QARMA-Beta-t-EGARCH versus ARMA-GARCH

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Motivation

- Hansen and Lunde (2005):
  - GARCH(1,1) is not outperformed by more sophisticated ARCH-type models.
- QARMA-Beta-*t*-EGARCH(1,1) (Harvey and Chakravarty, 2008; Harvey, 2013; Harvey and Sucarrat, 2014)

- **Is QARMA-Beta-*t*-EGARCH(1,1) superior to ARMA-GARCH(1,1)?**

Data

- Daily stock price and the S&P 500 index data for period 17 May 2006 to 27 July 2010 (source: Bloomberg)
- We use daily log returns:
- $y_t = \ln(s_t/s_{t-1})$, where s_t is price or index level at the end of day t.
- We use a random sample of 50 stocks from the S&P 500.

Definition of subperiods

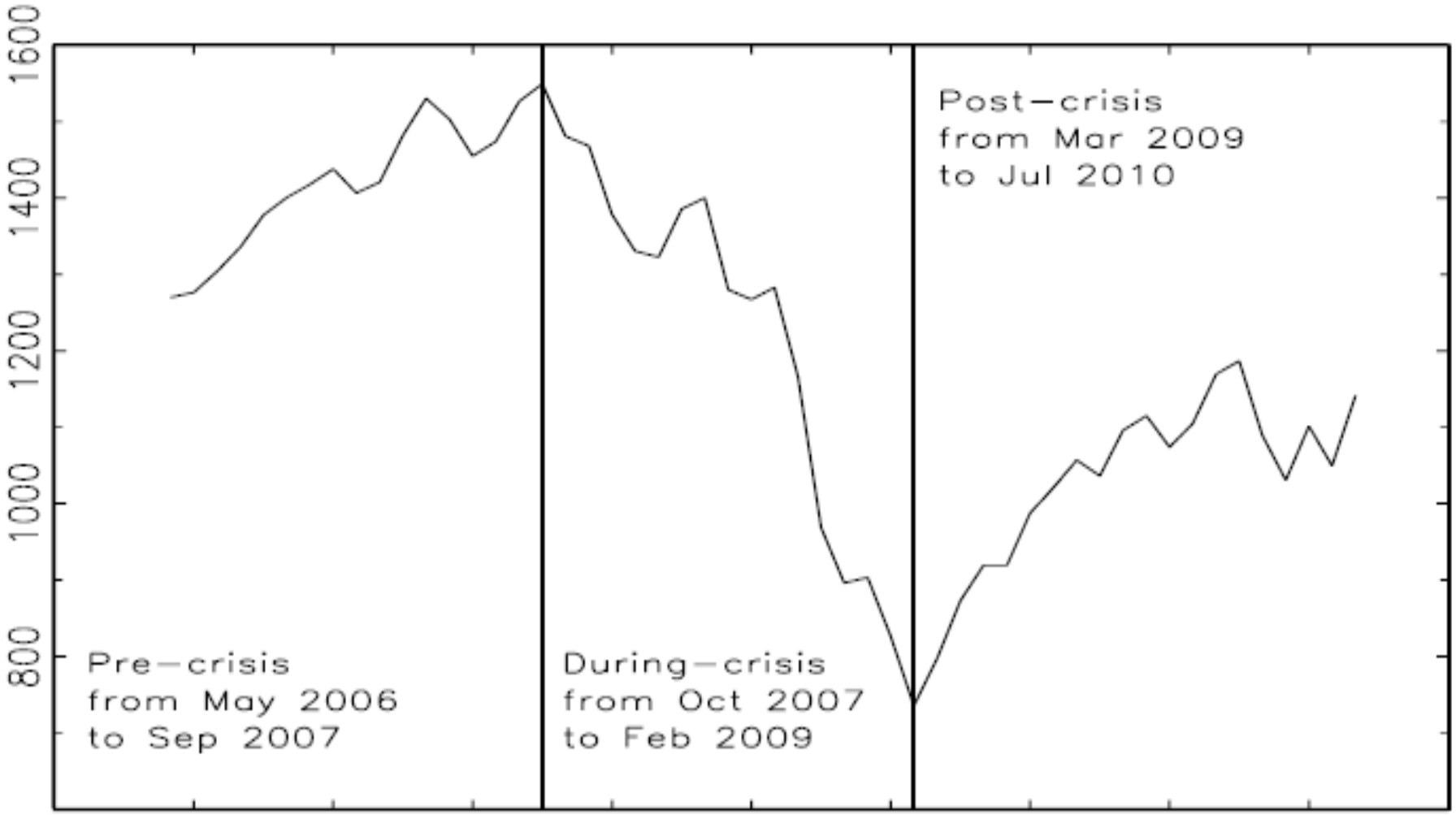

Fig. 1. S&P 500 index for period May 2006 to July 2010

Data

- An equal number of observations, T=352, is used in each subperiod in order to ensure that statistical estimates are comparable.

ARMA(p,q)-GARCH(1,1) (Box and Jenkins 1970; Taylor 1986; Bollerslev 1986)

$$y_t = \mu_t + v_t$$

$$v_t = \sigma_t \epsilon_t$$

$$\mu_t = \omega + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j v_{t-j}$$

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 v_{t-1}^2$$

- μ_t conditional expectation; σ_t conditional volatility; $\epsilon_t \sim N(0,1)$ with t=1,…,T error terms (motivated by Hansen and Lunde, 2005).

QARMA(p,q)-Beta-t-EGARCH (Harvey 2013)

$$y_t = \mu_t + v_t$$

$$v_t = \exp(\lambda_t)\epsilon_t$$

$$\mu_t = \omega + \sum_{i=1}^{p} \phi_i \mu_{t-i} + \sum_{j=1}^{q} \theta_j u_{\mu,t-j}$$

$$u_{\mu,t} = \left[1 + \frac{v_t^2}{\nu \exp(2\lambda_t)}\right]^{-1} v_t$$

$$\lambda_t = \alpha_0 + \beta_1 \lambda_{t-1} + \alpha_1 u_{\lambda,t-1}$$

$$u_{\lambda,t} = \frac{(\nu+1)v_t^2}{\nu \exp(2\lambda_t) + v_t^2} - 1$$

- $\epsilon_t \sim t(v)$; $u_{\mu,t}$ and $u_{\lambda,t}$ are conditional scores with respect to μ_t and λ_t

Specification of conditional location

- We compare:
  - constant conditional location $\mu_t = \omega$
  - autoregressive (AR) model of order *p*=1,2
  - moving average (*MA*) model of order *q*=1,…,5
  - ARMA model of orders *p*=1,2 and *q*=1,…,5
- by using the Log-Likelihood (LL) based model performance metric, **Bayesian Information Criterion**, BIC = *K*ln(*T*) – 2LL.
- **Lower BIC indicates more parsimonious model.**

In-Sample Statistical Performance

- Two likelihood-based model selection metrics:
  - Log-likelihood, LL
  - Bayesian Information Criterion, BIC
- Result: QARMA-Beta-*t*-EGARCH dominates in the pre-crisis and during-crisis periods.
- *See tables on LL and BIC.*

Out- of-Sample Density Forecast Performance

- We use the statistical test of Amisano and Giacomini (2007).
- $\ln f(\cdot\,|\Theta)$ and $\ln g\left(\cdot\,\middle|\widetilde{\Theta}\right)$ denote the log density function of QARMA-Beta-*t*-EGARCH and ARMA-GARCH, respectively.
- We estimate parameters for the 'estimation window': Pre-Crisis and During-Crisis.
- We compute log densities for the 'forecast evaluation window': During-Crisis and Post-Crisis.

Out- of-Sample Density Forecast Performance

- If $\frac{1}{T}\sum_{t=1}^{T} \ln f(y_t|\Theta) > \frac{1}{T}\sum_{t=1}^{T} \ln g\left(y_t\middle|\widetilde{\Theta}\right)$
- then QARMA-Beta-t-EGARCH will be superior density predictor to ARMA-GARCH.
- Result: QARMA-Beta-t-EGARCH is more effective in the during-crisis and post-crisis periods.
- *See table on density forecast evaluation.*

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Thank you for your attention!

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