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Estimation and statistical performance of Markov-switching score-driven volatility models: the case of G20 stock markets

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Keywords: Dynamic conditional score; Generalized autoregressive score; EGARCH; Markov regime switching; Maximum likelihood estimation; Statistical performance.

JEL classification codes: C22, C51, C52, C58

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1. Introduction

A growing literature following the seminal works of Creal et al. (2008), Harvey and Chakravarty (2008), Creal et al. (2013) and Harvey (2013) indicates that modelling asymmetric dependence between pairs of investment alternatives and tail-dependent distributions has increasingly becoming more relevant considering the current financial markets' context (Frahm et al., 2005; Bernardi and Catania, 2019). The model framework proposed by those authors is either called generalized autoregressive score (GAS) or dynamic conditional score (DCS) (hereinafter, the present paper uses the DCS abbreviation).

In the DCS framework, the model parameters are time-varying, driven by the score of the observation density. This type of models is capable of capturing distribution asymmetries with time series shifts/jumps/outliers detection. One of the advantages of the use of DCS models is that they provide optimal filters according to the Kullback–Leibler divergence measure (Blasques et al., 2015). Moreover, due to its conditional score dynamics, the maximum likelihood (ML) may be straightforwardly estimated. Further extensions to time series data idiosyncrasies such as long memory, asymmetry, and other more complex data dynamics may be relatively easily implemented as well. The DCS models have been applied to a wide variety of empirical studies exploring problems in finance and economics, such as the modeling of credit default swap (CDS) spread, stock market return volatility and correlation, credit rating/credit risk, and systemic risk/financial stability (Creal et al., 2011; Creal et al., 2013; Harvey and Lange, 2017; Babatunde, 2019).

As the DCS models were relatively recently introduced, from a theoretical vantage point, its ML estimation still consists of a developing and work-in-progress subject matter (Ardia et al., 2019). The capacity of financial time series models to effectively capture the impacts derived from extreme negative events greatly depends on the respective model framework flexibility and adaptability to asymmetric dependent and non-linear financial returns data. The present paper compares several score-driven Markov regime-switching (MS) volatility models and their single-regime versions, and the following three main contributions to the existing literature are provided:

Firstly, a new Meixner (MXN) probability distribution-based DCS volatility model, named MXN-DCS-EGARCH (exponential generalized autoregressive conditional heteroscedasticity) is proposed. The MXN probability distribution is introduced in the work of Schoutens (2002), in which it is applied to pricing financial derivatives by using MXN distribution-based stochastic volatility models. Further related works based on the MXN distribution, for example, are Grigoletto and Provasi (2008), Madan and Yor (2008), Bozejko and Demni (2010) and Kawai (2012). However, score-driven conditional volatility models for the MXN distribution have not been considered yet in the body of literature on time series models. Therefore, in this paper, the statistical performances of MXN-DCS-EGARCH and MS-MXN-DCS-EGARCH, both including leverage effects, are compared to the classical t-GARCH model with leverage effects (Bollerslev, 1987; Glosten et al., 1993) and to alternative DCS-EGARCH models with leverage effects that are found in the literature.

The aforementioned alternative score-driven models are Beta-t-EGARCH, proposed by Harvey and Chakravarty (2008); GED-EGARCH (general error distribution EGARCH) proposed by Harvey (2013); Beta-Skew-Gen-t-EGARCH (skewed generalized t-distribution EGARCH, henceforth only

Skew-Gen-t-EGARCH), proposed by Harvey and Sucarrat (2014); EGB2-EGARCH (exponential generalized beta distribution of the second kind EGARCH), proposed by Caivano and Harvey (2014); and NIG-EGARCH (normal-inverse Gaussian distribution EGARCH), proposed by Blazsek et al. (2018). For all alternative volatility models explored in the present paper, single-regime and MS versions are considered, statistical performances are compared, and model diagnostics are analyzed.

Secondly, new ML conditions for the MS-DCS-EGARCH model are introduced. Those conditions ensure the covariance stationarity of stock index log-returns, the covariance stationarity of the contributions to the gradient vector of the ML estimator, the covariance stationarity of the contributions to the information matrix of the ML estimator, and the invertibility of the score-driven MS-EGARCH model. For the first three conditions, arguments from the works of Abramson and Cohen (2007) and Harvey (2013) are extended. For the latter condition, the results from the work of Blasques et al. (2018) are applied to all DCS-EGARCH models that are explored in this paper. The ML estimation method that is used in the present paper is introduced in the work of Klaassen (2002) for the estimation of a non-path-dependent MS-GARCH model. We apply the Klaassen's method to the estimation of all MS volatility models. Nevertheless, there are other non-path-dependent (e.g. Gray, 1996; Haas et al., 2004) and path-dependent (e.g. Dueker, 1997; Bauwens et al., 2010; Henneke et al., 2011) estimation methods in the MS-GARCH literature that may be considered as alternatives to the present paper. In the literature on DCS models, the works of Blazsek and Ho (2017) and Blazsek et al. (2018) apply the Klaassen's method for MS-DCS-EGARCH models. Therefore, the present paper extends both the ML conditions and the MS-DCS-EGARCH specifications of those papers.

Thirdly, the empirical analyses involving MS-DCS-EGARCH models are performed on a large and international dataset of daily log-returns for 20 of the most relevant stock markets from the G20 economies, covering the maximum available historical data period for each stock market. Data from the following countries are analyzed: Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Mexico, Russia, Saudi Arabia, South Africa, South Korea, Spain, Turkey, United Kingdom (UK) and United States (US). In the context of MS-DCS-EGARCH models, the works of Blazsek and Ho (2017) and Blazsek et al. (2018) focus on US data. Nonetheless, the present paper provides the broadest international comparison of the statistical performances and model diagnostics of MS-DCS-EGARCH models in the existing literature on DCS models.

The main empirical results are as follows. All single-regime and MS volatility models are estimated for all stock markets in the sample. For Argentina, Brazil, China, India, Indonesia, Saudi Arabia, South Africa, South Korea and Turkey, the different regimes of the MS volatility models are not separated effectively. For those cases, the statistical performance of single-regime MXN-EGARCH is compared to single-regime alternatives. According to likelihood-based model performance metrics, MXN-EGARCH is superior to the classical t-GARCH model for the stock markets in China, Indonesia, Saudi Arabia and South Africa. However, according to the same metrics, Beta-t-EGARCH or Skew-Gen-t-EGARCH are superior to MXN-EGARCH for all stock markets. The model diagnostics statistic for the single-regime volatility models of this paper, which verifies the covariance stationarity of log-return, never suggests rejection of the asymptotic properties of the ML estimator.

For Australia, Canada, France, Germany, Italy, Japan, Mexico, Russia, Spain, UK and US, the separation of regimes for MS models is effective. With the exception of Russia, the MS-DCS-EGARCH specifications are superior to MS-t-GARCH for all stock markets. For each best-performing MS-DCS-EGARCH specification, the smoothed probabilities of the 'high-volatility regime' are presented with the estimates of the conditional volatility series. According to likelihood-based model performance metrics, the novel MS-MXN-EGARCH specification is superior to MS-t-GARCH for the stock markets in Australia, Canada, Italy, Japan and US. However, according to likelihood-based model performance, the best MS-DCS-EGARCH specifications are MS-Beta-t-EGARCH, MS-Skew-Gen-t-EGARCH and MS-NIG-EGARCH. The model diagnostics that verify the covariance stationarity of log-returns, the covariance stationarity of the contributions to the gradient vector, the covariance stationarity of the contributions to the information matrix, and invertibility for those MS specifications for which the separation of regimes is effective, never suggest rejection of the asymptotic properties of ML.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the dataset. Section 4 presents all MS-DCS-EGARCH specifications, including MS-MXN-EGARCH. Section 5 presents the statistical inferences. Section 6 summarizes the empirical results. Finally, Section 7 concludes and suggests future research opportunities.

2. Literature review

The state of the existing literature is divided into three subsections. Firstly, it is reviewed the literature of DCS models in general. Secondly, models that combine the DCS framework with EGARCH models are explored. Thirdly, the contributions to the literature by adding the MS structure to DCS and DCS-EGARCH models are discussed as well.

2.1. DCS models

Time series models based on parameters that dynamically vary through time are commonly grouped in the existing literature into two model categories, consisting of the observation-driven and the parameter-driven models (Cox et al., 1981; Creal et al., 2013). The observation-driven models have been developed to exploit large changes, also known as shifts or jumps, and distributional asymmetries frequently present in financial time series, such as stock market returns. Such model category includes the ARCH model, originally introduced by Engle (1982), and its subsequent relevant extension, the GARCH model proposed by Bollerslev (1986), which have been widely applied to empirical studies (e.g. Engle and Sheppard, 2001; Bauwens et al., 2006).

However, motivated by difficulties of such established models in estimating stochastic volatility by not properly capturing conditional distribution properties of the input data as well as lack of robustness in treating outlier effects, more recently and two and half decades after the seminal contributions of Engle (1982) and Bollerslev (1986), a class of score-driven volatility models, named DCS or GAS, was introduced by Harvey and Chakravarty (2008) and Creal et al. (2008), respectively.

This third and new class of score-driven models is, in fact, a modification of the GARCH model, proposed with the purpose of capturing time large changes/shifts/jumps and mitigate negative impact of outliers. As in the DCS model it is assumed that the innovations follow a non-normal distribution and its second central moment is modelled through a GARCH-type equation based on the conditional score of the assumed distribution regarding the variance, then such a distinct approach elevates DCS models to a distinctive model category compared to volatility models previously introduced in the literature (Harvey, 2013; Ardia et al., 2019). Due to its peculiar framework characteristics, more specifically by scaling the score function in an appropriate manner, the DCS models are able to capture properties of existing observation-driven models, such as the GARCH model (Engle, 1982; Bollerslev, 1986), autoregressive conditional duration (ACD) model of Engle and Russell (1998), the autoregressive conditional intensity (ACI) model of Russell (1999), the dynamic conditional correlation (DCC) model of Engle (2002), the GARMA models of Benjamin et al. (2003), autoregressive conditional multinomial (ACM) model of Russell and Engle (2005), and the dynamic copula models of Patton (2006), among others well-known related models (Creal et al., 2013; Blasques et al., 2014).

Subsequently, the DCS model framework was further developed into a number of model extensions, such as the observation-driven mixed measurement dynamic factor models proposed by Creal et al. (2014), the dynamic models for location, volatility and multivariate dependence for fat-tailed densities introduced by Creal et al. (2011), the exponential DCS proposed by Harvey and Chakravarty (2008) and Harvey (2013), and the asymmetric exponential DCS introduced by Creal et al. (2013), among other extensions (Harvey and Luati, 2014; Babatunde et al., 2019).

The practical usefulness of DCS models is tested and demonstrated through several studies on different topics. For example, the DCS modelling framework is applied to market risk forecasting (Harvey and Sucarrat, 2014), systematic risk forecasting (Oh and Patton, 2013; Cerrato et al., 2017; Eckernkemper, 2017; Bernardi and Catania, 2019), credit risk analysis (Creal et al., 2014), dependence modelling (Harvey and Thiele, 2016; Janus et al., 2014; Opschoor et al., 2018), spatial econometrics (Blasques et al., 2014; Catania and Billé, 2017), CDS spread modelling (Lange et al., 2017; Oh and Patton, 2018), high-frequency data modelling (Gorgi et al., 2018; Opschoor and Lucas, 2019), among other empirical applications (e.g. Ardia et al., 2019; Lazar and Xue, 2019; Patton et al., 2019).

2.2. DCS-EGARCH models

The EGARCH model was introduced by Nelson (1991), being proposed to model the natural logarithm of the conditional variance. The original EGARCH proposition attempts to accommodate the asymmetric relation between stock returns and volatility changes. One the main motivations behind its proposition is to treat the stylized fact known as 'leverage effect' (Black, 1976; Pierre, 1998; Figlewski and Wang, 2000).

Almost two decades after the seminal contribution of Nelson (1991) and respective model extensions, a further EGARCH model improvement is introduced by Harvey and Chakravarty (2008), Creal et al. (2008), Creal et al. (2011) and Harvey (2013), named Beta-t-EGARCH model. Such model consists of an unrestricted version of the DCS model as in the framework introduced by Creal et al. (2013). In the Beta-t-EGARCH, the second central moment is modelled by an equation that depends on the conditional score of the most recent observation. A direct consequence of the application of the conditional score in the equation of the dynamic volatility is that data points considered as outliers under a Gaussian distribution vantage point, have a relatively lower impact than the remaining observations. Therefore, the Beta-t-EGARCH consists of a robust volatility model (Harvey and Sucarrat, 2014). One of the additional advantages of the Beta-t-EGARCH in comparison with competing established models is that through an exponential function, stationarity conditions as well as positive scale may be reached in a relatively simple and straightforward manner. Moreover, it is possible to derive volatility forecasts expressions, being their respective conditional variances properly calculated, and simulating straightforwardly the related conditional distribution. Finally, general analytic expressions of the serial correlation function of squared observations may be appropriately calculated as well. As a consequence of such framework characteristics, the Beta-t-EGARCH frequently demonstrates a superior performance compared to a number of competing GARCH-type models in empirical applications involving financial return time series (Harvey, 2013; Sucarrat, 2013).

In addition to the Beta-t-EGARCH model, Caivano and Harvey (2014) propose another member of the EGARCH class of models, in which a DCS model is based on the EGB2 distribution. In such model, the signal is a linear function of past values of the score of the conditional distribution. This model is complementary to the model version under the Student's t-distribution explored by Creal et al. (2011) and Harvey (2013). The proposed model is then modelled to macroeconomic data, which empirical results demonstrating that the exponential generalized beta distribution of the second kind may provide a superior fit when applied to certain macroeconomic time series - e.g. exchange rates (Caivano and Harvey, 2014).

More recently, a further EGARCH-type model which dynamic equation for the natural logarithm of its scale is based on the conditional score following a generalized Student's t-distribution was introduced by Harvey and Lange (2017). Such a model configuration is able to encompass asymmetry and/or skewness present in the input data, and the expression regarding the respective information matrix is derived by the authors. The model usefulness is tested through empirical analyses on stock market and commodity return series, which results potentially provide a flexible volatility modeling framework that is robust to outliers as well (Harvey and Lange, 2017).

2.3. MS-DCS models

Since the influential study of Hamilton (1989), MS models have been extensively applied to economic data. Depending on the sample period and data characteristics, the dependence series structure may suffer breaks, which are more frequent and noticeable in turbulent times, such as during financial crisis and/or financial contagion episodes (e.g. Global Financial Crisis of 2007-2008). In such cases, MS models are considered as an effective approach to properly capture abrupt changes in the volatility and correlations temporal progress (Creal et al., 2013; Bazzi et al., 2017; Bernardi and Catania, 2019). A regime switching approach is introduced by Boudt et al. (2012) in order to model and forecast the volatility and correlation of financial time series. The proposed model exploits the state variables to predict the switching probabilities, being the state transition dynamics dependent upon the score of the density function. Through the application of this model to data of deposit bank holding companies in the US from 1994 to 2011, it is found evidence of temporal variation of the state switching probabilities as well as the within-regime volatility of most financial institutions in the sample, whereas the equicorrelation within-state dynamics apparently remained unchanged through the sample period.

A MS augmented Dickey–Fuller (MS-ADF) unit root test, to identify covariance stationarity and unit root subsample periods, is introduced by Holmes (2008). That study is then extended to the DCS literature by Ayala et al. (2016), who propose a Monte Carlo simulation-based MS unit root test, for which the data generating process is score-driven. Those authors use a MS-DCS model with unit root for conditional location under the null hypothesis, and a MS autoregressive moving average (MS-ARMA) model under the alternative hypothesis. Score-driven volatility dynamics are used both under the null and alternative hypotheses. The unit root test is applied to the real effective exchange rates (REERs) of several Latin America economies to test purchasing power theory.

Subsequently, Bazzi et al. (2017) propose a MS observation-driven model with state/regime transition probabilities that vary through time, being the temporal varying probability innovation produced by the score of the likelihood function. The authors estimate the time-varying probabilities conditions for stationarity and ergodicity convergence of the respective model. Based on empirical findings, the authors argue that their model may be applied as a benchmark to regime/state switching models which transition probabilities change through time.

Blazsek and Ho (2017) introduce MS-Beta-t-EGARCH to the DCS literature. More recently, Blazsek et al. (2018) introduce alternative MS-DCS-EGARCH models, including MS-NIG-EGARCH. Those authors show that the statistical performances of the proposed models are superior to that of MS-Beta-t-EGARCH. Moreover, an empirical application is also provided in the work of Blazsek et al. (2018), in which value-at-rik (VaR) and expected shortfall (ES) are studied for the US stock market.

3. Data and summary statistics

The empirical analyses in the present paper are based on daily G20 stock market returns, totalling 181,383 data points. As the empirical analyses are performed on each individual G20 stock market return series p_t (i.e. univariate analysis), then the sample period varies according to data availability, being preferred the longest time period available for each stock market. Overall, the sample covers a period at least longer than 20 years for every stock market. The end date is October 25, 2019 for all countries except for Saudi Arabia, which stock market does not operate on Fridays and then it ends on the last Thursday of the sample period (i.e. October 24, 2019). The start date varies from January 3, 1928 (in the case of the US) to January 2, 1998 (in the case of Italy).

More specifically, the input dataset consists of the following 20 stock markets return series based on closing positions of daily price of each the G20 stock market benchmark index p_t: MERVAL Index (Argentina), S&P/ASX 300 (Australia), Ibovespa (Brazil), S&P/TSX Composite Index (Canada), Shanghai Stock Exchange Composite Index (China), CAC 40 (France), DAX (Germany), S&P BSE SENSEX Index (India), Jakarta Stock Exchange Composite Index (Indonesia), FTSE MIB Index (Italy), Nikkei 225 (Japan), S&P/BMV IPC (Mexico), MOEX Russia Index (Russia), Tadawul All Share Index (Saudi Arabia), FTSE/JSE Africa All Share Index (South Africa), Korea Stock Exchange KOSPI Index (South Korea), IBEX 35 Index (Spain), Borsa Istanbul 100 Index (Turkey), FTSE 100 Index (UK) and S&P 500 Index (US). All data are from Bloomberg.

In addition, in terms of data pre-processing, the raw stock market input dataset was prepared in order to transform the stock market indices p_t into daily log-returns $y_t = \ln(p_t/p_{t-1})$. Further detailed information of the input dataset and related log returns summary statistics are reported in Table 1.

Following stylized facts reported in the literature, most of the series has mean around zero, negative skewness and excess kurtosis (i.e. fat-tailedness). Therefore, according to the normality test outputs reported in Table 1, there is statistical evidence that none of the series should follow a Gaussian distribution. Moreover, the results of the correlation between the squared returns in time t and its respective previous non-squared return in time $t-1$ indicate that they are most frequently negatively correlated, being the correlation coefficient usually very low (i.e. around -0.06 on average).

4. Model specification and properties

In this section, the model specifications as well as respective data distributions are formulated for all MS-DCS-EGARCH models of this paper. The model specification and statistical inference of the non-path-dependent MS-t-GARCH model are not presented in the present paper, but we refer to the works of Bollerslev (1987), Glosten et al. (1993), Klaassen (2002) and Abramson and Cohen (2007).

4.1. MS models

In the empirical part of this paper, MS volatility models with two regimes are used, where the stochastic process of $s_t \in \{1, 2\}$ for $t = 1, \ldots, T$, is defined by using the following transition probability matrix:

$$P = \left[\begin{array}{cc} \Pr(s_t = 1|s_{t-1} = 1) & \Pr(s_t = 2|s_{t-1} = 1) \\ \Pr(s_t = 1|s_{t-1} = 2) & \Pr(s_t = 2|s_{t-1} = 2) \end{array} \right] = \left(\begin{array}{cc} p & 1-p \\ 1-q & q \end{array} \right) \tag{1}$$

where p and q are the transition probability parameters. The two-regime MS volatility models of this paper can be extended to multi-regime MS volatility models in a straightforward manner. The Markov chain $(s_1, \ldots, s_T)$ is strictly stationary with time-invariant probabilities $\pi^*(1) = \Pr(s_t = 1) = (1-q)/(2-p-q)$ and $\pi^*(2) = 1 - \pi^*(1)$. In the statistical inference, the following filtered probabilities are also used: $\pi_t(1) = \Pr(s_t = 1|y_1, \ldots, y_{t-1})$ as well as $\pi_t(2) = \Pr(s_t = 2|y_1, \ldots, y_{t-1})$, and $\tilde{\pi}_t(1) = \Pr(s_t = 1|y_1, \ldots, y_t)$ as well as $\tilde{\pi}_t(2) = \Pr(s_t = 2|y_1, \ldots, y_t)$. These filtered probabilities are computed recursively for $t = 1, \ldots, T$, as follows:

$$\pi_t(1) = p\tilde{\pi}_{t-1}(1) + (1-q)\tilde{\pi}_{t-1}(2) \tag{2}$$

$$\tilde{\pi}_t(1) = \frac{f(y_t|y_1, \ldots, y_{t-1}, s_t = 1; \Theta)\pi_t(1)}{f(y_t|y_1, \ldots, y_{t-1}, s_t = 1; \Theta)\pi_t(1) + f(y_t|y_1, \ldots, y_{t-1}, s_t = 2; \Theta)\pi_t(2)} \tag{3}$$

where $\pi_t(2) = 1 - \pi_t(1)$, $\tilde{\pi}_t(2) = 1 - \tilde{\pi}_t(1)$, $\tilde{\pi}_0(1) = \pi^*(1)$ and $\tilde{\pi}_0(2) = \pi^*(2)$ are used for initialization. Given the ML parameter estimates of the MS score-driven volatility model, statistical inferences on s_t are also made by using the following smoothed probabilities of regimes $\Pr(s_t|y_1, \ldots, y_T)$:

$$\Pr(s_t = j|y_1, \ldots, y_T) = \sum_{k=1,2} \Pr(s_t = j, s_{t+1} = k|y_1, \ldots, y_T) \tag{4}$$

for $t = 1, \ldots, T$, where:

$$\Pr(s_t = j, s_{t+1} = k | y_1, \ldots, y_T) \simeq \frac{\Pr(s_{t+1} = k | y_1, \ldots, y_T) \Pr(s_{t+1} = k | s_t = j) \tilde{\pi}_t(j)}{\pi_{t+1}(k)} \tag{5}$$

for $j = 1, 2$ and $k = 1, 2$ (Kim and Nelson, 1999). The smoothed probabilities are computed recursively for $t = T, \ldots, 1$. The recursion is started at $t = T$, by using the ML estimates of p, q, $\pi_T(k)$ and $\tilde{\pi}_T(k)$.

4.2. MS-EGARCH model

The general form of all MS-EGARCH models, for the daily log-return of a financial asset, is:

$$y_t = c(s_t) + \exp[\lambda_t(s_t)]\epsilon_t(s_t) \tag{6}$$

for days $t = 1, \ldots, T$, where $y_t = \ln(p_t/p_{t-1})$; for p_0 pre-sample data are used. Parameter $c(s_t)$ is a regime-dependent constant parameter, and $\epsilon_t(s_t)$ is the regime-dependent error term for which the regime dependence is due to the regime-switching shape parameters. The log of the regime-dependent scaling parameter is specified as:

$$\begin{aligned} \lambda_t(s_t) &= \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + \alpha(s_t)u_{t-1}(s_t) + \alpha^*(s_t)\text{sgn}[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t) + 1] \\ &= \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + g_{t-1}(s_t) \end{aligned} \tag{7}$$

for days $t = 2, \ldots, T$; for day $t = 1$ parameters $\lambda_1(1)$ and $\lambda_1(2)$ are used for initialization. The latter equation refers to a score-driven MS-EGARCH model with leverage effects, in which the following variables are used: $\lambda_{t-1}(s_t) \equiv E[\lambda_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t]$, $u_{t-1}(s_t) \equiv E[u_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t]$, and $\epsilon_{t-1}(s_t) \equiv E[\epsilon_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t]$. These expectations are computed with respect to s_{t-1}, by conditioning on $(y_1, \ldots, y_{t-1}, s_t)$ (Klaassen, 2002); further details are presented in Appendix A. An advantage of the use of these expectations in EGARCH models is that in this way path-dependence on regimes is avoided, hence the statistical inference of MS score-driven volatility models can be performed as in classical MS models (e.g. Hamilton, 1989; Kim and Nelson, 1999). In the second expectation $u_t(s_t)$ is used, which is the regime-dependent score function of the log-likelihood (LL) with respect to $\lambda_t(s_t)$. More formally, it is defined as: $u_t(s_t) = \partial \ln f(y_t|y_1, \ldots, y_{t-1}, s_t)/\partial \lambda_t(s_t)$, where the log-conditional density of y_t is specified in the following sections for alternative probability distributions. In addition, the regime-dependent conditional means $\mu_t(s_t)$ and conditional standard deviations (i.e. conditional volatility) $\sigma_t(s_t)$ of those probability distributions are detailed in Appendix B.

4.2.1. Skew-Gen-t distribution and its special cases

The error term follows the Skew-Gen-t distribution, as shown below:

$$\epsilon_t(s_t) \sim \text{Skew-Gen-}t\{0, 1, \tanh[\delta_1(s_t)], \exp[\delta_2(s_t)] + 2, \exp[\delta_3(s_t)]\} \tag{8}$$

where $\tanh(x)$ is the hyperbolic tangent function. Furthermore, $\delta_1(s_t)$, $\delta_2(s_t)$ and $\delta_3(s_t)$ with $s_t = 1, 2$ are regime-dependent shape parameters that influence asymmetry, tail-heaviness and peakedness of $\epsilon_t(s_t)$, respectively. The degrees of freedom parameter $\{\exp[\delta_1(s_t)] + 2\}$ is greater than two, hence, the conditional variance of y_t is finite. The asymmetry parameter is given by $\tanh(\tau_t) \in (-1, 1)$, as required for Skew-Gen-t. The log conditional density of y_t is detailed below:

$$\ln f(y_t|y_1,\ldots,y_{t-1},s_t) = \delta_3(s_t) - \lambda_t(s_t) - \ln(2) - \frac{\ln\{\exp[\delta_2(s_t)\}+2]}{\exp[\delta_3(s_t)]} \tag{9}$$

$$-\ln\Gamma\left\{\frac{\exp[\delta_2(s_t)]+2}{\exp[\delta_3(s_t)]}\right\} - \ln\Gamma\{\exp[-\delta_3(s_t)]\} + \ln\Gamma\left\{\frac{\exp[\delta_2(s_t)]+3}{\exp[\delta_3(s_t)]}\right\}$$

$$-\frac{\exp[\delta_2(s_t)]+3}{\exp[\delta_3(s_t)]}\ln\left\{1+\frac{|\epsilon_t(s_t)|^{\exp[\delta_3(s_t)]}}{\{1+\tanh[\delta_1(s_t)]\mathrm{sgn}[\epsilon_t(s_t)]\}^{\exp[\delta_3(s_t)]}\times\{\exp[\delta_2(s_t)]+2\}}\right\}$$

The regime-dependent score function is given by:

$$u_t(s_t) = \frac{|\epsilon_t(s_t)|^{\exp[\delta_3(s_t)]}\{\exp[\delta_2(s_t)]+3\}}{|\epsilon_t(s_t)|^{\exp[\delta_3(s_t)]}+\{1+\tanh[\delta_1(s_t)]\mathrm{sgn}[\epsilon_t(s_t)]\}^{\exp[\delta_3(s_t)]}\{\exp[\delta_2(s_t)]+2\}} - 1 \tag{10}$$

The definition of the updating term shown in Equation (10) provides the framework of MS-Skew-Gen-t-EGARCH. The following two special cases of MS-Skew-Gen-t-EGARCH are also estimated: (a) MS-Beta-t-EGARCH for the Student's t-distribution, where $\tanh[\delta_1(s_t)] = 0$ and $\exp[\delta_3(s_t)] = 2$; (b) MS-GED-EGARCH for GED, where $\tanh[\delta_1(s_t)] = 0$ and $\exp[\delta_2(s_t)+2] \to \infty$.

Each score function is a different nonlinear transformation of $\epsilon_t(s_t)$. In Figure 1(a)-(c) those transformations are presented, by using the estimates of single-regime Beta-t-EGARCH, GED-EGARCH and Skew-Gen-t-EGARCH, respectively. The score functions of Figure 1 are estimated by using stock index data from the US. The corresponding non-linear transformations are presented for the interval $\epsilon_t \in [-50, 50]$ to illustrate the asymptotic properties of those transformations. According to Figure 1(a) and 1(c), Beta-t-EGARCH uses symmetric asymptotic Winsorizing and Skew-Gen-t-EGARCH uses asymmetric asymptotic Winsorizing, respectively, as $|\epsilon_t| \to \infty$. According to Figure 1(b), GED-EGARCH uses an increasing symmetric transformation of ϵ_t as $|\epsilon_t| \to \infty$.

4.2.2. EGB2 distribution

The error term follows the EGB2 distribution, as shown below:

$$\epsilon_t(s_t) \sim \mathrm{EGB2}\{0, 1, \exp[\delta_1(s_t)], \exp[\delta_2(s_t)]\} \tag{11}$$

where $\delta_1(s_t)$ and $\delta_2(s_t)$ with $s_t = 1, 2$ are shape parameters. Parameters $\delta_1(s_t)$ and $\delta_2(s_t)$ influence both asymmetry and tail-heaviness of $\epsilon_t(s_t)$. The log conditional density of y_t is detailed below:

$$\ln f(y_t|y_1,\ldots,y_{t-1},s_t) = \exp[\delta_1(s_t)]\epsilon_t(s_t) - \lambda_t(s_t) - \ln\Gamma\{\exp[\delta_1(s_t)]\} \tag{12}$$

$$-\ln\Gamma\{\exp[\delta_2(s_t)]\} + \ln\Gamma\{\exp[\delta_1(s_t)]+\exp[\delta_2(s_t)]\}$$

$$-\{\exp[\delta_1(s_t)]+\exp[\delta_2(s_t)]\}\ln\{1+\exp[\epsilon_t(s_t)]\}$$

The regime-dependent score function is given by:

$$u_t(s_t) = \{\exp[\delta_1(s_t)] + \exp[\delta_2(s_t)]\}\frac{\epsilon_t(s_t)\exp[\epsilon_t(s_t)]}{\exp[\epsilon_t(s_t)]+1} - \exp[\delta_1(s_t)]\epsilon_t(s_t) - 1 \tag{13}$$

The definition of the updating term shown in Equation (13) provides the framework of MS-EGB2-EGARCH. In Figure 1(d), the non-linear transformation of ϵ_t is presented for US data. EGB2-EGARCH uses an approximately linearly increasing transformation of ϵ_t that is asymmetric around zero as $|\epsilon_t| \to \infty$.

4.2.3. NIG distribution

The error term follows the NIG distribution, as shown below:

$$\epsilon_t(s_t) \sim \text{NIG}\{0, 1, \exp[\delta_1(s_t)], \exp[\delta_1(s_t)]\tanh[\delta_2(s_t)]\} \tag{14}$$

where $\delta_1(s_t)$ and $\delta_2(s_t)$ with $s_t = 1, 2$ are regime-dependent shape parameters. Parameters $\delta_1(s_t)$ and $\delta_2(s_t)$ influence tail-heaviness and asymmetry of $\epsilon_t(s_t)$; the asymmetry parameter is given by $\exp[\delta_1(s_t)]\tanh[\delta_2(s_t)]$. The log conditional density of y_t is detailed below:

$$\ln f(y_t|y_1,\ldots,y_{t-1},s_t) = \delta_1(s_t) - \lambda_t(s_t) - \ln(\pi) + \exp[\delta_1(s_t)]\{1-\tanh^2[\delta_2(s_t)]\}^{1/2} \tag{15}$$

$$+\exp[\delta_1(s_t)]\tanh[\delta_2(s_t)]\epsilon_t(s_t) + \ln K^{(1)}\left\{\exp[\delta_1(s_t)]\sqrt{1+\epsilon_t^2(s_t)}\right\} - \frac{1}{2}\ln[1+\epsilon_t^2(s_t)]$$

where $K^{(1)}(x)$ is the modified Bessel function of the second kind of order 1. The regime-dependent score function is given by:

$$u_t(s_t) = -1 - \exp[\delta_1(s_t)]\tanh[\delta_2(s_t)]\epsilon_t(s_t) + \frac{\epsilon_t^2(s_t)}{1+\epsilon_t^2(s_t)} \tag{16}$$

$$+\frac{\exp[\delta_1(s_t)]\epsilon_t^2(s_t)}{\sqrt{1+\epsilon_t^2(s_t)}} \times \frac{K^{(0)}\left\{\exp[\delta_1(s_t)]\sqrt{1+\epsilon_t^2(s_t)}\right\} + K^{(2)}\left\{\exp[\delta_1(s_t)]\sqrt{1+\epsilon_t^2(s_t)}\right\}}{2K^{(1)}\left\{\exp[\delta_1(s_t)]\sqrt{1+\epsilon_t^2(s_t)}\right\}}$$

where $K^{(j)}(x)$ is the modified Bessel function of the second kind of order j. Such definition of the updating term provides the framework of MS-NIG-EGARCH. In Figure 1(e), the non-linear transformation of ϵ_t is presented for US data. NIG-EGARCH uses an approximately linearly increasing transformation of ϵ_t that is asymmetric around zero as $|\epsilon_t| \to \infty$.

4.2.4. MXN distribution

The error term follows the MXN distribution, as shown below:

$$\epsilon_t(s_t) \sim \text{MXN}\{0, 1, \pi\tanh[\delta_1(s_t)], \exp[\delta_2(s_t)]\} \tag{17}$$

where $\delta_1(s_t)$ and $\delta_2(s_t)$ with $s_t = 1, 2$ are regime-dependent shape parameters, which influence tail-heaviness and asymmetry of $\epsilon_t(s_t)$. The log conditional density of y_t is detailed below:

$$\ln f(y_t|y_1,\ldots,y_{t-1},s_t) = -\lambda_t(s_t) + 2\exp[\delta_2(s_t)]\ln\{2\cos\{\pi\tanh[\delta_1(s_t)]/2\}\} - \ln(2\pi) \tag{18}$$

$$-\ln\Gamma\{2\exp[\delta_2(s_t)]\} + \pi\tanh[\delta_1(s_t)]\epsilon_t(s_t) + 2\ln|\Gamma\{\exp[\delta_2(s_t)] + i\epsilon_t(s_t)\}|$$

where $\cos(x)$ is the cosine function and i is the imaginary unit. The following notation is introduced: $g[\lambda_t(s_t)] = \Gamma\{\exp[\delta_2(s_t)] + i[y_t - c(s_t)]\exp[-\lambda(s_t)]\}$, being parameter $\lambda_t(s_t)$ a real number. Therefore, $\partial \ln|g[\lambda_t(s_t)]|/\partial\lambda_t(s_t) = \mathrm{Re}\{g'[\lambda_t(s_t)]/g[\lambda_t(s_t)]\}$, where $\mathrm{Re}(z)$ is the real part of complex number z. Considering the fact that $\Gamma'(x) = \Gamma(x)\Psi^{(0)}(x)$, where $\Psi^{(0)}(x)$ is the digamma function, the regime-dependent score function is given by:

$$u_t(s_t) = 2\mathrm{Re}\left\{-i\epsilon_t(s_t)\Psi^{(0)}[\exp[\delta_2(s_t)] + i\epsilon_t(s_t)]\right\} - \pi\tanh[\delta_1(s_t)]\epsilon_t(s_t) - 1 \tag{19}$$

The definition above of the updating term provides the framework of MS-MXN-EGARCH. In Figure 1(e), the non-linear transformation of ϵ_t is presented for US data. MXN-EGARCH uses an approximately linearly increasing transformation of ϵ_t that is asymmetric around zero as $|\epsilon_t| \to \infty$.

5. Statistical inference

Score-driven MS-EGARCH models are estimated by using the ML method, as follows:

$$\hat{\Theta}_{\mathrm{ML}} = \arg\max_{\Theta} \mathrm{LL}(y_1,\ldots,y_T;\Theta) = \arg\max_{\Theta}\sum_{t=1}^{T}\ln\left[\sum_{i=1}^{2}\pi_t(i) f(y_t|y_1,\ldots,y_{t-1},s_t = i;\Theta)\right] \tag{20}$$

Estimations are performed by using alternative start values of parameters, in order to find a global maximum. In the numerical maximization of the LL function, the convergence tolerance for gradient is 10^{-5} for all the parameters. The inverse information matrix is used for the estimation of the standard errors of $\hat{\Theta}_{\mathrm{ML}}$ (Harvey, 2013; Creal et al., 2011, 2013). The standard errors of transformed parameters are computed by using the delta method (e.g. Davidson and MacKinnon, 2004).

The following conditions ensure the covariance stationarity of $\lambda_t(s_t)$ (Condition 1), the covariance stationarity of the contributions to the gradient vector (Conditions 1 to 3), the covariance stationarity of the contributions to the information matrix (Conditions 1 to 4), and the invertibility of the score-driven MS-EGARCH model (Condition 5):

Condition 1 is the covariance stationarity of $\lambda_t(s_t)$, which is expressed as:

$$\begin{aligned}
\lambda_t(s_t) &= \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_t) + g_{t-1}(s_t) \\
&= \omega(s_t) + \beta(s_t)E[\lambda_{t-1}(s_{t-1})|y_1,\ldots,y_{t-1},s_t] + E[g_{t-1}(s_{t-1})|y_1,\ldots,y_{t-1},s_t] \\
&= \omega(s_t) + \beta(s_t)\lambda_{t-1}(s_{t-1}=1)\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t) \\
&\quad +\beta(s_t)\lambda_{t-1}(s_{t-1}=2)\Pr(s_{t-1}=2|y_1,\ldots,y_{t-1},s_t) \\
&\quad +g_{t-1}(s_{t-1}=1)\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t) \\
&\quad +g_{t-1}(s_{t-1}=2)\Pr(s_{t-1}=2|y_1,\ldots,y_{t-1},s_t)
\end{aligned} \tag{21}$$

To show the condition of covariance stationarity of $\lambda_t(s_t)$, the conditional mean $E[\lambda_t(s_t)|y_1,\ldots,y_{t-1},s_t]$

is evaluated for both sides of Equation (21), from which we consider the following terms:

$$
\begin{aligned}
&E[\lambda_{t-1}(s_{t-1})\Pr(s_{t-1}|y_1,\ldots,y_{t-1},s_t)|y_1,\ldots,y_{t-1},s_t] \\
&= \int_{\{y_1,\ldots,y_{t-1}\}} \lambda_{t-1}(s_{t-1})\Pr(s_{t-1}|y_1,\ldots,y_{t-1},s_t)h(y_1,\ldots,y_{t-1}|s_t)d(y_1,\ldots,y_{t-1}) \\
&= \int_{\{y_1,\ldots,y_{t-1}\}} \lambda_{t-1}(s_{t-1})\Pr(s_{t-1}|s_t)h(y_1,\ldots,y_{t-1}|s_{t-1},s_t)d(y_1,\ldots,y_{t-1}) \\
&= \Pr(s_{t-1}|s_t)E[\lambda_{t-1}(s_{t-1})|y_1,\ldots,y_{t-2},s_{t-1}]
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
&E[g_{t-1}(s_{t-1})\Pr(s_{t-1}|y_1,\ldots,y_{t-1},s_t)|y_1,\ldots,y_{t-1},s_t] \\
&= \int_{\{y_1,\ldots,y_{t-1}\}} g_{t-1}(s_{t-1})\Pr(s_{t-1}|y_1,\ldots,y_{t-1},s_t)h(y_1,\ldots,y_{t-1}|s_t)d(y_1,\ldots,y_{t-1}) \\
&= \int_{\{y_1,\ldots,y_{t-1}\}} g_{t-1}(s_{t-1})\Pr(s_{t-1}|s_t)h(y_1,\ldots,y_{t-1}|s_{t-1},s_t)d(y_1,\ldots,y_{t-1}) \\
&= \Pr(s_{t-1}|s_t)E[g_{t-1}(s_{t-1})|y_1,\ldots,y_{t-2},s_{t-1}]
\end{aligned}
\tag{23}
$$

where h is a joint density function and the value of s_{t-1} is not specified as the expectation is with respect to $\{y_1,\ldots,y_{t-1}\}$. Thus, $E[\lambda_t(s_t)|y_1,\ldots,y_{t-1},s_t]$ can be recursively constructed as follows:

$$
\begin{aligned}
E[\lambda_t(s_t)|y_1,\ldots,y_{t-1},s_t] &= \beta(s_t)\Pr(s_{t-1}|s_t)E[\lambda_{t-1}(s_{t-1})|y_1,\ldots,y_{t-2},s_{t-1}] \\
&\quad + \Pr(s_{t-1}|s_t)E[g_{t-1}(s_{t-1})|y_1,\ldots,y_{t-2},s_{t-1}]
\end{aligned}
\tag{24}
$$

The probability $\Pr(s_{t-1}|s_t)$ in Equation (24) for $s_t = 1,2$ and $s_{t-1} = 1,2$ is given by:

$$
\begin{aligned}
&\Pr(s_{t-1}=1|s_t=1) = \Pr(s_t=1|s_{t-1}=1) = p \\
&\Pr(s_{t-1}=2|s_t=1) = \tfrac{\pi^*(2)}{\pi^*(1)}\Pr(s_t=1|s_{t-1}=2) = \tfrac{\pi^*(2)}{\pi^*(1)}(1-q) \\
&\Pr(s_{t-1}=1|s_t=2) = \tfrac{\pi^*(1)}{\pi^*(2)}\Pr(s_t=2|s_{t-1}=1) = \tfrac{\pi^*(1)}{\pi^*(2)}(1-p) \\
&\Pr(s_{t-1}=2|s_t=2) = \Pr(s_t=2|s_{t-1}=2) = q
\end{aligned}
\tag{25}
$$

Based on Equations (24) and (25) and by considering all possible values of s_t and s_{t-1} in Equation (24), the following matrix is defined:

$$
E_1 = \begin{bmatrix} \beta(1)p & \beta(1)\frac{\pi^*(2)}{\pi^*(1)}(1-q) \\ \beta(2)\frac{\pi^*(1)}{\pi^*(2)}(1-p) & \beta(2)q \end{bmatrix}
\tag{26}
$$

If the maximum modulus of eigenvalues, C_1, of matrix E_1 is less than one, then Condition 1 is satisfied. Moreover, Condition 2, denoted as C_2, holds if $E[u_{t-1}^{2-i}(s_t)[\partial u_{t-1}(s_t)/\partial\lambda_{t-1}(s_t)]^i] < \infty$, where $i = 0,1,2$ for $s_t = 1,2$, and define the derivative of the score function as:

$$
\frac{\partial u_{t-1}(s_t)}{\partial\lambda_{t-1}(s_t)} = E\left[\frac{\partial u_{t-1}(s_{t-1})}{\partial\lambda_{t-1}(s_{t-1})}|y_1,\ldots,y_{t-1},s_t\right]
\tag{27}
$$

where the expected value is with respect to s_{t-1}. For Condition 3, the score function uses the partial derivative of $\lambda_t(s_t)$ with respect to $\alpha^*(s_t)$ as follows:

$$\frac{\partial \lambda_t(s_t)}{\partial \alpha^*(s_t)} = \beta(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha^*(s_t)} + \alpha(s_t)\frac{\partial u_{t-1}(s_t)}{\partial \alpha^*(s_t)} + \alpha^*(s_t)\text{sgn}[-\epsilon_{t-1}(s_t)]\frac{\partial u_{t-1}(s_t)}{\partial \alpha^*(s_t)} \tag{28}$$

$$+\text{sgn}[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t)+1]$$

Equation (28) can be written as:

$$\frac{\partial \lambda_t(s_t)}{\partial \alpha^*(s_t)} = X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha^*(s_t)} + \text{sgn}[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t)+1] \tag{29}$$

where

$$X_{t-1}(s_t) = \beta(s_t) + \{\alpha(s_t) + \alpha^*(s_t)\text{sgn}[-\epsilon_{t-1}(s_t)]\}\frac{\partial u_{t-1}(s_t)}{\partial \lambda_{t-1}(s_t)} \tag{30}$$

For Condition 3, the same arguments hold as for Condition 1. Thus, we define the following:

$$E_3 = \left\{ \begin{array}{ll} |E[X_{t-1}(s_t=1)]|p & |E[X_{t-1}(s_t=1)]|\frac{\pi^*(2)}{\pi^*(1)}(1-q) \\ |E[X_{t-1}(s_t=2)]|\frac{\pi^*(1)}{\pi^*(2)}(1-p) & |E[X_{t-1}(s_t=2)]|q \end{array} \right\} \tag{31}$$

where the expectations are estimated by using sample average. If the maximum modulus of eigenvalues, C_3, of matrix E_3 is less than one, then Condition 3 is satisfied. For the partial derivatives with respect to $\omega(s_t)$, $\beta(s_t)$ and $\alpha(s_t)$, the following results are obtained:

$$\frac{\partial \lambda_t(s_t)}{\partial \omega(s_t)} = X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \omega(s_t)} + 1 \tag{32}$$

$$\frac{\partial \lambda_t(s_t)}{\partial \beta(s_t)} = X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \beta(s_t)} + \lambda_{t-1}(s_t) \tag{33}$$

$$\frac{\partial \lambda_t(s_t)}{\partial \alpha(s_t)} = X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha(s_t)} + u_{t-1}(s_t) \tag{34}$$

Thus, C_3 ensures the stability of the contributions to the score function. For Condition 4, the contributions to the information matrix use the outer product of the contributions to the score function. Thus, the information matrix uses the following:

$$\left[\frac{\partial \lambda_t(s_t)}{\partial \alpha^*(s_t)}\right]^2 = \left\{X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha^*(s_t)} + \text{sgn}[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t)+1]\right\}^2 \tag{35}$$

$$= X_{t-1}^2(s_t)\left[\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha^*(s_t)}\right]^2 + 2X_{t-1}(s_t)\frac{\partial \lambda_{t-1}(s_t)}{\partial \alpha^*(s_t)}\text{sgn}[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t)+1]$$

$+\text{sgn}^2[-\epsilon_{t-1}(s_t)][u_{t-1}(s_t)+1]^2$

For Condition 4, the same arguments hold as for Condition 1. Thus, the following matrix is defined:

$$E_4 = \left\{ \begin{array}{ll} E[X_{t-1}^2(s_t=1)]p & E[X_{t-1}^2(s_t=1)]\frac{\pi^*(2)}{\pi^*(1)}(1-q) \\ E[X_{t-1}^2(s_t=2)]\frac{\pi^*(1)}{\pi^*(2)}(1-p) & E[X_{t-1}^2(s_t=2)]q \end{array} \right\} \tag{36}$$

where the expectations are estimated by using sample average. If the maximum modulus of eigenvalues, C_4, of matrix E_4 is less than one, then Condition 4 is satisfied. For all combinations of the partial derivatives with respect to $\omega(s_t)$, $\beta(s_t)$, $\alpha^*(s_t)$ and $\alpha(s_t)$, the same condition ensures the stability of the contributions to the information matrix. Conditions 2 to 4 are extensions of the corresponding conditions of the work of Harvey (2013) for score-driven MS models. For Condition 5, the result on invertibility of single-regime observation-driven models from the work of Blasques et al. (2018) is extended to the MS-DCS-EGARCH model. For the empirical version of the regime-dependent Lyapunov condition for all MS DCS-EGARCH models of this paper, the following statistic is defined:

$$C_5(s_t) = \frac{\partial \lambda_t(s_t)}{\partial \lambda_{t-1}(s_t)} = \beta(s_t) + \left\{ \alpha + \alpha^* \text{sgn}[-\epsilon_{t-1}(s_t)] \frac{\partial u_{t-1}(s_t)}{\partial \lambda_{t-1}(s_t)} \right\} \tag{37}$$

for $t = 2, \ldots, T$. By using the stationary probabilities for s_t, Condition 5 holds if:

$$C_5 = \frac{1}{T-1} \sum_{t=2}^{T} \ln |C_5(1) \times \pi_t(1) + C_5(2) \times \pi_t(2)| < 0 \tag{38}$$

Conditions 1 and 3 to 5 are empirically studied in the next section of empirical results, while Condition 2 is used as a maintained assumption in this paper.

6. Empirical results

In this section, empirical results for the ML estimates of single-regime and MS versions of t-GARCH, Beta-t-EGARCH, GED-EGARCH, Skew-Gen-t-EGARCH, EGB2-EGARCH, NIG-EGARCH and MXN-EGARCH are presented for the G20 dataset. For the comparison of statistical performances of different models, the LL, Akaike information criterion (AIC), Bayesian information criterion (BIC) and Hannan–Quinn information criterion (HQC) likelihood-based models performance metrics are used (e.g. Hamilton, 1994; Davidson and MacKinnon, 2004). The use of these metrics for DCS models is supported by the work of Harvey (2013). For all stock markets, both single-regime and MS specifications are estimated in order to validate the use of the regime-switching volatility models. To show the quality of the ML estimates, for the single-regime models only the estimate of C_1 is reported, while for the MS models the estimates of C_1 and C_3 to C_5 are reported.

The first result of this paper is that the regimes of MS volatility model are not separated effectively for several stock markets (i.e. the two regimes switch very frequently in order to meaningfully identify periods of high and low volatility). Therefore, for those stock markets, the statistical performances of single-regime volatility models are compared (Table 2).

The estimation results in Table 2, with the exceptions of Argentina and Brazil, support the use of a DCS-EGARCH model as an alternative to t-GARCH. The estimation results for the remaining stock markets are presented in Table 2 suggest that the best volatility specifications are Beta-t-EGARCH and Skew-Gen-t-EGARCH. For India and Turkey, Beta-t-EGARCH is superior to Skew-Gen-t-EGARCH, according to the AIC, BIC and HQC metrics. For Indonesia, Saudi Arabia and South Africa, Skew-Gen-t-EGARCH is superior to Beta-t-EGARCH, according to the AIC, BIC and HQC metrics. For China and South Korea, the AIC-, BIC- and HQC-based results are mixed for Beta-t-EGARCH and Skew-Gen-t-EGARCH, suggesting that the statistical performances of those models are similar. GED-EGARCH is not supported for any of the stock markets for single-regime models (Table 2). According to the LL, AIC, BIC and HQC metrics, the novel MXN-EGARCH specification is superior to t-GARCH for the stock markets in China, Indonesia, Saudi Arabia and South Africa. However, for all stock markets in Table 2, Beta-t-EGARCH or Skew-Gen-t-EGARCH is superior to MXN-EGARCH, according to the LL, AIC, BIC and HQC metrics. Regarding the quality of the ML estimates, C_1 never suggests rejection of the asymptotic properties of the ML estimator for single-regime volatility models, as detailed in Table 2.

In Table 3(a)-(d), for the stock markets for which the two switching regimes are effectively separated, the likelihood-based models performance metrics and model diagnostics results are presented for the single-regime and MS volatility specifications. With the exception of Russia, the DCS-EGARCH specifications are superior to t-GARCH for all stock markets. The estimation results for the remaining stock markets suggest that the best volatility specifications are single-regime or MS versions of Beta-t-EGARCH, Skew-Gen-t-EGARCH and NIG-EGARCH. According to the LL, AIC, BIC and HQC metrics, a MS-DCS-EGARCH specification is supported for the stock markets in: Canada (MS-Skew-Gen-t-EGARCH or MS-NIG-EGARCH), France (MS-Beta-t-EGARCH or MS-Skew-Gen-t-EGARCH), Germany (MS-Skew-Gen-t-EGARCH), Japan (MS-Skew-Gen-t-EGARCH), UK (MS-Skew-Gen-t-EGARCH) and US (MS-Skew-Gen-t-EGARCH or MS-NIG-EGARCH).

On the other hand, some LL-based model performance metrics support a possible single-regime DCS-EGARCH alternative for: Australia (Skew-Gen-t-EGARCH), Italy (Skew-Gen-t-EGARCH), Mexico (Beta-t-EGARCH) and Spain (Skew-Gen-t-EGARCH). However, for the stock markets in those countries other model performance metrics support MS-DCS-EGARCH specifications. MS-GED-EGARCH is not supported for any of the stock markets for MS models (Table 3). According to the LL, AIC, BIC and HQC metrics, the novel MS-MXN-EGARCH specification is superior to MS-t-GARCH for the stock markets in: Australia, Canada, Italy, Japan and US. However, according to the LL, AIC, BIC and HQC metrics, MS-Beta-t-EGARCH, MS-Skew-Gen-t-EGARCH or MS-NIG-EGARCH is superior to MS-MXN-EGARCH for all stock markets in Table 3. Regarding the quality of the ML estimates, C_1 and C_3 to C_5 never suggest rejection of the asymptotic properties of the ML estimator for single-regime and MS volatility models, as shown in Table 3.

In Table 4(a)-(d), the ML parameter estimates are presented for the best-performing single-regime and MS DCS-EGARCH specifications for the stock markets in: Australia, Canada, Germany, France, Italy, Japan, Mexico, Spain, UK and US. For the same stock markets, Figures 2 to 5 present the smoothed probabilities for $s_t = 1$, which can be associated with the 'high-volatility' period of the sample. The same figures also present the conditional volatility estimates, i.e. the ML estimates of $\pi_t(1)\sigma_t(1) + \pi_t(2)\sigma_t(2)$.

7. Conclusions and future research

In the present paper, we have provided, for the first time in the literature, the proposition and application of the novel score-driven MXN-EGARCH volatility model, for which we have considered both single-regime and MS specifications. The likelihood-based model selection metrics have shown that the proposed MXN-EGARCH model, for several stock markets, provides better model performance than the classical t-EGARCH model. Nevertheless, the statistical performance of the single-regime or MS version of Beta-t-EGARCH, Skew-Gen-t-EGARCH or NIG-EGARCH has consistently been superior to the statistical performance of MXN-EGARCH.

The statistical performance metrics have indicated that, for most of the stock markets, the Winsorizing-type transformation of the error term that is provided by Beta-t-EGARCH and Skew-Gen-t-EGARCH seems to be more appropriate than the GRACH-type quadratic transformation that is provided by t-GARCH, or the quasi-linearly increasing transformations that are provided by GED-EGARCH, EGB2-EGARCH, NIG-EGARCH or MXN-EGARCH; although for some stock markets, NIG-EGARCH has worked effectively. For all MS-DCS-EGARCH specifications, new conditions of the asymptotic properties of the ML estimator have been presented, by extending relevant works from the existing literature on dynamic volatility models. Those conditions have been supported for all models and all stock markets in our empirical application. We have applied all models of conditional volatility to an internationally broad stock market index historical dataset from the G20 countries, by using all available data, and providing the most general application of MS-DCS-EGARCH to the literature.

There are a number of future research opportunities related to the present paper: (a) Future works may consider alternative data periods for each index, which may be motivated by application-specific considerations. In this paper, historical data is used due to statistical generality of the results and also for the particular property of DCS filters that they learn in an optimal way from the data according to the Kullback–Leibler divergence measure (i.e. the longer the sample period, the better more effective learning is provided by the filter).

(b) Moreover, future works may consider alternative estimation methods for MS-DCS-EGARCH models, for example, the aforementioned non-path-dependent or the path-dependent estimation methods for MS models. The use of those alternative methods will also require the proofs of the corresponding conditions of the asymptotic properties of the statistical estimators.

(c) Future works may also consider extended versions of the DCS-EGARCH and MS-DCS-EGARCH models of this paper and use more than two switching regimes in empirical applications. The model specification and ML estimation procedures can be extended in straightforward ways for such multi-regime MS-DCS-EGARCH models. Those extended models might be effective for the separation of regimes for those stock markets in which the two-regime volatility models have been ineffective.

(d) Future works may consider probability distributions alternative to the ones that are used in this paper. Those distributions may provide more parsimonious estimates due to simplicity or better model fit. In this paper, Beta-t-EGARCH and GED-EGARCH have been used due to relative model simplicity, and Skew-Gen-t-EGARCH, EGB2-EGARCH, NIG-EGARCH and MXN-EGARCH have been used due to relatively complex tail shape and asymmetric properties.

(e) Finally, future works may also compare the forecasting performances of those models in portfolio management applications, or use VaR backtests to study the performances of MS-DCS-EGARCH models in financial risk management applications.

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Appendix A: Updating terms in score-driven MS-EGARCH

For all MS models of this paper, the conditional distribution of y_t depends on s_t. This idea was suggested by Gray (1996) for MS-GARCH, who used $\lambda_{t-1} = E[\lambda_{t-1}(s_{t-1})|y_1, \ldots, y_{t-2}]$ to integrate out s_{t-1} from $\lambda_{t-1}(s_{t-1})$. Klaassen (2002) extended the work of Gray (1996) for MS-GARCH, and used $\lambda_{t-1}(s_t) = E[\lambda_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t]$. The variables that update the score-driven MS-EGARCH models of the present paper are computed as follows:

$$\begin{aligned}\lambda_{t-1}(s_t = 1) &= E[\lambda_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t = 1] \\ &= \lambda_{t-1}(s_{t-1} = 1)\Pr(s_{t-1} = 1|y_1, \ldots, y_{t-1}, s_t = 1) + \\ &\quad \lambda_{t-1}(s_{t-1} = 2)\Pr(s_{t-1} = 2|y_1, \ldots, y_{t-1}, s_t = 1)\end{aligned} \tag{A.1}$$

$$\begin{aligned}\lambda_{t-1}(s_t = 2) &= E[\lambda_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t = 2] \\ &= \lambda_{t-1}(s_{t-1} = 1)\Pr(s_{t-1} = 1|y_1, \ldots, y_{t-1}, s_t = 2) + \\ &\quad \lambda_{t-1}(s_{t-1} = 2)\Pr(s_{t-1} = 2|y_1, \ldots, y_{t-1}, s_t = 2)\end{aligned} \tag{A.2}$$

$$\begin{aligned}u_{t-1}(s_t = 1) &= E[u_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t = 1] \\ &= u_{t-1}(s_{t-1} = 1)\Pr(s_{t-1} = 1|y_1, \ldots, y_{t-1}, s_t = 1) + \\ &\quad u_{t-1}(s_{t-1} = 2)\Pr(s_{t-1} = 2|y_1, \ldots, y_{t-1}, s_t = 1)\end{aligned} \tag{A.3}$$

$$\begin{aligned}u_{t-1}(s_t = 2) &= E[u_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t = 2] \\ &= u_{t-1}(s_{t-1} = 1)\Pr(s_{t-1} = 1|y_1, \ldots, y_{t-1}, s_t = 2) + \\ &\quad u_{t-1}(s_{t-1} = 2)\Pr(s_{t-1} = 2|y_1, \ldots, y_{t-1}, s_t = 2)\end{aligned} \tag{A.4}$$

$$\begin{aligned}\epsilon_{t-1}(s_t = 1) &= E[\epsilon_{t-1}(s_{t-1})|y_1, \ldots, y_{t-1}, s_t = 1] \\ &= \epsilon_{t-1}(s_{t-1} = 1)\Pr(s_{t-1} = 1|y_1, \ldots, y_{t-1}, s_t = 1) + \\ &\quad \epsilon_{t-1}(s_{t-1} = 2)\Pr(s_{t-1} = 2|y_1, \ldots, y_{t-1}, s_t = 1)\end{aligned} \tag{A.5}$$

$$\begin{aligned}\epsilon_{t-1}(s_t=2) &= E[\epsilon_{t-1}(s_{t-1})|y_1,\ldots,y_{t-1},s_t=2] \\ &= \epsilon_{t-1}(s_{t-1}=1)\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t=2)+ \\ &\quad \epsilon_{t-1}(s_{t-1}=2)\Pr(s_{t-1}=2|y_1,\ldots,y_{t-1},s_t=2)\end{aligned} \tag{A.6}$$

The conditional probabilities of the previous equations are computed as:

$$\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t=1)=\frac{p\tilde{\pi}_{t-1}(1)}{p\tilde{\pi}_{t-1}(1)+(1-q)\tilde{\pi}_{t-1}(2)} \tag{A.7}$$

$$\Pr(s_{t-1}=2|y_1,\ldots,y_{t-1},s_t=1)=1-\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t=1) \tag{A.8}$$

$$\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t=2)=\frac{(1-p)\tilde{\pi}_{t-1}(1)}{(1-p)\tilde{\pi}_{t-1}(1)+q\tilde{\pi}_{t-1}(2)} \tag{A.9}$$

$$\Pr(s_{t-1}=2|y_1,\ldots,y_{t-1},s_t=2)=1-\Pr(s_{t-1}=1|y_1,\ldots,y_{t-1},s_t=2) \tag{A.10}$$

and for the first period we use $\tilde{\pi}_0(1)=\pi^*(1)$ and $\tilde{\pi}_0(2)=\pi^*(2)$, as initial probabilities.

Appendix B: Regime-dependent conditional mean and volatility

For the Skew-Gen-t distribution, the conditional mean of y_t is calculated as follows:

$$\mu_t(s_t)=c(s_t)+2\exp[\lambda_t(s_t)]\tanh[\delta_1(s_t)]\{\exp[\delta_2(s_t)]+2\}^{\exp[-\delta_3(s_t)]}\times\frac{B\left\{\frac{2}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]+1}{\exp[\delta_3(s_t)]}\right\}}{B\left\{\frac{1}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]+2}{\exp[\delta_3(s_t)]}\right\}} \tag{B.1}$$

and the conditional volatility of y_t is given by:

$$\sigma_t(s_t)=\exp[\lambda_t(s_t)]\{\exp[\delta_2(s_t)]+2\}^{\exp[-\delta_3(s_t)]}\times \tag{B.2}$$

$$\times\left\{\frac{\{3\tanh^2[\delta_1(s_t)]+1\}B\left\{\frac{3}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]}{\exp[\delta_3(s_t)]}\right\}}{B\left[\frac{1}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]+2}{\exp[\delta_3(s_t)]}\right]}-\frac{4\tanh^2[\delta_1(s_t)]B^2\left\{\frac{2}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]+1}{\exp[\delta_3(s_t)]}\right\}}{B^2\left\{\frac{1}{\exp[\delta_3(s_t)]},\frac{\exp[\delta_2(s_t)]+2}{\exp[\delta_3(s_t)]}\right\}}\right\}^{1/2}$$

where $B(x,y)$ is the Beta function.

For the EGB2 distribution, the conditional mean of y_t is calculated as follows:

$$\mu_t(s_t)=c(s_t)+\exp[\lambda_t(s_t)]\left\{\Psi^{(0)}\{\exp[\delta_1(s_t)]\}-\Psi^{(0)}\{\exp[\delta_2(s_t)]\}\right\} \tag{B.3}$$

and the conditional volatility of y_t refers to the following:

$$\sigma_t(s_t) = \exp[\lambda_t(s_t)] \left\{ \Psi^{(1)}\{\exp[\delta_1(s_t)]\} + \Psi^{(1)}\{\exp[\delta_2(s_t)]\} \right\}^{1/2} \tag{B.4}$$

where $\Psi^{(0)}(x)$ and $\Psi^{(1)}(x)$ are digamma and trigamma functions, respectively.

For the NIG distribution, the conditional mean of y_t is calculated as follows:

$$\mu_t(s_t) = c(s_t) + \frac{\exp[\lambda_t(s_t)]\tanh[\delta_2(s_t)]}{\{1 - \tanh^2[\delta_2(s_t)]\}^{1/2}} \tag{B.5}$$

and the conditional volatility of y_t refers to the following:

$$\sigma_t(s_t) = \left\{ \frac{\exp[2\lambda_t(s_t) - \delta_1(s_t)]}{\{1 - \tanh^2[\delta_2(s_t)]\}^{3/2}} \right\}^{1/2} \tag{B.6}$$

For the MXN distribution, the conditional mean of y_t is

$$\mu_t(s_t) = c(s_t) + \exp[\lambda_t(s_t) + \delta_2(s_t)]\tan\left\{ \frac{\pi\tanh[\delta_1(s_t)]}{2} \right\} \tag{B.7}$$

and the conditional volatility of y_t is

$$\sigma_t(s_t) = \left\{ \frac{\exp[2\lambda_t(s_t) + \delta_2(s_t)]}{\cos\{\pi\tanh[\delta_1(s_t)]\} + 1} \right\}^{1/2} \tag{B.8}$$

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Table 1. Descriptive statistics

Country	Index name	Ticker	Start date	T	Mean	SD	Min	Max	Skewness	Kurtosis	SF test	Corr
Argentina	S&P MERVAL TR (ARS)	MERVAL:IND	4-Apr-89	7, 530	0.0015	0.0305	−0.7571	0.3295	−1.4609	72.0144	0.8051***	0.0515
Australia	S&P/ASX 300	AS52:IND	1-Jun-92	6, 941	0.0002	0.0093	−0.0870	0.0577	−0.4791	8.7237	0.9484***	−0.1035
Brazil	Ibovespa Brasil Sao Paulo Stock Exchange	IBOV:IND	1-Mar-93	6, 593	0.0014	0.0223	−0.1723	0.2882	0.5011	13.4941	0.9212***	−0.0991
Canada	S&P/TSX Composite	SPTSX:IND	4-Jan-77	10, 774	0.0003	0.0093	−0.1179	0.0937	−0.8664	16.7922	0.8881***	−0.0977
China	Shanghai Stock Exchange Composite	SHCOMP:IND	20-Dec-90	7, 050	0.0005	0.0227	−0.1791	0.7192	5.3269	160.3742	0.7435***	0.0161
France	CAC 40	CAC:IND	10-Jul-87	8, 184	0.0002	0.0136	−0.1014	0.1059	−0.1743	8.4698	0.9446***	−0.0953
Germany	Deutsche Boerse AG German Stock	DAX:IND	2-Oct-59	15, 109	0.0002	0.0122	−0.1371	0.1200	−0.1583	10.3751	0.9345***	−0.1075
India	S&P BSE SENSEX	SENSEX:IND	4-Apr-79	9, 341	0.0006	0.0158	−0.1366	0.1599	0.0044	9.4813	0.9368***	−0.0462
Indonesia	Jakarta Stock Exchange Composite	JCI:IND	5-Apr-83	8, 908	0.0005	0.0150	−0.2253	0.4031	2.5833	86.7828	0.7519***	0.1435
Italy	FTSE MIB	FTSEMIB:IND	2-Jan-98	5, 537	0.0000	0.0153	−0.1333	0.1087	−0.2017	7.6288	0.9523***	−0.1180
Japan	Nikkei 225	NKY:IND	6-Jan-70	12, 273	0.0002	0.0129	−0.1614	0.1323	−0.4181	12.3871	0.9202***	−0.1125
Mexico	S&P/BMV IPC	MEXBOL:IND	6-Apr-94	6, 429	0.0005	0.0143	−0.1431	0.1215	0.0655	10.3592	0.9262***	−0.0826
Russia	MOEX Russia	IMOEX:IND	23-Sep-97	5, 528	0.0006	0.0249	−0.2334	0.2750	0.1185	20.7765	0.8340***	−0.0136
Saudi Arabia	Tadawul All Share	SASEIDX:IND	30-Jan-94	6, 050	0.0002	0.0136	−0.1349	0.1640	−0.9515	19.7255	0.8146***	−0.1359
South Africa	FTSE/JSE Africa All Share	JALSH:IND	3-Jul-95	6, 078	0.0004	0.0119	−0.1263	0.0727	−0.4374	9.0585	0.9465***	−0.1119
South Korea	Korea Stock Exchange KOSPI	KOSPI:IND	5-Jan-80	10, 703	0.0003	0.0145	−0.1280	0.1128	−0.2382	8.8375	0.9281***	−0.0547
Spain	IBEX 35	IBEX:IND	7-Jan-87	8, 275	0.0002	0.0137	−0.1319	0.1348	−0.2048	9.3596	0.9416***	−0.0864
Turkey	Borsa Istanbul 100	XU100:IND	5-Jan-88	7, 948	0.0012	0.0252	−0.1998	0.1777	0.0231	7.5831	0.9432***	−0.0076
UK	FTSE 100	UKX:IND	3-Jan-84	9, 068	0.0002	0.0108	−0.1303	0.0938	−0.4773	12.4317	0.9295***	−0.1287
US	S&P 500	SPX:IND	3-Jan-28	23, 064	0.0002	0.0117	−0.2290	0.1537	−0.4391	21.9710	0.8678***	−0.0971

Notes: United Kingdom (UK); United States (US); 'Ticker' indicates the Bloomberg ticker; for all counties the end date of the sample period is 25-Oct-2019, with the exception of Saudi Arabia for which the end date of the sample period is 24-Oct-2019; T indicates the sample size; standard deviation (SD); For the Shapiro–Francia (SF) normality test statistic (Shapiro and Francia, 1972), *** indicates the rejection of the normal distribution null hypothesis the 1% level; Corr indicates the estimate of $\mathrm{Corr}(y_t^2, y_{t-1})$.

Table 2. Model performance and diagnostics; single-regime models for those countries for which MS is not identified

		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Argentina	LL	**2.4328**	2.4312	2.4231	2.4321	2.4293	2.4303	2.4291
	AIC	**−4.8637**	−4.8606	−4.8443	−4.8619	−4.8565	−4.8585	−4.8560
	BIC	**−4.8572**	−4.8541	−4.8378	−4.8536	−4.8492	−4.8512	−4.8486
	HQC	**−4.8615**	−4.8584	−4.8420	−4.8590	−4.8540	−4.8560	−4.8535
	C_1	0.9445	0.7989	0.7776	0.8466	0.8007	0.8024	0.8006
Brazil	LL	**2.5970**	2.5969	2.5928	2.5970	2.5961	2.5961	2.5957
	AIC	**−5.1919**	−5.1916	−5.1835	−5.1912	−5.1898	−5.1897	−5.1889
	BIC	**−5.1847**	−5.1844	−5.1763	−5.1820	−5.1816	−5.1815	−5.1806
	HQC	**−5.1894**	−5.1892	−5.1810	−5.1880	−5.1870	−5.1869	−5.1860
	C_1	0.9722	0.8530	0.8532	0.8955	0.8529	0.8530	0.8532
China	LL	2.7885	2.7938	2.7834	**2.7950**	2.7879	2.7919	2.7894
	AIC	−5.5751	−5.5856	−5.5648	**−5.5873**	−5.5735	−5.5816	−5.5766
	BIC	−5.5683	**−5.5788**	−5.5579	−5.5786	−5.5657	−5.5738	−5.5688
	HQC	−5.5728	−5.5833	−5.5624	**−5.5843**	−5.5708	−5.5789	−5.5739
	C_1	0.9426	0.8118	0.8157	0.8429	0.8132	0.8064	0.8095
India	LL	2.9288	2.9298	2.9247	**2.9298**	2.9283	2.9286	2.9279
	AIC	−5.8562	**−5.8580**	−5.8480	−5.8577	−5.8550	−5.8556	−5.8541
	BIC	−5.8508	**−5.8527**	−5.8426	−5.8508	−5.8489	−5.8494	−5.8480
	HQC	−5.8543	**−5.8562**	−5.8462	−5.8554	−5.8529	−5.8535	−5.8521
	C_1	0.9641	0.8260	0.8375	0.8647	0.8292	0.8285	0.8304
Indonesia	LL	3.2617	3.2785	3.2695	**3.2807**	3.2655	3.2732	3.2694
	AIC	−6.5217	−6.5554	−6.5376	**−6.5594**	−6.5293	−6.5446	−6.5370
	BIC	−6.5162	−6.5498	−6.5329	**−6.5523**	−6.5229	−6.5382	−6.5306
	HQC	−6.5198	−6.5535	−6.5360	**−6.5570**	−6.5271	−6.5424	−6.5348
	C_1	0.9300	0.7830	0.8115	0.8048	0.8172	0.8041	0.8185
Saudi Arabia	LL	3.2840	3.2863	3.2843	**3.2888**	3.2861	3.2879	3.2871
	AIC	−6.5660	−6.5703	−6.5664	**−6.5747**	−6.5696	−6.5731	−6.5716
	BIC	−6.5593	−6.5626	−6.5586	**−6.5647**	−6.5607	−6.5642	−6.5627
	HQC	−6.5636	−6.5677	−6.5637	**−6.5712**	−6.5665	−6.5700	−6.5685
	C_1	0.9084	0.7249	0.7516	0.7785	0.7478	0.7324	0.7383
South Africa	LL	3.1761	3.1772	3.1737	**3.1791**	3.1777	3.1777	3.1773
	AIC	−6.3499	−6.3521	−6.3450	**−6.3553**	−6.3528	−6.3527	−6.3520
	BIC	−6.3422	−6.3444	−6.3373	**−6.3453**	−6.3440	−6.3439	−6.3432
	HQC	−6.3472	−6.3494	−6.3424	**−6.3518**	−6.3498	−6.3496	−6.3489
	C_1	0.9684	0.8502	0.8478	0.8885	0.8506	0.8505	0.8499
South Korea	LL	3.0542	3.0553	3.0489	**3.0556**	3.0534	3.0540	3.0531
	AIC	−6.1070	−6.1094	−6.0965	**−6.1096**	−6.1054	−6.1066	−6.1046
	BIC	−6.1023	**−6.1046**	−6.0918	−6.1035	−6.1000	−6.1011	−6.0992
	HQC	−6.1054	**−6.1078**	−6.0949	−6.1075	−6.1036	−6.1047	−6.1028
	C_1	0.9665	0.8205	0.8545	0.8669	0.8315	0.8281	0.8342
Turkey	LL	2.4609	2.4610	2.4576	**2.4610**	2.4608	2.4609	2.4605
	AIC	−4.9201	**−4.9202**	−4.9135	−4.9198	−4.9196	−4.9198	−4.9190
	BIC	−4.9139	**−4.9141**	−4.9074	−4.9119	−4.9126	−4.9128	−4.9120
	HQC	−4.9180	**−4.9181**	−4.9114	−4.9171	−4.9172	−4.9174	−4.9166
	C_1	0.9625	0.8061	0.8206	0.8505	0.8098	0.8089	0.8109

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics. $C_1 = \beta + \alpha + \alpha^*/2$ for t-GARCH and $C_1 = |\beta|$ for the rest of the volatility models. $C_1 < 1$ suggests covariance stationarity for y_t. Metrics C_3 to C_5 are not reported for DCS models.

Table 3(a). Model performance and diagnostics for single-regime (SR) and Markov-switching (MS) volatility models

		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Australia SR	LL	3.4147	3.4164	3.4132	3.4188	3.4174	3.4173	3.4169
	AIC	−6.8273	−6.8308	−6.8244	−6.8350	−6.8324	−6.8323	−6.8316
	BIC	−6.8204	−6.8239	−6.8175	**−6.8262**	−6.8245	−6.8244	−6.8237
	HQC	−6.8249	−6.8284	−6.8220	−6.8320	−6.8297	−6.8296	−6.8288
	C_1	0.9705	0.8824	0.8725	0.9150	0.8863	0.8863	0.8854
Australia MS	LL	3.4190	3.4195	3.4172	**3.4227**	3.4213	3.4213	3.4202
	AIC	−6.8333	−6.8344	−6.8298	**−6.8396**	−6.8375	−6.8375	−6.8353
	BIC	−6.8175	−6.8186	−6.8140	−6.8199	−6.8197	−6.8197	−6.8175
	HQC	−6.8279	−6.8290	−6.8244	**−6.8328**	−6.8313	−6.8314	−6.8292
	C_1	0.9646	0.9727	0.9691	0.9796	0.9776	0.9783	0.9795
	C_3		0.9457	0.9374	0.9607	0.9488	0.9498	0.9453
	C_4		0.9015	0.8890	0.9265	0.9082	0.9099	0.9021
	C_5		−0.0731	−0.0818	−0.0490	−0.0750	−0.0738	−0.0738
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Canada SR	LL	3.5123	3.5124	3.5079	3.5157	3.5148	3.5149	3.5144
	AIC	−7.0233	−7.0236	−7.0145	−7.0297	−7.0280	−7.0283	−7.0273
	BIC	−7.0186	−7.0189	−7.0098	−7.0236	−7.0226	−7.0229	−7.0219
	HQC	−7.0217	−7.0220	−7.0129	−7.0276	−7.0262	−7.0265	−7.0255
	C_1	0.9631	0.8549	0.8588	0.8930	0.8558	0.8557	0.8562
Canada MS	LL	3.5164	3.5156	3.5119	**3.5203**	3.5195	3.5199	3.5187
	AIC	−7.0298	−7.0284	−7.0209	**−7.0369**	−7.0356	−7.0364	−7.0341
	BIC	−7.0189	−7.0182	−7.0101	−7.0234	−7.0234	**−7.0243**	−7.0219
	HQC	−7.0261	−7.0250	−7.0172	**−7.0324**	−7.0315	−7.0323	−7.0300
	C_1	0.9575	0.9772	0.9884	0.9810	0.9808	0.9813	0.9811
	C_3		0.9133	0.9378	0.9319	0.9064	0.9072	0.9276
	C_4		0.8403	0.8946	0.8711	0.8271	0.8301	0.8643
	C_5		−0.1276	−0.1081	−0.0952	−0.1011	−0.1317	−0.0903
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
France SR	LL	3.0516	3.0540	3.0462	3.0552	3.0529	3.0531	3.0521
	AIC	−6.1014	−6.1063	−6.0908	−6.1082	−6.1039	−6.1043	−6.1022
	BIC	−6.0954	−6.1003	−6.0848	−6.1005	−6.0970	−6.0974	−6.0954
	HQC	−6.0994	−6.1042	−6.0887	−6.1056	−6.1015	−6.1019	−6.0999
	C_1	0.9667	0.8664	0.8722	0.8998	0.8732	0.8726	0.8741
France MS	LL	3.0585	3.0603	3.0548	**3.0615**	3.0581	3.0597	3.0570
	AIC	−6.1131	−6.1166	−6.1057	**−6.1181**	−6.1119	−6.1150	−6.1096
	BIC	−6.0994	**−6.1029**	−6.0920	−6.1010	−6.0964	−6.0996	−6.0942
	HQC	−6.1084	−6.1119	−6.1010	**−6.1124**	−6.1066	−6.1097	−6.1043
	C_1	0.9681	0.9836	0.9841	0.9857	0.9859	0.9796	0.9866
	C_3		0.9398	0.9431	0.9616	0.9393	0.9235	0.9559
	C_4		0.8907	0.9094	0.9272	0.8885	0.8667	0.9183
	C_5		−0.0879	−0.0688	−0.0614	−0.0871	−0.0908	−0.0740

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics. $C_1 < 1$ suggests covariance stationarity for y_t. $C_3 < 1$ suggests covariance stationarity for the contributions to the gradient vector. $C_4 < 1$ suggests covariance stationarity for the contributions to the information matrix. $C_5 < 0$ suggests that the DCS model is invertible. Metrics C_3 to C_5 are not reported for single-regime DCS models.

Table 3(b). Model performance and diagnostics for single-regime (SR) and Markov-switching (MS) volatility models

		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Germany SR	LL	3.1760	3.1767	3.1672	3.1773	3.1749	3.1749	3.1738
	AIC	−6.3512	−6.3525	−6.3334	−6.3535	−6.3487	−6.3487	−6.3465
	BIC	−6.3476	−6.3490	−6.3299	−6.3489	−6.3446	−6.3447	−6.3424
	HQC	−6.3500	−6.3513	−6.3322	−6.3520	−6.3473	−6.3474	−6.3451
	C_1	0.9642	0.8280	0.8408	0.8686	0.8326	0.8321	0.8347
Germany MS	LL	3.1791	3.1805	3.1760	**3.1823**	3.1794	3.1804	3.1772
	AIC	−6.3560	−6.3589	−6.3499	**−6.3619**	−6.3565	−6.3584	−6.3519
	BIC	−6.3479	−6.3508	−6.3418	**−6.3518**	−6.3474	−6.3493	−6.3429
	HQC	−6.3533	−6.3562	−6.3472	**−6.3585**	−6.3535	−6.3554	−6.3489
	C_1	0.9549	0.9826	0.9820	0.9877	0.9881	0.9850	0.9803
	C_3		0.9216	0.9261	0.9530	0.9431	0.9317	0.9111
	C_4		0.8559	0.8713	0.9108	0.8975	0.8737	0.8417
	C_5		−0.1356	−0.1146	−0.0769	−0.1139	−0.1401	−0.0983
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Italy SR	LL	2.9492	2.9509	2.9470	2.9540	2.9520	2.9521	2.9517
	AIC	−5.8958	−5.8992	−5.8915	−5.9048	−5.9012	−5.9013	−5.9005
	BIC	−5.8874	−5.8908	−5.8831	**−5.8941**	−5.8916	−5.8917	−5.8910
	HQC	−5.8929	−5.8963	−5.8886	−5.9011	−5.8978	−5.8980	−5.8972
	C_1	0.9740	0.8646	0.8634	0.9039	0.8681	0.8681	0.8687
Italy MS	LL	2.9545	2.9557	2.9521	**2.9603**	2.9580	2.9572	2.9572
	AIC	−5.9032	−5.9056	−5.8983	**−5.9134**	−5.9096	−5.9079	−5.9080
	BIC	−5.8841	−5.8864	−5.8792	−5.8895	−5.8880	−5.8864	−5.8865
	HQC	−5.8965	−5.8989	−5.8917	**−5.9051**	−5.9021	−5.9004	−5.9005
	C_1	0.9576	0.9722	0.9884	0.9871	0.9912	0.9799	0.9886
	C_3		0.9271	0.9413	0.9541	0.9411	0.9379	0.9366
	C_4		0.8739	0.8944	0.9162	0.8954	0.8930	0.8922
	C_5		−0.0861	−0.0800	−0.0512	−0.0670	−0.0906	−0.0779
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Japan SR	LL	3.1597	3.1633	3.1561	3.1648	3.1631	3.1638	3.1627
	AIC	−6.3183	−6.3255	−6.3110	−6.3282	−6.3250	−6.3262	−6.3240
	BIC	−6.3141	−6.3213	−6.3067	−6.3227	−6.3201	−6.3214	−6.3192
	HQC	−6.3169	−6.3241	−6.3096	−6.3264	−6.3234	−6.3246	−6.3224
	C_1	0.9601	0.8277	0.8195	0.8660	0.8266	0.8268	0.8261
Japan MS	LL	3.1679	3.1700	3.1661	**3.1724**	3.1713	3.1698	3.1711
	AIC	−6.3331	−6.3374	−6.3297	**−6.3416**	−6.3397	−6.3367	−6.3392
	BIC	−6.3235	−6.3278	−6.3200	**−6.3295**	−6.3288	−6.3259	−6.3283
	HQC	−6.3299	−6.3342	−6.3264	**−6.3375**	−6.3361	−6.3331	−6.3356
	C_1	0.9523	0.9909	0.9891	0.9917	0.9920	0.9737	0.9914
	C_3		0.9339	0.9241	0.9525	0.9306	0.9011	0.9294
	C_4		0.8785	0.8719	0.9102	0.8738	0.8246	0.8714
	C_5		−0.1182	−0.1505	−0.1013	−0.1352	−0.1258	−0.1362

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics. $C_1 < 1$ suggests covariance stationarity for y_t. $C_3 < 1$ suggests covariance stationarity for the contributions to the gradient vector. $C_4 < 1$ suggests covariance stationarity for the contributions to the information matrix. $C_5 < 0$ suggests that the DCS model is invertible. Metrics C_3 to C_5 are not reported for single-regime DCS models.

Table 3(c). Model performance and diagnostics for single-regime (SR) and Markov-switching (MS) volatility models

		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Mexico SR	LL	3.0380	3.0389	3.0367	3.0393	3.0391	3.0391	3.0389
	AIC	−6.0739	−6.0757	−6.0713	−6.0758	−6.0757	−6.0758	−6.0753
	BIC	−6.0665	**−6.0683**	−6.0639	−6.0663	−6.0672	−6.0673	−6.0669
	HQC	−6.0713	−6.0731	−6.0688	−6.0725	−6.0728	−6.0728	−6.0724
	C_1	0.9721	0.8669	0.8618	0.9037	0.8653	0.8656	0.8653
Mexico MS	LL	3.0430	3.0434	3.0417	**3.0443**	3.0440	3.0440	3.0439
	AIC	−6.0811	−6.0819	−6.0784	−6.0824	−6.0824	**−6.0825**	−6.0822
	BIC	−6.0642	−6.0651	−6.0615	−6.0614	−6.0635	−6.0635	−6.0632
	HQC	−6.0752	**−6.0761**	−6.0725	−6.0751	−6.0759	−6.0759	−6.0756
	C_1	0.9648	0.9800	0.9783	0.9834	0.9828	0.9828	0.9824
	C_3		0.9466	0.9423	0.9619	0.9447	0.9449	0.9448
	C_4		0.9024	0.8991	0.9272	0.8993	0.8997	0.8999
	C_5		−0.0784	−0.0867	−0.0543	−0.0814	−0.0810	−0.0824
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Russia SR	LL	2.6672	2.6662	2.6578	2.6665	2.6643	2.6649	2.6636
	AIC	−5.3319	−5.3299	−5.3130	−5.3297	−5.3257	−5.3269	−5.3243
	BIC	**−5.3236**	−5.3216	−5.3046	−5.3189	−5.3162	−5.3173	−5.3147
	HQC	−5.3290	−5.3270	−5.3101	−5.3259	−5.3224	−5.3235	−5.3210
	C_1	0.9679	0.8387	0.8559	0.8767	0.8438	0.8423	0.8458
Russia MS	LL	2.6724	2.6716	2.6631	**2.6725**	2.6696	2.6704	2.6691
	AIC	**−5.3391**	−5.3374	−5.3204	−5.3378	−5.3328	−5.3344	−5.3317
	BIC	−5.3199	−5.3182	−5.3012	−5.3139	−5.3112	−5.3128	−5.3102
	HQC	**−5.3324**	−5.3307	−5.3137	−5.3294	−5.3253	−5.3269	−5.3242
	C_1	0.9784	0.9926	0.9785	0.9932	0.9812	0.9931	0.9861
	C_3		0.9413	0.9319	0.9573	0.9294	0.9430	0.9401
	C_4		0.8897	0.8736	0.9179	0.8689	0.8941	0.8893
	C_5		−0.1128	−0.1073	−0.0846	−0.1103	−0.1191	−0.1022
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
Spain SR	LL	3.0569	3.0603	3.0478	3.0615	3.0577	3.0582	3.0565
	AIC	−6.1121	−6.1188	−6.0939	−6.1209	−6.1134	−6.1145	−6.1111
	BIC	−6.1062	−6.1129	−6.0879	**−6.1132**	−6.1066	−6.1077	−6.1043
	HQC	−6.1101	−6.1168	−6.0919	−6.1183	−6.1111	−6.1122	−6.1088
	C_1	0.9666	0.8568	0.8680	0.8898	0.8633	0.8620	0.8645
Spain MS	LL	3.0647	3.0641	3.0576	**3.0660**	3.0632	3.0619	3.0623
	AIC	−6.1256	−6.1244	−6.1113	**−6.1272**	−6.1221	−6.1195	−6.1202
	BIC	−6.1121	−6.1108	−6.0978	−6.1103	−6.1068	−6.1042	−6.1049
	HQC	−6.1210	−6.1198	−6.1067	**−6.1214**	−6.1169	−6.1143	−6.1150
	C_1	0.9549	0.9687	0.9670	0.9889	0.9718	0.9841	0.9731
	C_3		0.9120	0.9013	0.9522	0.9065	0.9416	0.9148
	C_4		0.6982	0.8823	0.9098	0.8322	0.8973	0.8455
	C_5		−0.1277	−0.1233	−0.0668	−0.1282	−0.0957	−0.1309

Notes: Log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics. $C_1 < 1$ suggests covariance stationarity for y_t. $C_3 < 1$ suggests covariance stationarity for the contributions to the gradient vector. $C_4 < 1$ suggests covariance stationarity for the contributions to the information matrix. $C_5 < 0$ suggests that the DCS model is invertible. Metrics C_3 to C_5 are not reported for single-regime DCS models.

Table 3(d). Model performance and diagnostics for single-regime (SR) and Markov-switching (MS) volatility models

		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
UK SR	LL	3.2949	3.2941	3.2890	3.2958	3.2946	3.2946	3.2942
	AIC	−6.5882	−6.5866	−6.5764	−6.5896	−6.5875	−6.5873	−6.5865
	BIC	−6.5827	−6.5811	−6.5709	−6.5826	−6.5812	−6.5811	−6.5803
	HQC	−6.5863	−6.5848	−6.5745	−6.5872	−6.5853	−6.5852	−6.5844
	C_1	0.9653	0.8537	0.8684	0.8936	0.8581	0.8581	0.8601
UK MS	LL	3.2990	3.2992	3.2938	**3.3016**	3.2996	3.2996	3.2986
	AIC	−6.5945	−6.5949	−6.5841	**−6.5988**	−6.5952	−6.5952	−6.5933
	BIC	−6.5820	−6.5824	−6.5716	**−6.5831**	−6.5810	−6.5811	−6.5791
	HQC	−6.5903	−6.5906	−6.5799	**−6.5935**	−6.5904	−6.5904	−6.5885
	C_1	0.9660	0.9853	0.9836	0.9875	0.9880	0.9881	0.9881
	C_3		0.9438	0.9414	0.9569	0.9428	0.9432	0.9424
	C_4		0.8961	0.9058	0.9190	0.9014	0.9023	0.9020
	C_5		−0.0632	−0.0735	−0.0456	−0.0682	−0.0675	−0.0750
		t-GARCH	Beta-t-EGARCH	GED-EGARCH	Skew-Gen-t-EGARCH	EGB2-EGARCH	NIG-EGARCH	MXN-EGARCH
US SR	LL	3.3369	3.3386	3.3346	3.3404	3.3392	3.3395	3.3390
	AIC	−6.6732	−6.6767	−6.6686	−6.6801	−6.6777	−6.6784	−6.6773
	BIC	−6.6708	−6.6742	−6.6662	−6.6770	−6.6749	−6.6756	−6.6745
	HQC	−6.6724	−6.6759	−6.6678	−6.6791	−6.6768	−6.6775	−6.6764
	C_1	0.9671	0.8746	0.8763	0.9078	0.8762	0.8757	0.8763
US MS	LL	3.3425	3.3426	3.3380	**3.3447**	3.3432	3.3444	3.3439
	AIC	−6.6836	−6.6839	−6.6745	**−6.6877**	−6.6848	−6.6873	−6.6863
	BIC	−6.6780	−6.6783	−6.6690	−6.6807	−6.6785	**−6.6810**	−6.6800
	HQC	−6.6818	−6.6821	−6.6727	**−6.6854**	−6.6828	−6.6853	−6.6842
	C_1	0.9847	0.9945	0.9909	0.9953	0.9957	0.9956	0.9944
	C_3		0.9555	0.9875	0.9701	0.9609	0.9517	0.9574
	C_4		0.9178	0.9832	0.9436	0.9272	0.9112	0.9241
	C_5		−0.0831	−0.0747	−0.0623	−0.0729	−0.0883	−0.0496

Notes: United Kingdom (UK); United States (US); log-likelihood (LL); Akaike information criterion (AIC); Bayesian information criterion (BIC); Hannan–Quinn information criterion (HQC). Bold numbers indicate superior model performance metrics. $C_1 < 1$ suggests covariance stationarity for y_t. $C_3 < 1$ suggests covariance stationarity for the contributions to the gradient vector. $C_4 < 1$ suggests covariance stationarity for the contributions to the information matrix. $C_5 < 0$ suggests that the DCS model is invertible. Metrics C_3 to C_5 are not reported for single-regime DCS models.

Table 4(a). Parameter estimates for the best-performing single-regime (SR) and Markov-switching (MS) volatility models

Skew-Gen-t-EGARCH, Australia		Skew-Gen-t-EGARCH, Canada		NIG-EGARCH, Canada		Skew-Gen-t-EGARCH, Germany	
c	0.0012***(0.0002)	c	0.0013***(0.0001)	c	0.0014***(0.0001)	c	0.0006***(0.0001)
ω	−0.0634***(0.0118)	ω	−0.0608***(0.0095)	ω	−0.0548***(0.0087)	ω	−0.0852***(0.0091)
β	0.9876***(0.0024)	β	0.9883***(0.0019)	β	0.9883***(0.0019)	β	0.9820***(0.0019)
α	0.0316***(0.0028)	α	0.0464***(0.0027)	α	0.0473***(0.0028)	α	0.0481***(0.0022)
α^*	0.0245***(0.0021)	α^*	0.0133***(0.0017)	α^*	0.0124***(0.0018)	α^*	0.0193***(0.0015)
λ_0	−5.9223***(0.4195)	λ_0	−5.4566***(0.3701)	λ_0	−4.9227***(0.3628)	λ_0	−4.1004***(0.4465)
δ_1	−0.0843***(0.0153)	δ_1	−0.0943***(0.0115)	δ_1	0.7530***(0.0734)	δ_1	−0.0284***(0.0099)
δ_2	2.0321***(0.2278)	δ_2	1.9362***(0.1400)	δ_2	−0.1132***(0.0145)	δ_2	1.7428***(0.0907)
δ_3	0.7203***(0.0548)	δ_3	0.6342***(0.0379)			δ_3	0.8025***(0.0321)
LL	3.4188	LL	3.5157	LL	3.5149	LL	3.1773
MS-Skew-Gen-t-EGARCH, Australia		MS-Skew-Gen-t-EGARCH, Canada		MS-NIG-EGARCH, Canada		MS-Skew-Gen-t-EGARCH, Germany	
$c(1)$	0.0020***(0.0004)	$c(1)$	0.0022***(0.0002)	$c(1)$	0.0025***(0.0003)	$c(1)$	0.0015***(0.0002)
$c(2)$	0.0010***(0.0002)	$c(2)$	0.0011***(0.0001)	$c(2)$	0.0015***(0.0002)	$c(2)$	0.0002(0.0002)
$\omega(1)$	−0.1061***(0.0242)	$\omega(1)$	−0.0894***(0.0153)	$\omega(1)$	−0.0786***(0.0141)	$\omega(1)$	−0.0584***(0.0112)
$\omega(2)$	−0.1045***(0.0251)	$\omega(2)$	−0.3976***(0.0872)	$\omega(2)$	−0.3831***(0.0841)	$\omega(2)$	−0.1222***(0.0144)
$\beta(1)$	0.9779***(0.0052)	$\beta(1)$	0.9819***(0.0032)	$\beta(1)$	0.9823***(0.0033)	$\beta(1)$	0.9881***(0.0025)
$\beta(2)$	0.9805***(0.0049)	$\beta(2)$	0.9267***(0.0161)	$\beta(2)$	0.9215***(0.0173)	$\beta(2)$	0.9743***(0.0030)
$\alpha(1)$	0.0324***(0.0047)	$\alpha(1)$	0.0483***(0.0039)	$\alpha(1)$	0.0490***(0.0041)	$\alpha(1)$	0.0371***(0.0040)
$\alpha(2)$	0.0160***(0.0045)	$\alpha(2)$	0.0487***(0.0063)	$\alpha(2)$	0.0500***(0.0067)	$\alpha(2)$	0.0504***(0.0029)
$\alpha^*(1)$	0.0281***(0.0037)	$\alpha^*(1)$	0.0197***(0.0025)	$\alpha^*(1)$	0.0181***(0.0027)	$\alpha^*(1)$	0.0415***(0.0029)
$\alpha^*(2)$	0.0325***(0.0033)	$\alpha^*(2)$	0.0135***(0.0035)	$\alpha^*(2)$	0.0132***(0.0041)	$\alpha^*(2)$	0.0122***(0.0019)
$\lambda_0(1)$	−5.9179***(2.2664)	$\lambda_0(1)$	−5.4545***(1.7270)	$\lambda_0(1)$	−4.9196***(1.5177)	$\lambda_0(1)$	−4.3426**(1.9918)
$\lambda_0(2)$	−6.0813***(0.6963)	$\lambda_0(2)$	−5.2252***(1.3680)	$\lambda_0(2)$	−4.6011***(0.9895)	$\lambda_0(2)$	−3.9713***(1.1707)
$\delta_1(1)$	−0.1187***(0.0249)	$\delta_1(1)$	−0.1341***(0.0164)	$\delta_1(1)$	0.8892***(0.0984)	$\delta_1(1)$	−0.0888***(0.0160)
$\delta_1(2)$	−0.0881***(0.0247)	$\delta_1(2)$	−0.1013***(0.0210)	$\delta_1(2)$	0.8330***(0.1344)	$\delta_1(2)$	0.0057(0.0135)
$\delta_2(1)$	2.6756***(0.4583)	$\delta_2(1)$	1.9510***(0.1926)	$\delta_2(1)$	−0.1573***(0.0199)	$\delta_2(1)$	2.8367***(0.4828)
$\delta_2(2)$	1.8021***(0.3410)	$\delta_2(2)$	2.0455***(0.3054)	$\delta_2(2)$	−0.1631***(0.0303)	$\delta_2(2)$	1.3682***(0.0986)
$\delta_3(1)$	0.7340***(0.0822)	$\delta_3(1)$	0.6767***(0.0519)	p	0.9990***(0.0005)	$\delta_3(1)$	0.4820***(0.0598)
$\delta_3(2)$	0.6922***(0.0866)	$\delta_3(2)$	0.5956***(0.0671)	q	0.9990***(0.0005)	$\delta_3(2)$	1.0118***(0.0451)
p	0.9972***(0.0014)	p	0.9991***(0.0005)			p	0.9997***(0.0004)
q	0.9976***(0.0012)	q	0.9990***(0.0005)			q	0.9998***(0.0001)
LL	3.4227	LL	3.5203	LL	3.5199	LL	3.1823
$\overline{\sigma}(1)$	0.0089	$\overline{\sigma}(1)$	0.0084	$\overline{\sigma}(1)$	0.0084	$\overline{\sigma}(1)$	0.0114
$\overline{\sigma}(2)$	0.0079	$\overline{\sigma}(2)$	0.0068	$\overline{\sigma}(2)$	0.0068	$\overline{\sigma}(2)$	0.0107

Notes: Log-likelihood (LL); $\overline{\sigma}(s_t) = (1/T)\sum_{t=1}^{T} \sigma_t(s_t)$ is the regime-dependent mean volatility. Standard errors are reported in parentheses. ** and *** indicate parameter significance at the 5% and 1% levels, respectively.

Table 4(b). Parameter estimates for the best-performing single-regime (SR) and Markov-switching (MS) volatility models

Beta-t-EGARCH, France		Skew-Gen-t-EGARCH, France		Skew-Gen-t-EGARCH, Italy		Skew-Gen-t-EGARCH, Japan	
c	0.0003***(0.0001)	c	0.0011***(0.0002)	c	0.0015***(0.0002)	c	0.0011***(0.0001)
ω	−0.0847***(0.0103)	ω	−0.0712***(0.0106)	ω	−0.0469***(0.0107)	ω	−0.0789***(0.0094)
β	0.9818***(0.0022)	β	0.9849***(0.0023)	β	0.9901***(0.0024)	β	0.9837***(0.0020)
α	0.0367***(0.0028)	α	0.0374***(0.0028)	α	0.0419***(0.0034)	α	0.0578***(0.0029)
α^*	0.0306***(0.0021)	α^*	0.0308***(0.0021)	α^*	0.0280***(0.0024)	α^*	0.0279***(0.0019)
λ_0	−4.9224***(0.4541)	λ_0	−4.9111***(0.4530)	λ_0	−4.1315***(0.3243)	λ_0	−4.5451***(0.1923)
δ_1	1.9277***(0.0903)	δ_1	−0.0583***(0.0136)	δ_1	−0.0868***(0.0153)	δ_1	−0.0570***(0.0100)
		δ_2	1.9051***(0.1471)	δ_2	2.0724***(0.2460)	δ_2	1.7560***(0.1085)
		δ_3	0.7143***(0.0435)	δ_3	0.6648***(0.0585)	δ_3	0.6238***(0.0333)
LL	3.0540	LL	3.0552	LL	2.9540	LL	3.1648
MS-Beta-t-EGARCH, France		MS-Skew-Gen-t-EGARCH, France		MS-Skew-Gen-t-EGARCH, Italy		MS-Skew-Gen-t-EGARCH, Japan	
$c(1)$	0.0002(0.0002)	$c(1)$	0.0010***(0.0004)	$c(1)$	0.0035***(0.0013)	$c(1)$	0.0028***(0.0003)
$c(2)$	0.0003**(0.0001)	$c(2)$	0.0011***(0.0003)	$c(2)$	0.0016***(0.0003)	$c(2)$	0.0007***(0.0001)
$\omega(1)$	−0.0648***(0.0122)	$\omega(1)$	−0.0556***(0.0124)	$\omega(1)$	−0.1090***(0.0421)	$\omega(1)$	−0.6684***(0.0878)
$\omega(2)$	−0.2466***(0.0279)	$\omega(2)$	−0.2025***(0.0273)	$\omega(2)$	−0.0573***(0.0134)	$\omega(2)$	−0.0333***(0.0073)
$\beta(1)$	0.9853***(0.0028)	$\beta(1)$	0.9875***(0.0028)	$\beta(1)$	0.9737***(0.0102)	$\beta(1)$	0.8637***(0.0182)
$\beta(2)$	0.9497***(0.0058)	$\beta(2)$	0.9591***(0.0058)	$\beta(2)$	0.9890***(0.0030)	$\beta(2)$	0.9932***(0.0015)
$\alpha(1)$	0.0254***(0.0031)	$\alpha(1)$	0.0255***(0.0031)	$\alpha(1)$	0.0326***(0.0097)	$\alpha(1)$	0.0983***(0.0121)
$\alpha(2)$	0.0337***(0.0057)	$\alpha(2)$	0.0358***(0.0054)	$\alpha(2)$	0.0293***(0.0044)	$\alpha(2)$	0.0410***(0.0029)
$\alpha^*(1)$	0.0193***(0.0022)	$\alpha^*(1)$	0.0192***(0.0023)	$\alpha^*(1)$	0.0165***(0.0061)	$\alpha^*(1)$	0.0818***(0.0089)
$\alpha^*(2)$	0.0701***(0.0053)	$\alpha^*(2)$	0.0695***(0.0052)	$\alpha^*(2)$	0.0500***(0.0039)	$\alpha^*(2)$	0.0232***(0.0019)
$\lambda_0(1)$	−4.9022***(0.9977)	$\lambda_0(1)$	−4.9008***(1.1124)	$\lambda_0(1)$	−4.3407**(1.7686)	$\lambda_0(1)$	−3.7031(7.2104)
$\lambda_0(2)$	−4.9446**(2.3141)	$\lambda_0(2)$	−4.9353**(2.3116)	$\lambda_0(2)$	−4.1736***(0.6207)	$\lambda_0(2)$	−4.4979***(0.5448)
$\delta_1(1)$	2.9567***(0.2512)	$\delta_1(1)$	−0.0411**(0.0195)	$\delta_1(1)$	−0.0905*(0.0513)	$\delta_1(1)$	−0.1343***(0.0263)
$\delta_1(2)$	1.5152***(0.1112)	$\delta_1(2)$	−0.0784***(0.0217)	$\delta_1(2)$	−0.1227***(0.0191)	$\delta_1(2)$	−0.0455***(0.0112)
p	0.9982***(0.0007)	$\delta_2(1)$	3.2789***(0.6865)	$\delta_2(1)$	2.7061**(1.2051)	$\delta_2(1)$	1.4592***(0.2206)
q	0.9980***(0.0009)	$\delta_2(2)$	1.4967***(0.1669)	$\delta_2(2)$	1.7858***(0.2196)	$\delta_2(2)$	1.9450***(0.1857)
		$\delta_3(1)$	0.6566***(0.0685)	$\delta_3(1)$	0.7370***(0.1676)	$\delta_3(1)$	0.5981***(0.0788)
		$\delta_3(2)$	0.7285***(0.0619)	$\delta_3(2)$	0.7074***(0.0673)	$\delta_3(2)$	0.6495***(0.0468)
		p	0.9981***(0.0007)	p	0.9901***(0.0040)	p	0.9950***(0.0017)
		q	0.9979***(0.0010)	q	0.9967***(0.0018)	q	0.9984***(0.0006)
LL	3.0603	LL	3.0615	LL	2.9603	LL	3.1724
$\overline{\sigma}(1)$	0.0124	$\overline{\sigma}(1)$	0.0124	$\overline{\sigma}(1)$	0.0146	$\overline{\sigma}(1)$	0.0116
$\overline{\sigma}(2)$	0.0118	$\overline{\sigma}(2)$	0.0118	$\overline{\sigma}(2)$	0.0135	$\overline{\sigma}(2)$	0.0114

Notes: Log-likelihood (LL); $\overline{\sigma}(s_t) = (1/T)\sum_{t=1}^{T} \sigma_t(s_t)$ is the regime-dependent mean volatility. Standard errors are reported in parentheses. ** and *** indicate parameter significance at the 5% and 1% levels, respectively.

Table 4(c). Parameter estimates for the best-performing single-regime (SR) and Markov-switching (MS) volatility models

Beta-t-EGARCH, Mexico		Skew-Gen-t-EGARCH, Mexico		NIG-EGARCH, Mexico		Skew-Gen-t-EGARCH, Spain	
c	0.0004***(0.0001)	c	0.0007***(0.0002)	c	0.0007***(0.0003)	c	0.0011***(0.0002)
ω	−0.0526***(0.0091)	ω	−0.0498***(0.0098)	ω	−0.0460***(0.0089)	ω	−0.0634***(0.0098)
β	0.9887***(0.0020)	β	0.9893***(0.0021)	β	0.9888***(0.0022)	β	0.9865***(0.0021)
α	0.0421***(0.0034)	α	0.0430***(0.0035)	α	0.0435***(0.0035)	α	0.0425***(0.0030)
α^*	0.0234***(0.0022)	α^*	0.0235***(0.0022)	α^*	0.0235***(0.0023)	α^*	0.0221***(0.0019)
λ_0	−3.9531***(0.3905)	λ_0	−3.9611***(0.3975)	λ_0	−3.3726***(0.3982)	λ_0	−4.0635***(0.3037)
δ_1	1.7257***(0.1152)	δ_1	−0.0206(0.0144)	δ_1	0.8420***(0.1105)	δ_1	−0.0539***(0.0131)
		δ_2	2.1041***(0.2478)	δ_2	−0.0189(0.0178)	δ_2	1.6157***(0.1180)
		δ_3	0.5918***(0.0532)			δ_3	0.7681***(0.0439)
LL	3.0389	LL	3.0393	LL	3.0391	LL	3.0615
MS-Beta-t-EGARCH, Mexico		MS-Skew-Gen-t-EGARCH, Mexico		MS-NIG-EGARCH, Mexico		MS-Skew-Gen-t-EGARCH, Spain	
$c(1)$	0.0009***(0.0002)	$c(1)$	0.0014***(0.0003)	$c(1)$	0.0017***(0.0004)	$c(1)$	0.0041***(0.0011)
$c(2)$	0.0001(0.0002)	$c(2)$	0.0007**(0.0003)	$c(2)$	0.0008*(0.0004)	$c(2)$	0.0011***(0.0002)
$\omega(1)$	−0.0923***(0.0158)	$\omega(1)$	−0.0858***(0.0165)	$\omega(1)$	−0.0750***(0.0148)	$\omega(1)$	−0.5270***(0.2027)
$\omega(2)$	−0.0986***(0.0215)	$\omega(2)$	−0.0826***(0.0225)	$\omega(2)$	−0.0756***(0.0221)	$\omega(2)$	−0.0490***(0.0086)
$\beta(1)$	0.9791***(0.0036)	$\beta(1)$	0.9807***(0.0038)	$\beta(1)$	0.9807***(0.0039)	$\beta(1)$	0.8797***(0.0463)
$\beta(2)$	0.9802***(0.0044)	$\beta(2)$	0.9836***(0.0046)	$\beta(2)$	0.9830***(0.0052)	$\beta(2)$	0.9899***(0.0019)
$\alpha(1)$	0.0431***(0.0047)	$\alpha(1)$	0.0444***(0.0048)	$\alpha(1)$	0.0448***(0.0048)	$\alpha(1)$	0.0335***(0.0121)
$\alpha(2)$	0.0215***(0.0047)	$\alpha(2)$	0.0221***(0.0047)	$\alpha(2)$	0.0229***(0.0051)	$\alpha(2)$	0.0335***(0.0032)
$\alpha^*(1)$	0.0278***(0.0030)	$\alpha^*(1)$	0.0281***(0.0031)	$\alpha^*(1)$	0.0279***(0.0031)	$\alpha^*(1)$	0.0211***(0.0078)
$\alpha^*(2)$	0.0320***(0.0041)	$\alpha^*(2)$	0.0324***(0.0041)	$\alpha^*(2)$	0.0319***(0.0043)	$\alpha^*(2)$	0.0285***(0.0022)
$\lambda_0(1)$	−4.0048***(0.5137)	$\lambda_0(1)$	−4.0176***(0.5190)	$\lambda_0(1)$	−3.4022***(0.5215)	$\lambda_0(1)$	−3.9410(7.3809)
$\lambda_0(2)$	−3.4302***(0.9826)	$\lambda_0(2)$	−3.4233***(0.9177)	$\lambda_0(2)$	−2.7860***(0.9411)	$\lambda_0(2)$	−4.0107***(1.0395)
$\delta_1(1)$	1.7741***(0.1428)	$\delta_1(1)$	−0.0317*(0.0185)	$\delta_1(1)$	0.8843***(0.1373)	$\delta_1(1)$	−0.1158***(0.0483)
$\delta_1(2)$	1.8787***(0.2165)	$\delta_1(2)$	−0.0590**(0.0269)	$\delta_1(2)$	0.9948***(0.2202)	$\delta_1(2)$	−0.0679***(0.0140)
p	0.9998***(0.0002)	$\delta_2(1)$	2.2230***(0.3321)	$\delta_2(1)$	−0.0464**(0.0234)	$\delta_2(1)$	5.7566(20.4684)
q	0.9998***(0.0003)	$\delta_2(2)$	2.2334***(0.5651)	$\delta_2(2)$	−0.0558(0.0361)	$\delta_2(2)$	1.4812***(0.1261)
		$\delta_3(1)$	0.5730***(0.0668)	p	0.9998***(0.0002)	$\delta_3(1)$	0.7113***(0.1617)
		$\delta_3(2)$	0.6208***(0.1006)	q	0.9998***(0.0004)	$\delta_3(2)$	0.7786***(0.0493)
		p	0.9998***(0.0002)			p	0.9937***(0.0028)
		q	0.9998***(0.0004)			q	0.9989***(0.0005)
LL	3.0434	LL	3.0443	LL	3.0440	LL	3.0660
$\overline{\sigma}(1)$	0.0134	$\overline{\sigma}(1)$	0.0134	$\overline{\sigma}(1)$	0.0135	$\overline{\sigma}(1)$	0.0123
$\overline{\sigma}(2)$	0.0113	$\overline{\sigma}(2)$	0.0113	$\overline{\sigma}(2)$	0.0113	$\overline{\sigma}(2)$	0.0124

Notes: Log-likelihood (LL); $\overline{\sigma}(s_t) = (1/T)\sum_{t=1}^{T} \sigma_t(s_t)$ is the regime-dependent mean volatility. Standard errors are reported in parentheses. *, ** and *** indicate parameter significance at the 10%, 5% and 1% levels, respectively.

Table 4(d). Parameter estimates for the best-performing single-regime (SR) and Markov-switching (MS) volatility models

Skew-Gen-t-EGARCH, UK		Skew-Gen-t-EGARCH, US		NIG-EGARCH, US	
c	0.0011***(0.0002)	c	0.0009***(0.0001)	c	0.0010***(0.0001)
ω	−0.0768***(0.0110)	ω	−0.0459***(0.0050)	ω	−0.0410***(0.0046)
β	0.9843***(0.0023)	β	0.9910***(0.0010)	β	0.9912***(0.0010)
α	0.0396***(0.0027)	α	0.0437***(0.0018)	α	0.0436***(0.0018)
α^*	0.0220***(0.0018)	α^*	0.0245***(0.0013)	α^*	0.0238***(0.0013)
λ_0	−4.5854***(0.3108)	λ_0	−5.0305***(0.4878)	λ_0	−4.5606***(0.4689)
δ_1	−0.0712***(0.0129)	δ_1	−0.0556***(0.0073)	δ_1	0.5745***(0.0462)
δ_2	2.1263***(0.1227)	δ_2	2.0007***(0.0946)	δ_2	−0.0741***(0.0094)
δ_3	0.7249***(0.0361)	δ_3	0.5483***(0.0246)		
LL		LL	3.3404	LL	3.3395
MS-Skew-Gen-t-EGARCH, UK		MS-Skew-Gen-t-EGARCH, US		MS-NIG-EGARCH, US	
$c(1)$	0.0008***(0.0002)	$c(1)$	0.0018***(0.0002)	$c(1)$	0.0023***(0.0002)
$c(2)$	0.0018***(0.0003)	$c(2)$	0.0006***(0.0001)	$c(2)$	0.0008***(0.0001)
$\omega(1)$	−0.0569***(0.0117)	$\omega(1)$	−0.3597***(0.0419)	$\omega(1)$	−0.3704***(0.0417)
$\omega(2)$	−0.0804***(0.0218)	$\omega(2)$	−0.0147***(0.0038)	$\omega(2)$	−0.0120***(0.0034)
$\beta(1)$	0.9889***(0.0025)	$\beta(1)$	0.9286***(0.0084)	$\beta(1)$	0.9208***(0.0091)
$\beta(2)$	0.9834***(0.0045)	$\beta(2)$	0.9972***(0.0008)	$\beta(2)$	0.9975***(0.0008)
$\alpha(1)$	0.0302***(0.0036)	$\alpha(1)$	0.0697***(0.0066)	$\alpha(1)$	0.0729***(0.0069)
$\alpha(2)$	0.0253***(0.0042)	$\alpha(2)$	0.0279***(0.0018)	$\alpha(2)$	0.0284***(0.0018)
$\alpha^*(1)$	0.0503***(0.0033)	$\alpha^*(1)$	0.0611***(0.0055)	$\alpha^*(1)$	0.0639***(0.0057)
$\alpha^*(2)$	0.0021(0.0029)	$\alpha^*(2)$	0.0183***(0.0012)	$\alpha^*(2)$	0.0184***(0.0012)
$\lambda_0(1)$	−4.5111***(1.1007)	$\lambda_0(1)$	−5.0828(3.6010)	$\lambda_0(1)$	−4.6444(4.0487)
$\lambda_0(2)$	−4.6558***(1.0875)	$\lambda_0(2)$	−4.9841***(0.5411)	$\lambda_0(2)$	−4.4066***(0.5197)
$\delta_1(1)$	−0.0784***(0.0169)	$\delta_1(1)$	−0.1002***(0.0166)	$\delta_1(1)$	0.2159***(0.0801)
$\delta_1(2)$	−0.0920***(0.0232)	$\delta_1(2)$	−0.0404***(0.0089)	$\delta_1(2)$	0.9264***(0.0672)
$\delta_2(1)$	1.7566***(0.1626)	$\delta_2(1)$	1.7569***(0.1892)	$\delta_2(1)$	−0.1881***(0.0251)
$\delta_2(2)$	2.3268***(0.4207)	$\delta_2(2)$	2.4118***(0.1984)	$\delta_2(2)$	−0.0480***(0.0105)
$\delta_3(1)$	0.7489***(0.0558)	$\delta_3(1)$	0.4547***(0.0514)	p	0.9940***(0.0015)
$\delta_3(2)$	0.7909***(0.0856)	$\delta_3(2)$	0.5772***(0.0361)	q	0.9979***(0.0005)
p	0.9977***(0.0010)	p	0.9951***(0.0012)		
q	0.9977***(0.0009)	q	0.9979***(0.0005)		
LL	3.3016	LL	3.3447	LL	3.3444
$\overline{\sigma}(1)$	0.0097	$\overline{\sigma}(1)$	0.0101	$\overline{\sigma}(1)$	0.0105
$\overline{\sigma}(2)$	0.0093	$\overline{\sigma}(2)$	0.0096	$\overline{\sigma}(2)$	0.0094

Notes: United Kingdom (UK); United States (US); log-likelihood (LL); $\overline{\sigma}(s_t) = (1/T)\sum_{t=1}^{T} \sigma_t(s_t)$ is the regime-dependent mean volatility. Standard errors are reported in parentheses. *** indicates parameter significance at the 1% level.

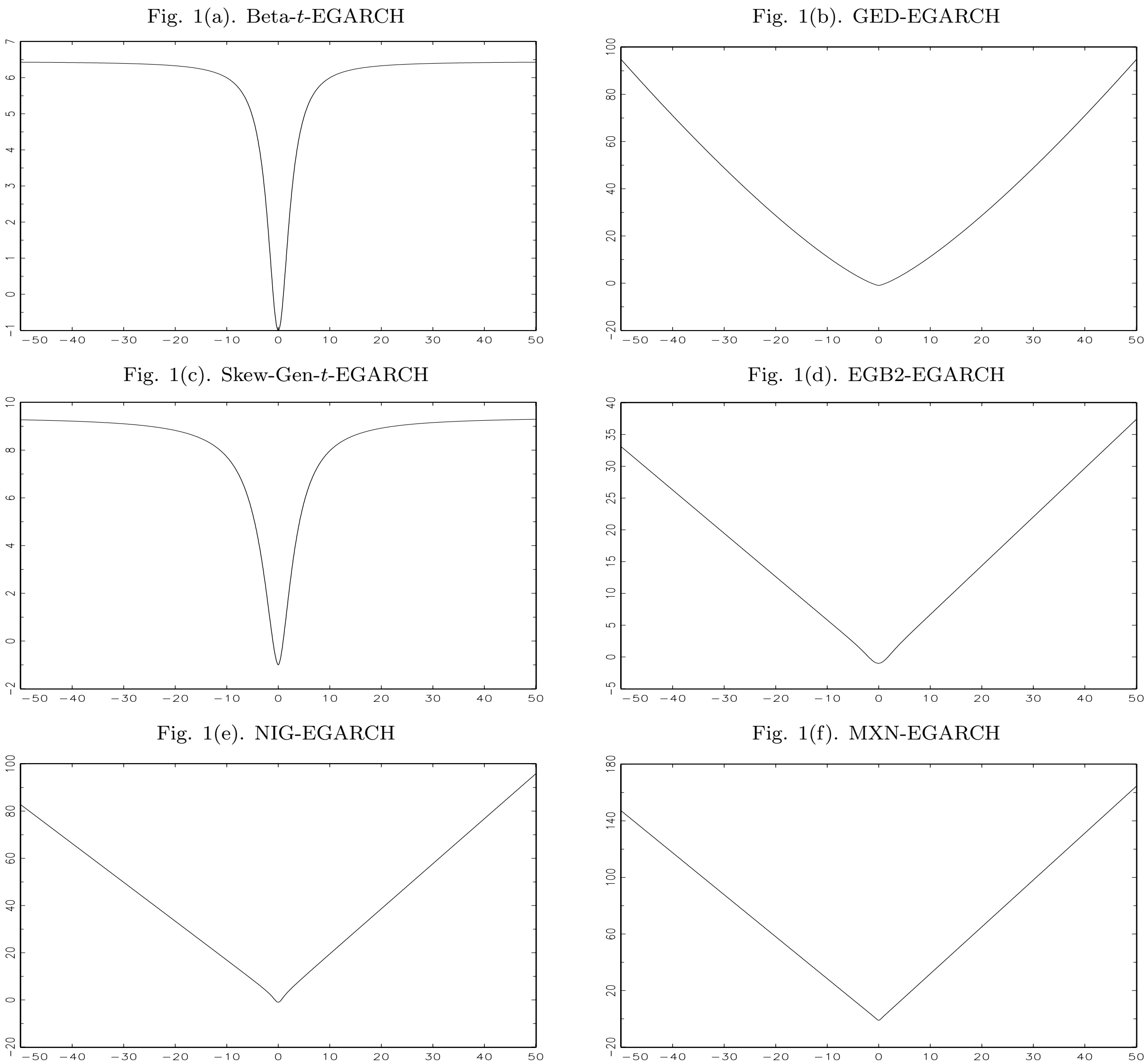

Fig. 1. Score functions as functions of ϵ_t; single-regime DCS-EGARCH estimates are presented for US data

Fig. 2(a). Australia, MS-Skew-Gen-t-EGARCH

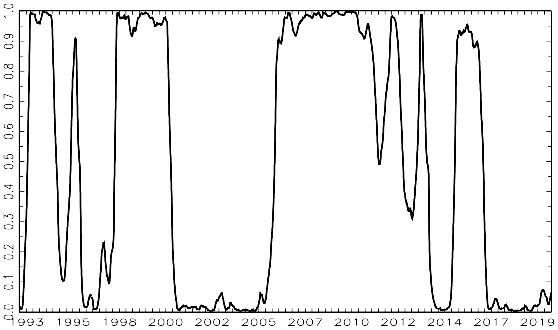

Fig. 2(b). Australia, MS-Skew-Gen-t-EGARCH

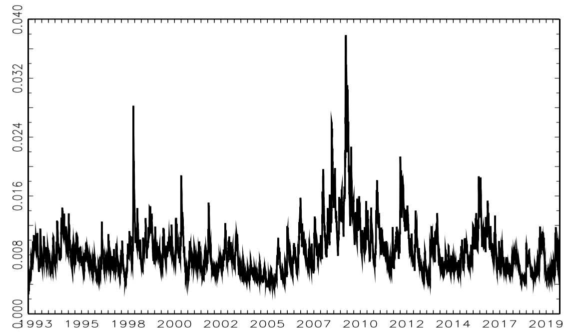

Fig. 2(c). Canada, MS-Skew-Gen-t-EGARCH

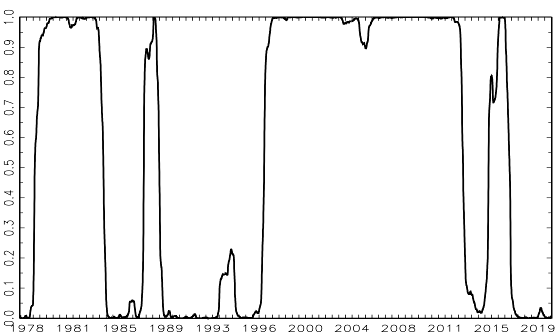

Fig. 2(d). Canada, MS-Skew-Gen-t-EGARCH

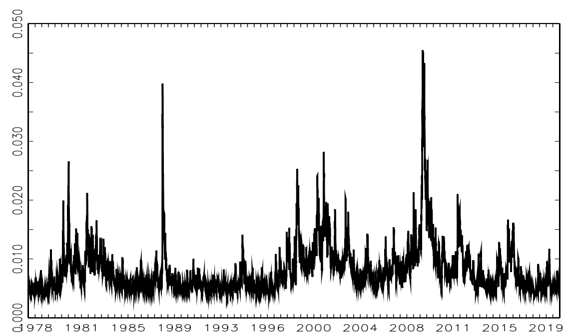

Fig. 2(e). Canada, MS-NIG-EGARCH

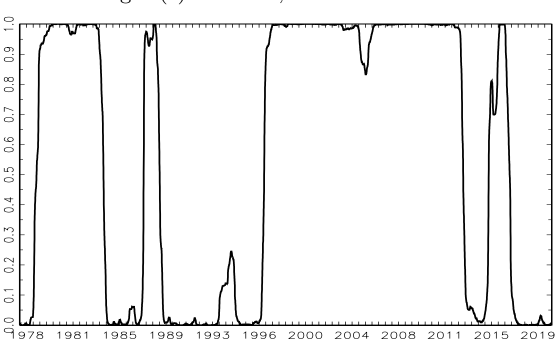

Fig. 2(f). Canada, MS-NIG-EGARCH

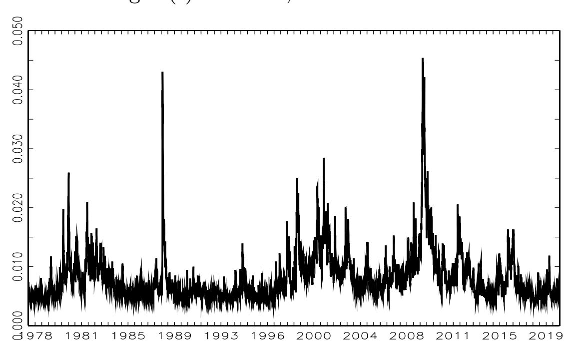

Fig. 2(g). Germany, MS-Skew-Gen-t-EGARCH

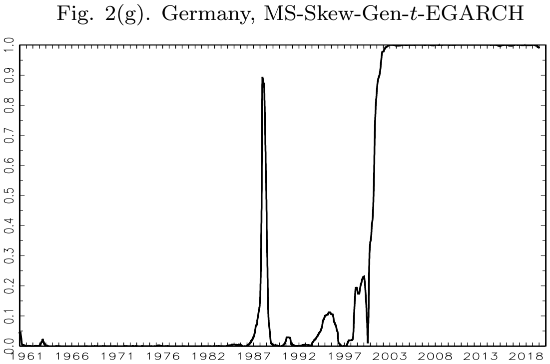

Fig. 2(h). Germany, MS-Skew-Gen-t-EGARCH

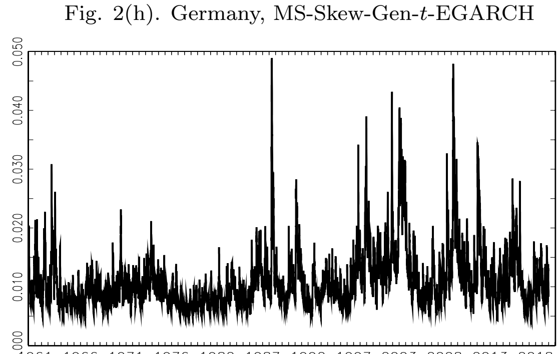

Fig. 2. Smoothed probability of regime $s_t = 1$ (left) and conditional volatility $\pi_t(1)\sigma_t(1) + \pi_t(2)\sigma_t(2)$ (right)

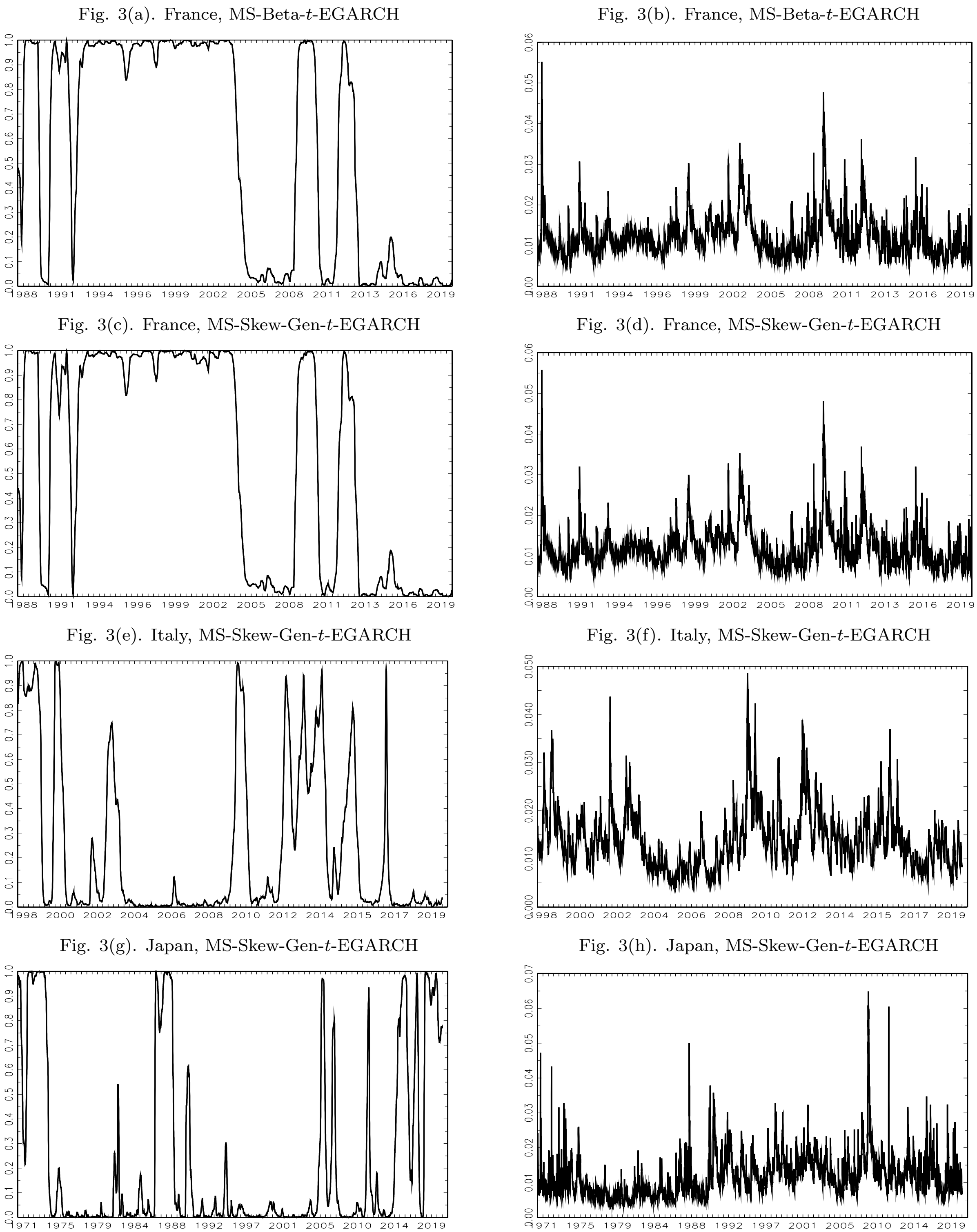

Fig. 3. Smoothed probability of regime $s_t = 1$ (left) and conditional volatility $\pi_t(1)\sigma_t(1) + \pi_t(2)\sigma_t(2)$ (right)

Fig. 4(a). Mexico, MS-Beta-t-EGARCH

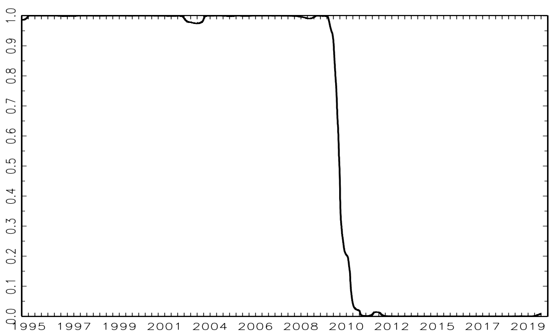

Fig. 4(b). Mexico, MS-Beta-t-EGARCH

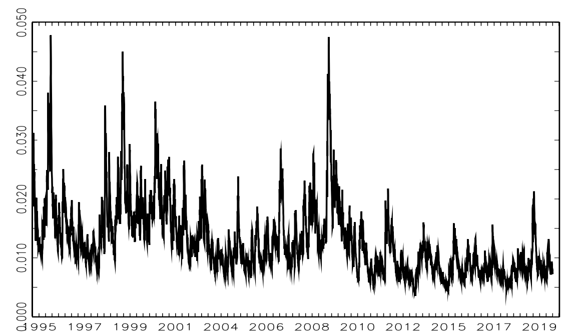

Fig. 4(c). Mexico, MS-Skew-Gen-t-EGARCH

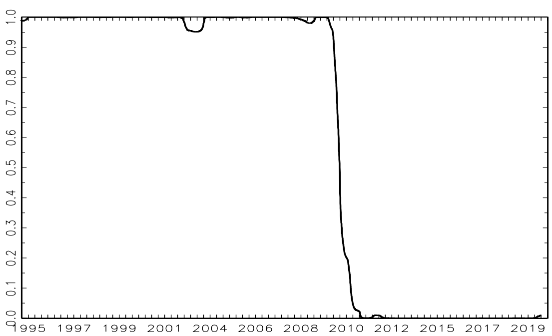

Fig. 4(d). Mexico, MS-Skew-Gen-t-EGARCH

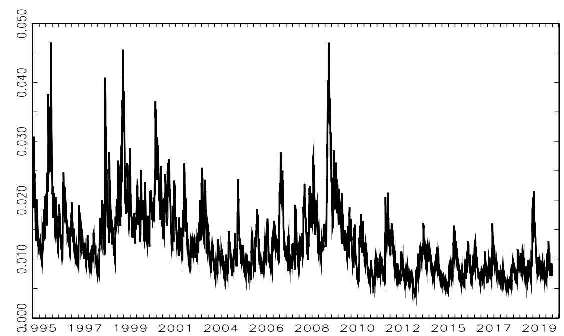

Fig. 4(e). Mexico, MS-NIG-EGARCH

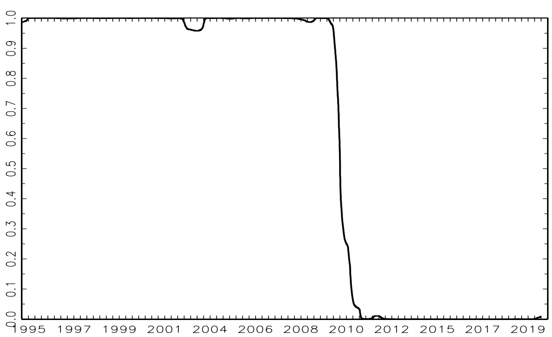

Fig. 4(f). Mexico, MS-NIG-EGARCH

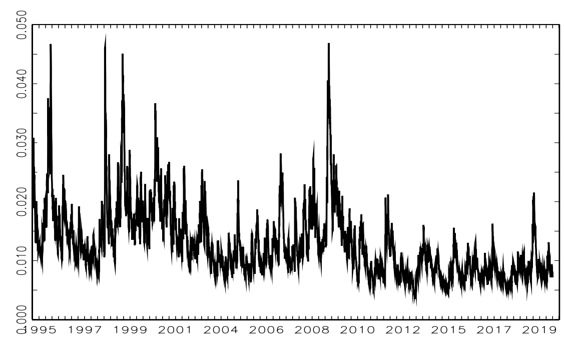

Fig. 4(g). Spain, MS-Skew-Gen-t-EGARCH

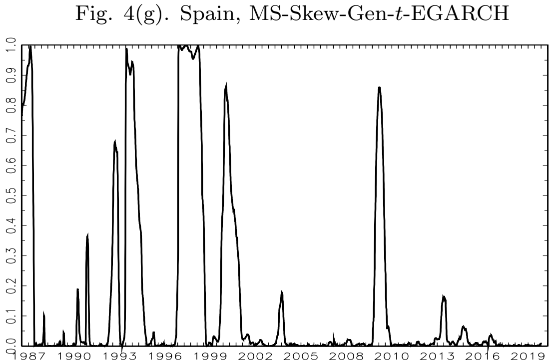

Fig. 4(h). Spain, MS-Skew-Gen-t-EGARCH

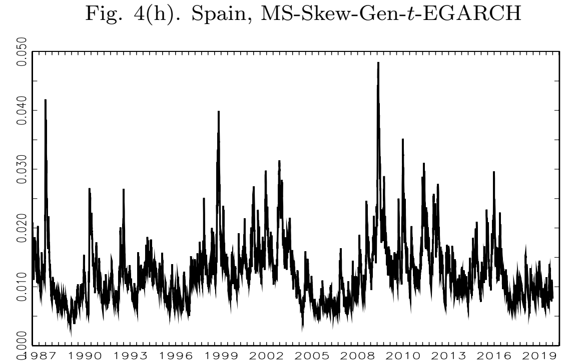

Fig. 4. Smoothed probability of regime $s_t = 1$ (left) and conditional volatility $\pi_t(1)\sigma_t(1) + \pi_t(2)\sigma_t(2)$ (right)

Fig. 5(a). UK, Skew-Gen-t-EGARCH

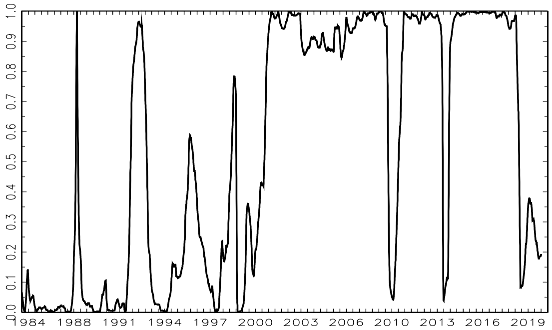

Fig. 5(b). UK, Skew-Gen-t-EGARCH

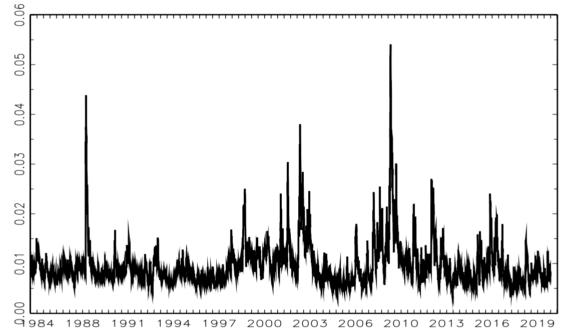

Fig. 5(c). US, Skew-Gen-t-EGARCH

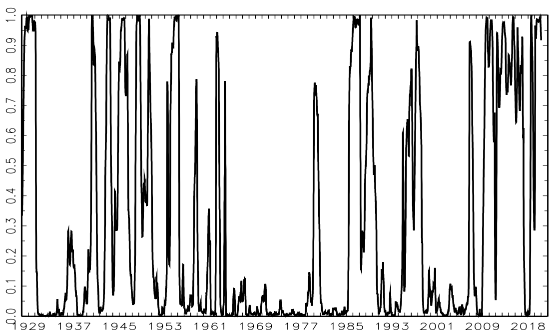

Fig. 5(d). US, Skew-Gen-t-EGARCH

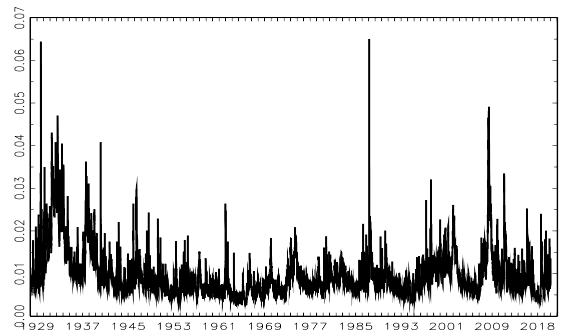

Fig. 5(e). US, NIG-EGARCH

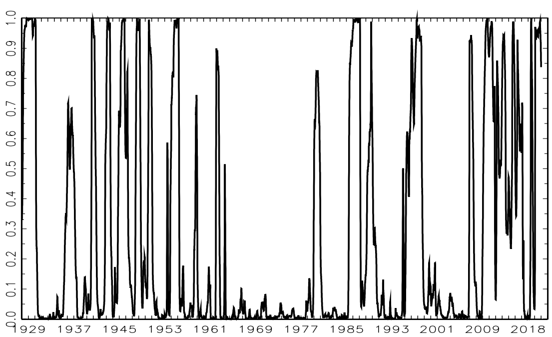

Fig. 5(f). US, NIG-EGARCH

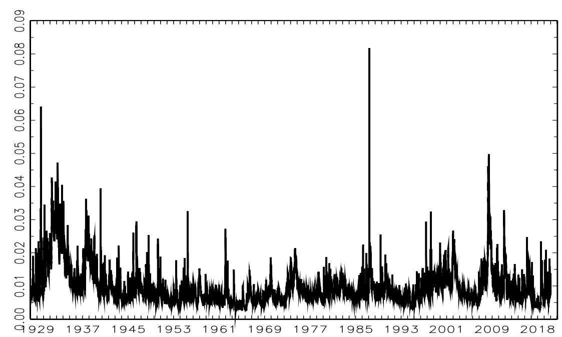

Fig. 5. Smoothed probability of regime $s_t = 1$ (left) and conditional volatility $\pi_t(1)\sigma_t(1) + \pi_t(2)\sigma_t(2)$ (right)