source*: Bloomberg
- We estimate competing models for *subperiods before, during and after the United States Financial Crisis of 2008*; $T = 396$ for each subperiod.


Before crisis
Jan 2003 to Sep 2007
After crisis
Mar 2009 to Mar 2012
S&P 500 index
During crisis
Oct 2007 to Feb 2009
1600
1400
1200
1000
800
600
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012

Summary of model specification

- *Location equation*:
  - AR(p)
  - MA(q)
  - ARMA(p, q)
  - Competing specifications with $p = 0,1,2$ and $q = 0,1, \ldots, 25$
  - Choose the most parsimonious formulation according to BIC
- *Scale equation*:
  - GARCH(1,1)-normal
  - Beta-t-EGARCH(1,1)

Summary of results

- *Statistical performance*: LL & BIC metrics. Beta-t-EGARCH(1,1) always has higher LL. Beta-t-EGARCH(1,1) does not always win according to BIC.
- *In-sample point forecast performance*: Diebold & Mariano (1996) test of volatility point forecast performance. Beta-t-EGARCH(1,1) has no clear in-sample superiority.
- *Out-of-sample density forecast performance*: Amisano & Giacomini (2007) test of density forecast performance. Mixed results. During crisis: GARCH(1,1) is clearly better; after crisis: Beta-t-EGARCH(1,1) is clearly better.

- Amisano & Giacomini (2007) Comparing density forecasts via weighted likelihood ratio tests, *Journal of Business and Economic Statistics*, 25, 177-190.
- Blazsek & Villatoro (2015) Is Beta-t-EGARCH(1,1) superior to GARCH(1,1)?, *Applied Economics*. http://dx.doi.org/10.1080/00036846.2014.1000536
- Bollerslev (1986) Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31, 307-327.
- Bollerslev (1987) A conditionally heteroskedastic time series model for security prices and rates of return data, *Review of Economics and Statistics*, 59, 542-547.
- Caivano & Harvey (2014) Time series models with an EGB2 conditional distribution, *Journal of Time Series Analysis*, 35, 558-571.
- Diebold & Mariano (1995) Comparing predictive accuracy, *Journal of Business and Economic Statistics*, 13, 253-263.

- Hansen & Lunde (2005) A forecast comparison of volatility models: does anything beat a GARCH(1,1)?, *Journal of Applied Econometrics*, 20, 873-889.
- Harvey (2013) *Dynamic Models for Volatility and Heavy Tails*, Cambridge Books, Cambridge University Press, Cambridge.
- Harvey (2014) Lecture notes, Dynamic Models for Volatility and Heavy Tails, Cass Business School, City University London, London, December 2014.
- Harvey, Ruiz & Shephard (1994) Multivariate stochastic variance models, *The Review of Economic Studies*, 61, 247-264.
- Harvey & Shephard (1996) Estimation of an asymmetric stochastic volatility model for asset returns, *Journal of Business & Economic Statistics*, 14, 429-434.
- Harvey & Chakravarty (2008) Beta-t-(E)GARCH, Cambridge Working Papers in Economics 0840, Faculty of Economics, University of Cambridge.
- Nelson (1991) Conditional heteroscedasticity in asset returns: a new approach, *Econometrica*, 59, 347-370.
- Taylor (1986) *Modelling Financial Time Series*, Wiley, Chichester.

Thank you for your attention!

sblazsek@ufm.edu