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Score-driven time series models with dynamic shape: An application to the Standard & Poor’s 500 index

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Motivation

The objective of this paper is to suggest new models of **conditional volatility of portfolio returns**.

Conditional volatility is **average gain or loss for the next period** given all the information that is available to the investor.

Gain or loss is measured in **% change** from current value.

This is interesting for investors, because a good forecasting model of volatility can provide estimates of potential gains or losses on the investment. Thus, **volatility is a measure of financial risk**.

Classical risk management metrics of banks, such as **value-at-risk** or **expected shortfall** can be estimated by using volatility models.

Motivation

Models of conditional volatility are very popular in practice, because **volatility** *to some extent* **is predictable**.

In other words, the **risk of an investment** *to some extent* **is predictable** (it is much more predictable than financial return).

To what extent? Well, this, at least partly, depends on the *correct choice of the volatility model*.

The body of literature on **conditional volatility models** is very extensive. *Our objective is to contribute to that literature.*

Portfolio return time series

In all models, we use data on the evolution of return that is obtained on changes in the value of a portfolio p_t, for consecutive time periods $t = 1, \dots, T$.

In practice, two types of returns are used alternatively:

Standard return:

$\tilde{y}_t = (p_t - p_{t-1})/p_{t-1}$ for $t = 1, \dots, T$ days, weeks or months.

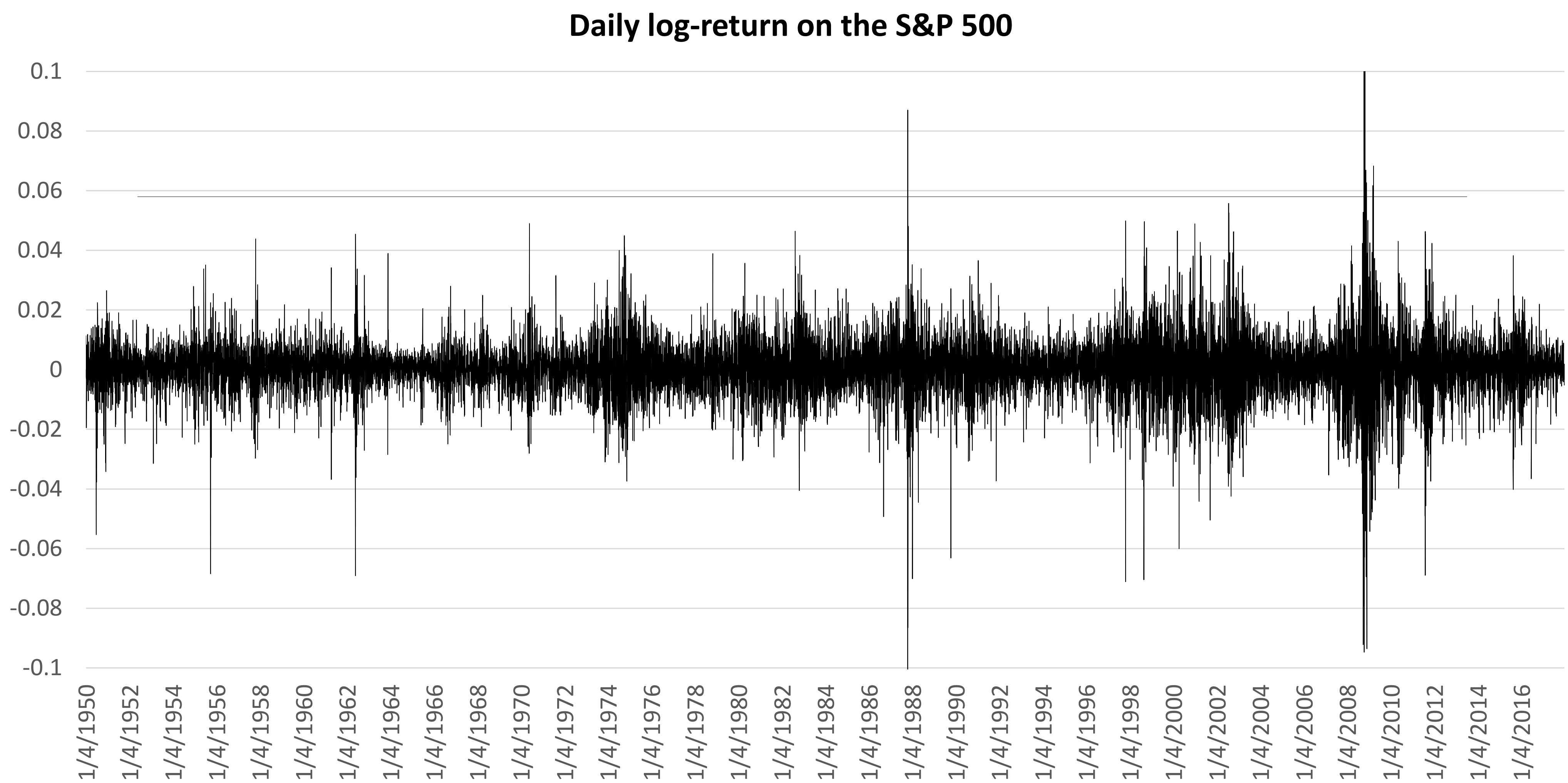
Log-return (we use this in our paper):

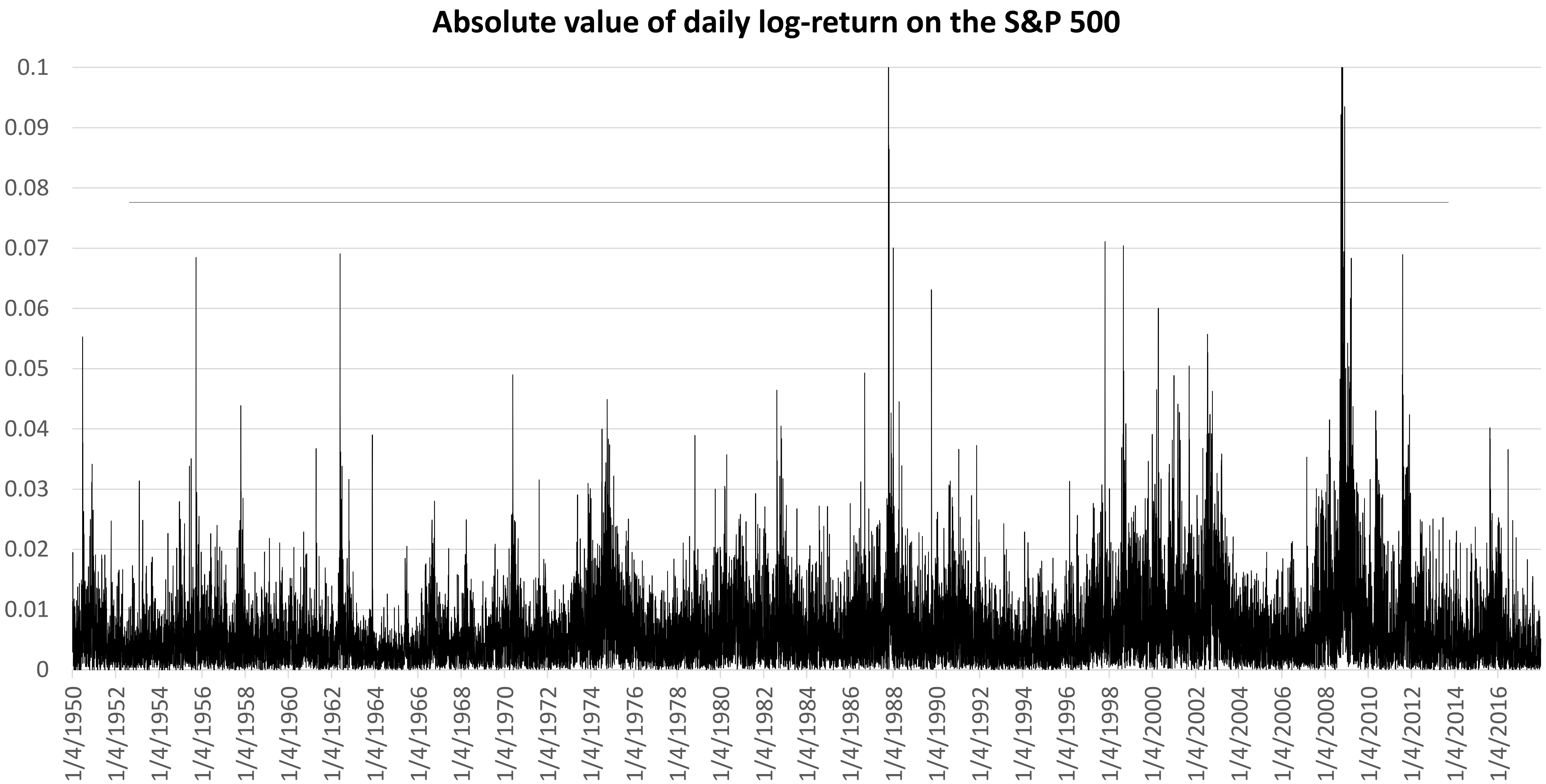
$y_t = \ln(p_t/p_{t-1})$ for $t = 1, \dots, T$ days, weeks or months.

Data

We use daily data on the S&P 500 index for the period of 1950 to 2017 (source: Bloomberg).

Start date	4-Jan-1950
End date	30-Dec-2017
Sample size T	$17,109$
Minimum	-0.2290
Maximum	0.1096
Mean	0.0003
Standard deviation	0.0096
Skewness	-1.0162
Excess kurtosis	27.4010
$\text{Corr}(y_t, y_{t-1})$	0.0269
$\text{Corr}(y_t^2, y_{t-1})$	-0.0877


Daily log-return on the S&P 500
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
1/4/1950
1/4/1952
1/4/1954
1/4/1956
1/4/1958
1/4/1960
1/4/1962
1/4/1964
1/4/1966
1/4/1968
1/4/1970
1/4/1972
1/4/1974
1/4/1976
1/4/1978
1/4/1980
1/4/1982
1/4/1984
1/4/1986
1/4/1988
1/4/1990
1/4/1992
1/4/1994
1/4/1996
1/4/1998
1/4/2000
1/4/2002
1/4/2004
1/4/2006
1/4/2008
1/4/2010
1/4/2012
1/4/2014
1/4/2016


Absolute value of daily log-return on the S&P 500
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
1/4/1950
1/4/1952
1/4/1954
1/4/1956
1/4/1958
1/4/1960
1/4/1962
1/4/1964
1/4/1966
1/4/1968
1/4/1970
1/4/1972
1/4/1974
1/4/1976
1/4/1978
1/4/1980
1/4/1982
1/4/1984
1/4/1986
1/4/1988
1/4/1990
1/4/1992
1/4/1994
1/4/1996
1/4/1998
1/4/2000
1/4/2002
1/4/2004
1/4/2006
1/4/2008
1/4/2010
1/4/2012
1/4/2014
1/4/2016

Data

From the latter figure:

Notice the **predictability** of absolute return.

Notice also the large number of **extreme observations**.

Robert Engle (1982, Econometrica) Nobel Prize in Economics, 2003

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

μ_t can be modeled as a constant parameter, or it can also include past values of y_t or other explanatory variables.

$v_t | (y_1, \ldots, y_{t-1}) \sim N(0, \lambda_t)$

$\lambda_t = \alpha_0 + \alpha_1 v_{t-1}^2 + \cdots + \alpha_q v_{t-q}^2$

This is the **ARCH(q) model** (autoregressive conditional heteroscedasticity, ARCH).

$\sqrt{\lambda_t}$ is the conditional standard deviation of unexpected return, which is also named **conditional volatility**.

ARCH(1) where $\mu_t = c$, denoted "const"

```
Model 1: GARCH, using observations 1950-01-04:2017-12-29 (T = 17109)
Dependent variable: logreturn
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  --------------------------------------------------------
  const      0.000434670   6.62049e-05    6.566   5.19e-011 ***

  alpha(0)   6.35423e-05   9.22820e-07   68.86    0.0000    ***
  alpha(1)   0.321735      0.0146128     22.02    1.96e-107 ***

Mean dependent var   0.000297   S.D. dependent var   0.009645
Log-likelihood       56226.17   Akaike criterion    -112444.3
Schwarz criterion   -112413.3   Hannan-Quinn        -112434.1

Unconditional error variance = 9.36836e-005
Likelihood ratio test for (G)ARCH terms:
  Chi-square(1) = 2188.72 [0]
```

ARCH(2) where $\mu_t = c$

```
Model 2: GARCH, using observations 1950-01-04:2017-12-29 (T = 17109)
Dependent variable: logreturn
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  ---------------------------------------------------------
  const      0.000494118   6.07700e-05    8.131   4.26e-016 ***

  alpha(0)   4.62756e-05   8.33039e-07   55.55    0.0000    ***
  alpha(1)   0.239235      0.0127299     18.79    8.59e-079 ***
  alpha(2)   0.259999      0.0121269     21.44    5.69e-102 ***

Mean dependent var    0.000297   S.D. dependent var   0.009645
Log-likelihood        57034.56   Akaike criterion    -114059.1
Schwarz criterion    -114020.4   Hannan-Quinn        -114046.4

Unconditional error variance = 9.24098e-005
Likelihood ratio test for (G)ARCH terms:
  Chi-square(2) = 3805.51 [0]
```

ARCH(3) where $\mu_t = c$

```
Model 3: GARCH, using observations 1950-01-04:2017-12-29 (T = 17109)
Dependent variable: logreturn
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  --------------------------------------------------------
  const      0.000510695   5.77511e-05    8.843   9.32e-019 ***

  alpha(0)   3.69428e-05   8.11247e-07   45.54    0.0000    ***
  alpha(1)   0.185940      0.0116426     15.97    2.05e-057 ***
  alpha(2)   0.227164      0.0114888     19.77    5.12e-087 ***
  alpha(3)   0.199721      0.0119364     16.73    7.65e-063 ***

Mean dependent var   0.000297   S.D. dependent var   0.009645
Log-likelihood       57388.70   Akaike criterion    -114765.4
Schwarz criterion   -114718.9   Hannan-Quinn        -114750.1

Unconditional error variance = 9.54162e-005
Likelihood ratio test for (G)ARCH terms:
  Chi-square(3) = 4513.77 [0]
```

ARCH(4) where $\mu_t = c$

Model 4: GARCH, using observations 1950-01-04:2017-12-29 (T = 17109)
Dependent variable: logreturn
Standard errors based on Hessian

	coefficient	std. error	z	p-value	
const	0.000550461	5.60966e-05	9.813	9.92e-023	***
alpha(0)	3.01933e-05	7.69738e-07	39.23	0.0000	***
alpha(1)	0.152501	0.0104929	14.53	7.40e-048	***
alpha(2)	0.177079	0.0107689	16.44	9.33e-061	***
alpha(3)	0.177680	0.0111533	15.93	3.88e-057	***
alpha(4)	0.179349	0.0107605	16.67	2.26e-062	***

Mean dependent var	0.000297	S.D. dependent var	0.009645
Log-likelihood	57714.58	Akaike criterion	-115415.2
Schwarz criterion	-115360.9	Hannan-Quinn	-115397.3

Unconditional error variance = 9.63438e-005
Likelihood ratio test for (G)ARCH terms:
  Chi-square(4) = 5165.55 [0]

ARCH – some conclusions

The results for ARCH(1) to ARCH(4) show that the model improves when more lags of v_t^2 are included in the conditional variance equation.

This is based on the **AIC, BIC (Schwarz) and HQC metrics**, which measure *model parsimoniousness* (i.e. model fit to data that is penalized by the number of estimated parameters).

We could continue including more and more lags in order to improve the model, but this would result in a large number of estimated parameters. *This causes imprecision of the estimates*.

A possible solution:
Tim Bollerslev (1986, Journal of Econometrics)

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

μ_t can be modeled as a constant parameter or it can also include past values of y_t or other explanatory variables.

$v_t | (y_1, \ldots, y_{t-1}) \sim N(0, \lambda_t)$

$\lambda_t = \alpha_0 + \alpha_1 v_{t-1}^2 + \cdots + \alpha_q v_{t-q}^2 + \beta_1 \lambda_{t-1} + \cdots + \beta_p \lambda_{t-p}$

This is the **GARCH(p, q) model** (generalized ARCH, GARCH).

$\sqrt{\lambda_t}$ is the conditional standard deviation of unexpected return, which is also named conditional volatility.

GARCH(1,1)

In the literature, several papers show that the volatility forecast performance of GARCH(1,1) is very difficult to beat with other volatility models.

For example, Lunde and Hansen (2005, Journal of Applied Econometrics): *"A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)?"*

GARCH(1,1)

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

μ_t can be modeled as a constant parameter or it can also include past values of y_t or other explanatory variables.

$v_t | (y_1, \dots, y_{t-1}) \sim N(0, \lambda_t)$

$\lambda_t = \alpha_0 + \alpha_1 v_{t-1}^2 + \beta_1 \lambda_{t-1}$

This is the **GARCH(1, 1) model**.

$\sqrt{\lambda_t}$ is the conditional standard deviation of unexpected return, which is also named conditional volatility.

GARCH(1,1) where $\mu_t = c$

```
Model 6: GARCH, using observations 1950-01-04:2017-12-29 (T = 17109)
Dependent variable: logreturn
Standard errors based on Hessian

             coefficient   std. error      z       p-value
  ---------------------------------------------------------
  const      0.000486014   5.33283e-05    9.114   7.97e-020 ***

  alpha(0)   9.00512e-07   1.06911e-07    8.423   3.67e-017 ***
  alpha(1)   0.0841087     0.00447230    18.81    6.67e-079 ***
  beta(1)    0.907942      0.00490421   185.1     0.0000    ***

Mean dependent var   0.000297   S.D. dependent var   0.009645
Log-likelihood       58370.12   Akaike criterion    -116730.2
Schwarz criterion   -116691.5   Hannan-Quinn        -116717.5

Unconditional error variance = 0.000113276
Likelihood ratio test for (G)ARCH terms:
  Chi-square(2) = 6476.62 [0]
```

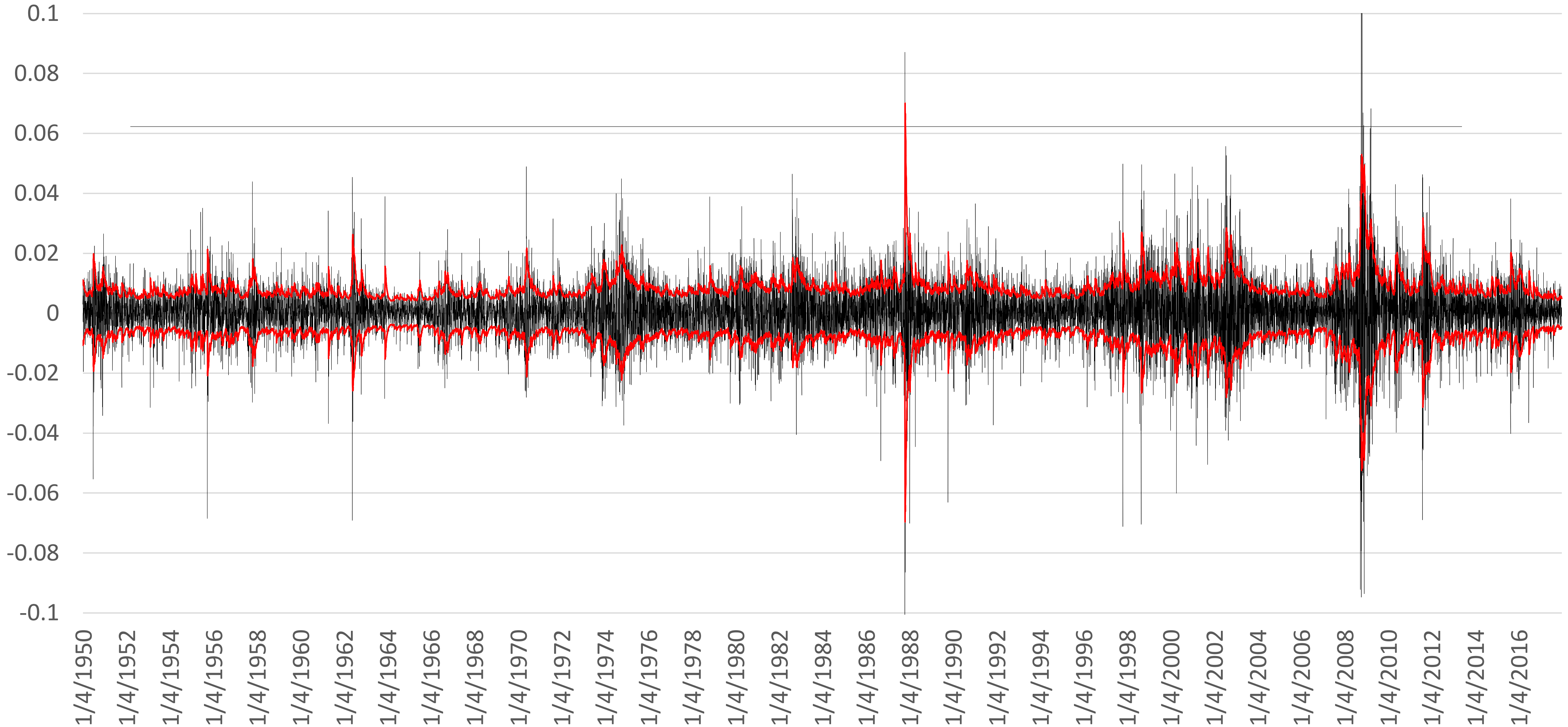
GARCH(1,1) – some conclusions

Notice that AIC, BIC (Schwarz) and HQC metrics improve with respect to all ARCH estimates (i.e. ARCH(1) to ARCH(4)).

The annualized expected return estimate is 252 x 0.000486=12.25%

We present the evolution of GARCH(1,1) conditional volatility of S&P 500 returns:

S&P 500 returns; expected return +/- conditional volatility



#1 Extensions of ARCH and GARCH

Bollerslev (1987, Review of Economics and Statistics):

t-GARCH model.

The error term has the Student's t-distribution.

The degrees of freedom parameter of the Student's t-distribution is jointly estimated with other parameters.

In this way the model is robust to extreme observations.

#1 Extensions of ARCH and GARCH

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

μ_t can be modeled as a constant parameter or it can also include past values of y_t or other explanatory variables.

$v_t|(y_1, \ldots, y_{t-1}) \sim t(0, \lambda_t, \nu)$

$\lambda_t = \omega + \alpha_1 v_{t-1}^2 + \beta_1 \lambda_{t-1}$

This is the **t-GARCH(1, 1) model**.

$\sqrt{\lambda_t \nu/(\nu-2)}$ is the conditional standard deviation of unexpected return, which is also named conditional volatility.

#1 Extensions of ARCH and GARCH

Bollerslev (1987, Review of Economics and Statistics):

t-GARCH model.

In this model two parameters influence volatility:

(1) The time-varying λ_t (higher λ_t → higher volatility).

(2) The constant degrees of freedom parameter ν (lower ν → higher volatility).

Changes in ν control extreme risk!

t-GARCH(1,1) where $\mu_t = c$

```
Model: GARCH(1,1) [Bollerslev] (Student's t)*
Dependent variable: logreturn
Sample: 1950-01-04 -- 2017-12-29 (T = 17109), VCV method: Robust

    Conditional mean equation

               coefficient   std. error      z      p-value
  ---------------------------------------------------------
  const      0.000573496   4.99622e-05   11.48   1.69e-030 ***

    Conditional variance equation

               coefficient   std. error       z       p-value
  -----------------------------------------------------------
  omega      6.37545e-07   1.06765e-07     5.971   2.35e-09  ***
  alpha      0.0771649     0.00608374     12.68    7.27e-037 ***
  beta       0.917896      0.00624018    147.1     0.0000    ***

    Conditional density parameters

               coefficient   std. error     z      p-value
  --------------------------------------------------------
  ni           6.56456      0.356946    18.39   1.55e-075 ***

Llik:  58861.80651 AIC: -117713.61302
BIC:  -117674.87622 HQC: -117700.84306
```

t-GARCH(1,1) – some conclusions

Notice that model performance improves with respect to GARCH(1,1), based on the AIC, BIC (Schwarz) and HQC metrics.

The degrees of freedom parameter estimate is **6.56**.

N(0,1) is a special case of the Student's t distribution for infinite degrees of freedom. In practice, if the degrees of freedom is higher than 30, then the two distributions coincide.

It is much lower than 30 for our degrees of freedom estimate (hence, the improved model performance).

#2a Extensions of ARCH and GARCH

Glosten, Jagannathan and Runkle (1993, Journal of Finance):
GARCH with leverage effects. For GARCH(1,1):

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

$v_t|(y_1, \dots, y_{t-1}) \sim N(0, \lambda_t)$

$\lambda_t = \omega + \alpha_1 v_{t-1}^2 + \beta_1 \lambda_{t-1} + \gamma_1 1(v_{t-1} < 0) v_{t-1}^2$

where $1(\cdot)$ is the indicator function (i.e. it takes the value 1 if the argument is true). This model introduces **asymmetry** into λ_t.

#2b Extensions of ARCH and GARCH

t-GARCH with leverage effects. For GARCH(1,1):

$y_t = \mu_t + v_t$ (**expected return** μ_t plus **unexpected return** v_t)

$v_t | (y_1, \ldots, y_{t-1}) \sim t(0, \lambda_t, \nu)$

$\lambda_t = \omega + \alpha_1 v_{t-1}^2 + \beta_1 \lambda_{t-1} + \gamma_1 1(v_{t-1} < 0) v_{t-1}^2$

where $1(\cdot)$ is the indicator function (i.e. it takes the value 1 if the argument is true).

This model also uses **asymmetry** in λ_t.

t-GARCH(1,1) with leverage effects

```
Model: GJR(1,1) [Glosten et al.] (Student's t)*
Dependent variable: logreturn
Sample: 1950-01-04 -- 2017-12-29 (T = 17109), VCV method: Robust

    Conditional mean equation

             coefficient    std. error       z       p-value
  ------------------------------------------------------------
  const       0.000462197   4.95625e-05   9.326   1.10e-020 ***

             coefficient    std. error       z        p-value
  -------------------------------------------------------------
  delta       8.08678e-07   1.27595e-07     6.338   2.33e-010 ***
  alpha       0.0261149     0.00379293      6.885   5.77e-012 ***
  gamma       0.0993859     0.00963896     10.31    6.29e-025 ***
  beta        0.914697      0.00665306    137.5     0.0000    ***

    Conditional density parameters

             coefficient    std. error     z       p-value
  ----------------------------------------------------------
  ni            6.98157      0.411971    16.95   2.03e-064 ***

Llik:  58979.90625 AIC: -117947.81250
BIC:  -117901.32834 HQC: -117932.48855
```

t-GARCH(1,1) with leverage effects – some conclusions

Notice that the model performance improves with respect to t-GARCH(1,1), based on the AIC, BIC (Schwarz) and HQC metrics.

The leverage effects parameter is significantly different from zero (hence, the improvement).

The leverage effects parameter is **positive**. This means that if we have a negative unexpected return on the previous day, then the risk of the portfolio increases. Explanation: *If the value of the equity falls, then the debt to equity ratio (i.e. leverage) of the firm increases. Hence, the risk of the shareholders increases.*

#3 Extensions of ARCH and GARCH

Harvey, Ruiz and Shephard (1994, The Review of Economic Studies): **stochastic volatility model**.

$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\varepsilon_t$ where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ i.i.d.

$\lambda_t = \omega + \beta_1 \lambda_{t-1} + \alpha_1 u_t$ where $u_t \sim N(0, \sigma_u^2)$ i.i.d.

This model is estimated by using the Kalman filter (Kalman 1960).

More recent stochastic volatility models include leverage effects, and consider other distributions for the error term than the normal distribution (more complicated, simulation-based estimation).

Some conclusions from previous models

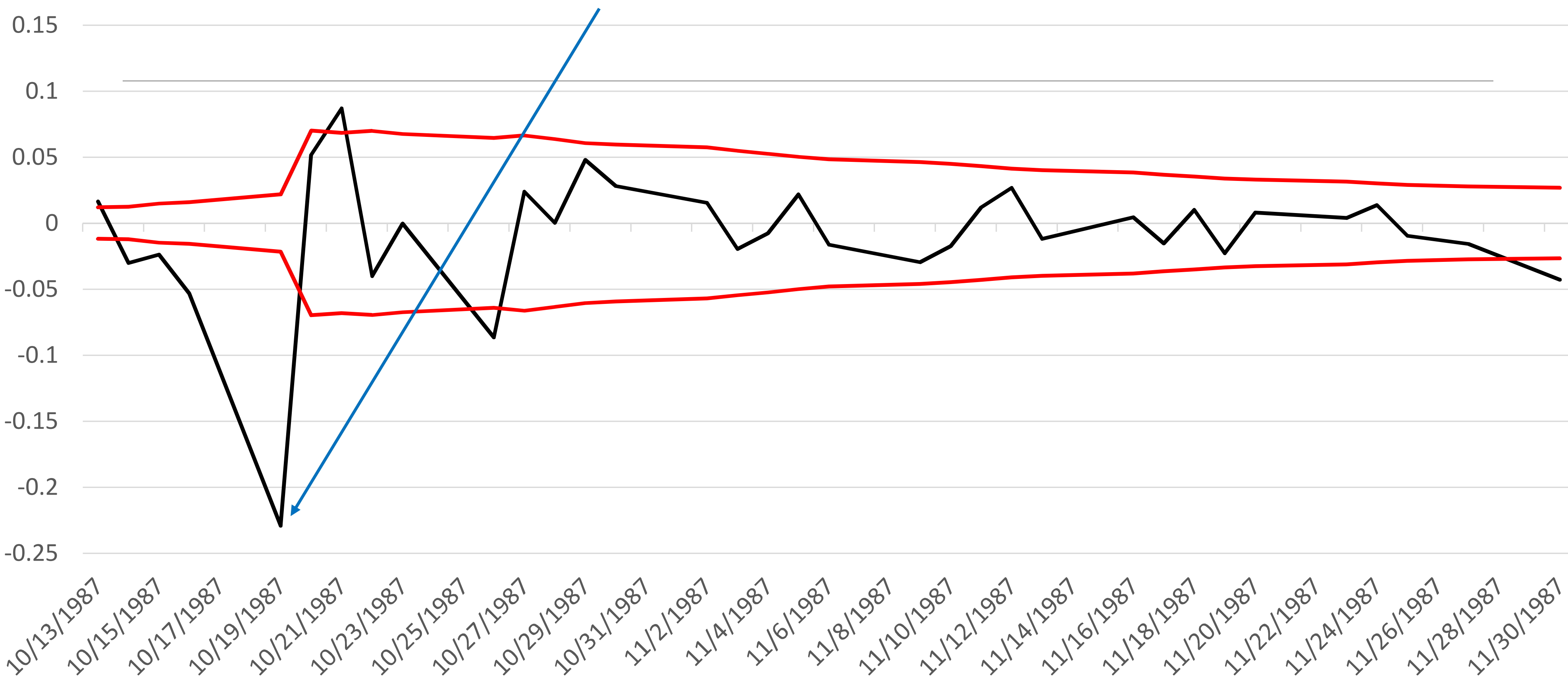
The previous extensions are effective, because they are robust to extreme observations (i.e. Student's t-distribution) and they also incorporate the possibility of asymmetries in volatility.

Nevertheless, for ARCH and GARCH models, volatility is updated by lags of the squared unexpected return, i.e. $v_{t-1}^2, \dots, v_1^2$.

This means that if in the previous period we have a large price fall (e.g. a stock market crash), then volatility may be overestimated for the days following the crash.

See the following estimate for GARCH(1,1):

13rd October 1987 to 30th November 1987
Black Monday: 19th October 1987



Conclusions from the figure

Clearly, the GARCH(1,1) model overestimates volatility for the period after the stock market crash:

Volatility is **average** price change, but the figure shows that the mean +/- volatility lines clearly *bound* the realized returns on the S&P 500.

This is due to the property of GARCH that is uses squared unexpected returns to update the variance equation.

Why not to use an alternative function to the square function?

#4 Extensions of ARCH and GARCH

Harvey and Chakravarty (2008, University of Cambridge WP), **Beta-t-EGARCH model**:

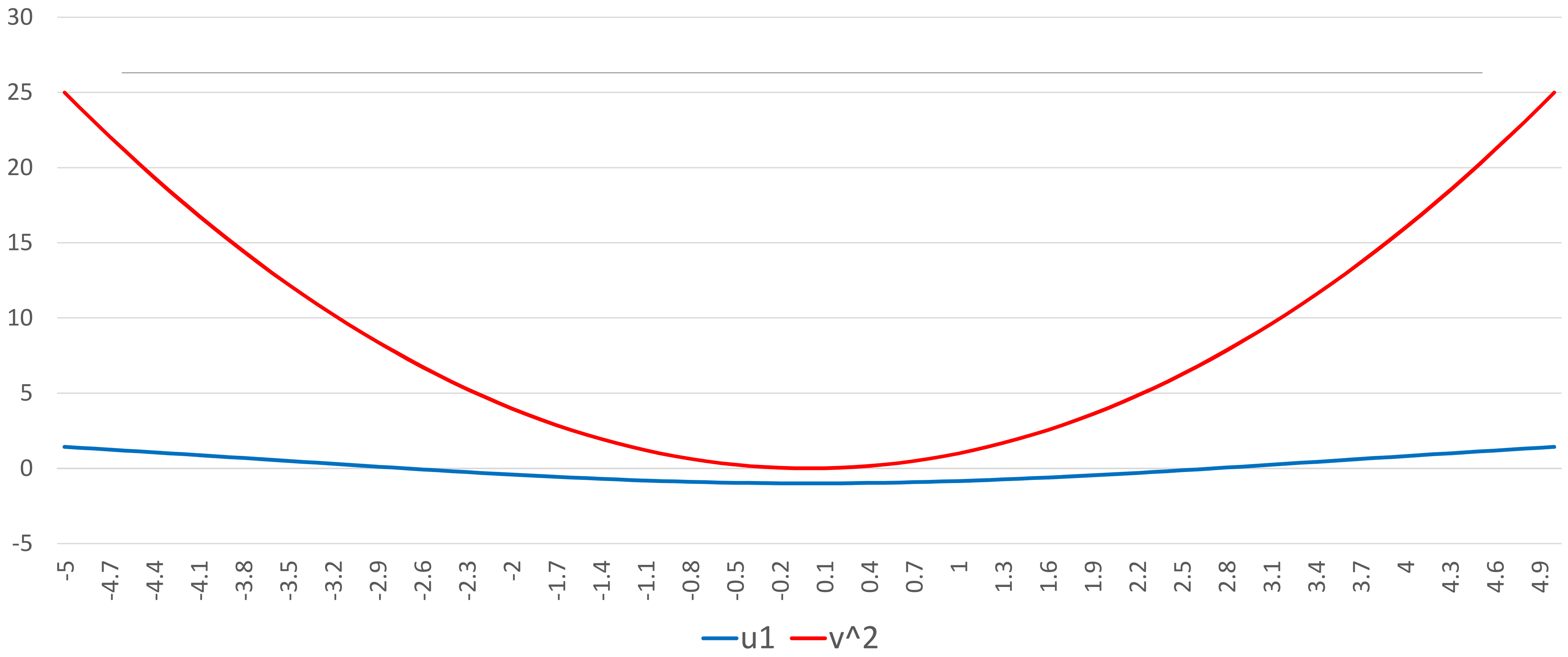
$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\varepsilon_t$ where $\varepsilon_t \sim t(\nu)$ i.i.d.

$\lambda_t = \omega + \beta_1 \lambda_{t-1} + \alpha_1 u_{t-1}$ (leverage effects can be included)

$$u_t = \frac{(\nu+1)v_t^2}{\nu \exp(2\lambda_t) + v_t^2} - 1$$

Compared to GARCH(1,1), the v_{t-1}^2 updating term *is replaced by* u_{t-1}, which is a nonlinear transformation of v_{t-1}^2.

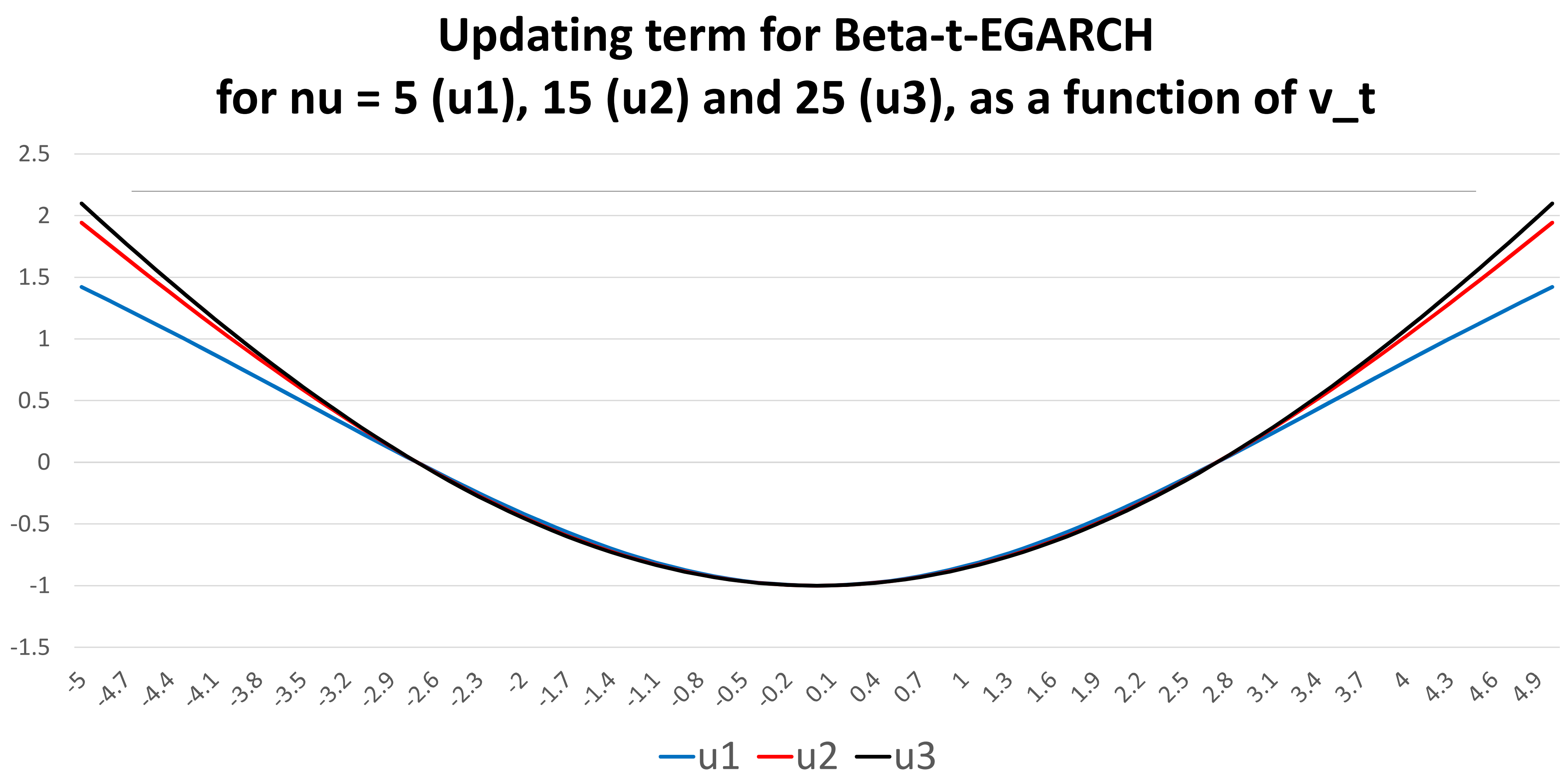
Comparison of updating terms for GARCH (red), and Beta-t-EGARCH (blue) for nu=5, as a function of v_t



Some conclusions on Beta-t-EGARCH

The updating term for Beta-t-EGARCH discounts extreme observations when volatility is updated.

The degree of discounting is estimated for the data. It is directly related to the degrees of freedom parameter. If the degrees of freedom parameter goes to infinity then the u_t converges to a quadratic function of v_t, i.e. the GARCH-type update.


Updating term for Beta-t-EGARCH
for nu = 5 (u1), 15 (u2) and 25 (u3), as a function of v_t
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-5
-4.7
-4.4
-4.1
-3.8
-3.5
-3.2
-2.9
-2.6
-2.3
-2
-1.7
-1.4
-1.1
-0.8
-0.5
-0.2
0.1
0.4
0.7
1
1.3
1.6
1.9
2.2
2.5
2.8
3.1
3.4
3.7
4
4.3
4.6
4.9
u1
u2
u3

Several studies show that Beta-t-EGARCH is superior to GARCH:

Blazsek and Villatoro (2015, Applied Economics)

Blazsek, Chavez and Mendez (2016, Applied Economics Letters)

Blazsek and Mendoza (2016, Applied Economics)

Blazsek and Monteros (2017a, Applied Economics)

Blazsek, Carrizo, Eskildsen and Gonzalez (2018, Finance Research Letters)

Blazsek and Hernandez (2018, Empirical Economics)

Blazsek and Licht (2018, Financial Statistical Journal)

Ayala and Blazsek (2018a and 2019, both in Applied Economics)

Extensions of Beta-t-EGARCH

#1 Use other distributions for the error term (other than the Student's t-distribution):

Ayala and Blazsek (2018b Applied Economics; 2018c The European Journal of Finance)

#2 Use Markov-switching (MS) dynamics for model parameters:

Blazsek and Ho (2017, Applied Economics): MS-Beta-t-EGARCH

Blazsek, Ho and Liu (2018, Applied Economics): MS-EGARCH for Skew-Gen-t, NIG and EGB2 distributions.

Extensions of Beta-t-EGARCH

#3 Use time-varying shape parameters in addition to the time-varying scale parameters:

Blazsek and Monteros (2017b, Applied Economics): Beta-t-EGARCH with time-varying degrees of freedom parameter.

Ayala, Blazsek and Escribano (**2016**, **the present paper**):

Skew-Gen-t-EGARCH, NIG-EGARCH and EGB2-EGARCH with time-varying shape parameters.

Beta-t-EGARCH with time-varying degrees of freedom; Blazsek and Monteros (2017b)

$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\varepsilon_t$ where $\varepsilon_t \sim t(\nu_t)$

$\lambda_t = \omega + \beta_1 \lambda_{t-1} + \alpha_1 u_{\lambda,t-1}$

$\nu_t = \delta + \rho \nu_{t-1} + \kappa u_{\nu,t-1}$

$$u_{\lambda,t} = \frac{\partial \ln f(y_t | y_1, \ldots, y_{t-1})}{\partial \lambda_t} = \frac{[\exp(\nu_t) + 3]\varepsilon_t^2}{\exp(\nu_t) + 2 + \varepsilon_t^2} - 1$$

$$u_{\nu,t} = \frac{\exp(\nu_t)}{2} \Psi^{(0)} \left[\frac{\exp(\nu_t)+3}{2}\right] - \frac{\exp(\nu_t)}{2} \Psi^{(0)} \left[\frac{\exp(\nu_t)+2}{2}\right] - \frac{\exp(\nu_t)}{2\exp(\nu_t)+4} + \frac{\exp(\nu_t)[\exp(\nu_t)+3]\varepsilon_t^2}{2[\exp(\nu_t)+2]\left[\varepsilon_t^2+\exp(\nu_t)+2\right]} - \frac{\exp(\nu_t)}{2} \times \ln\left[1 + \frac{\varepsilon_t^2}{\exp(\nu_t)+2}\right]$$

Ayala, Blazsek and Escribano (2019) EGARCH with several dynamic shape parameters

$$y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\epsilon_t$$

$$\mu_t = c + \phi\mu_{t-1} + \theta u_{\mu,t-1}$$

$$\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$$

$$\rho_{k,t} = \delta_k + \gamma_k \rho_{k,t-1} + \kappa_k u_{\rho,k,t-1}$$

The latter equation is for the *k*-th shape parameter of ε_t.

Ayala, Blazsek and Escribano (2019)

Alternatives for the error term:

EGB2 (exponential generalized beta of the second kind):

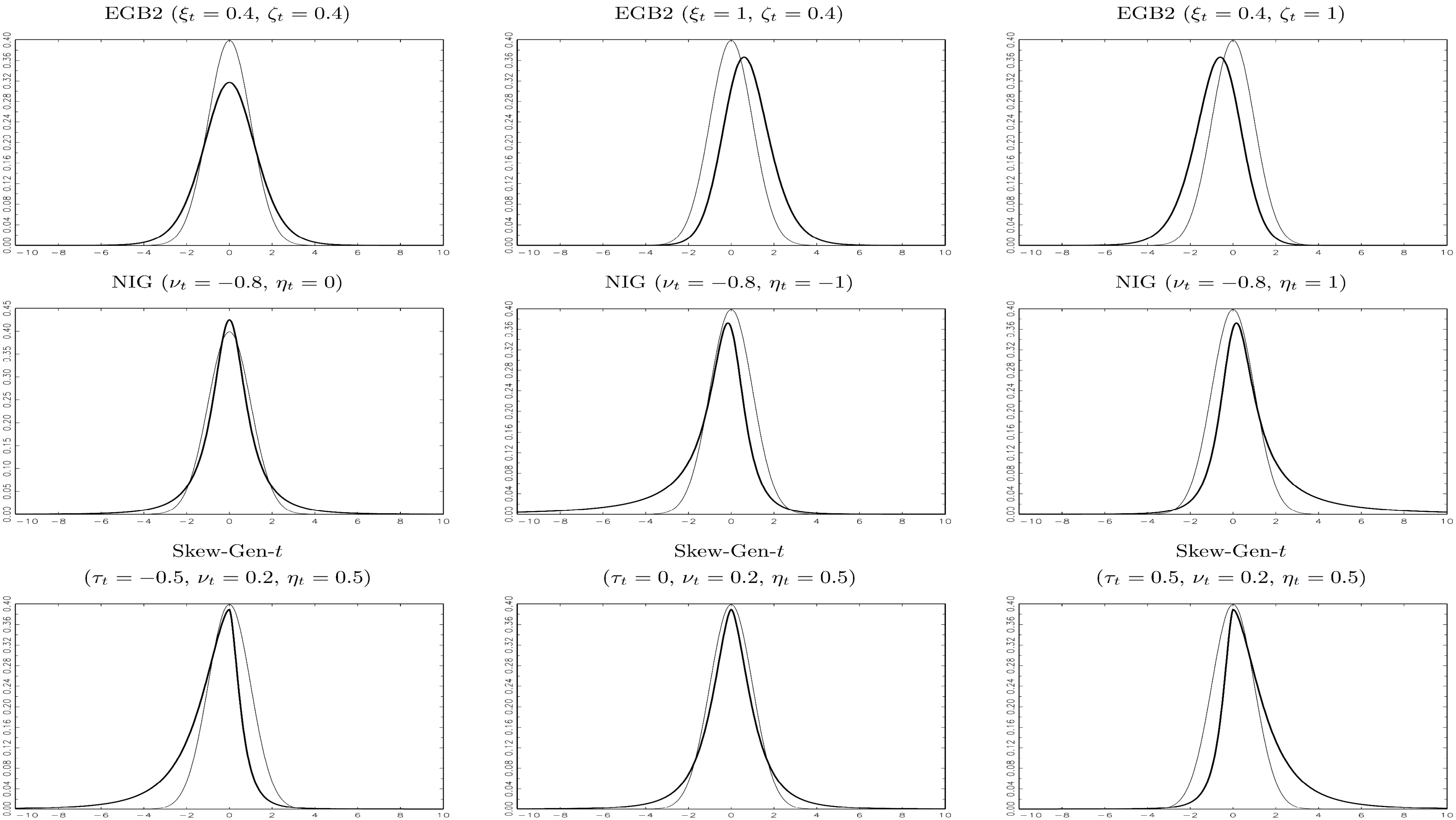
2 shape parameters (skewness, tail thickness)

NIG (normal-inverse Gaussian):

2 shape parameters (skewness, tail thickness)

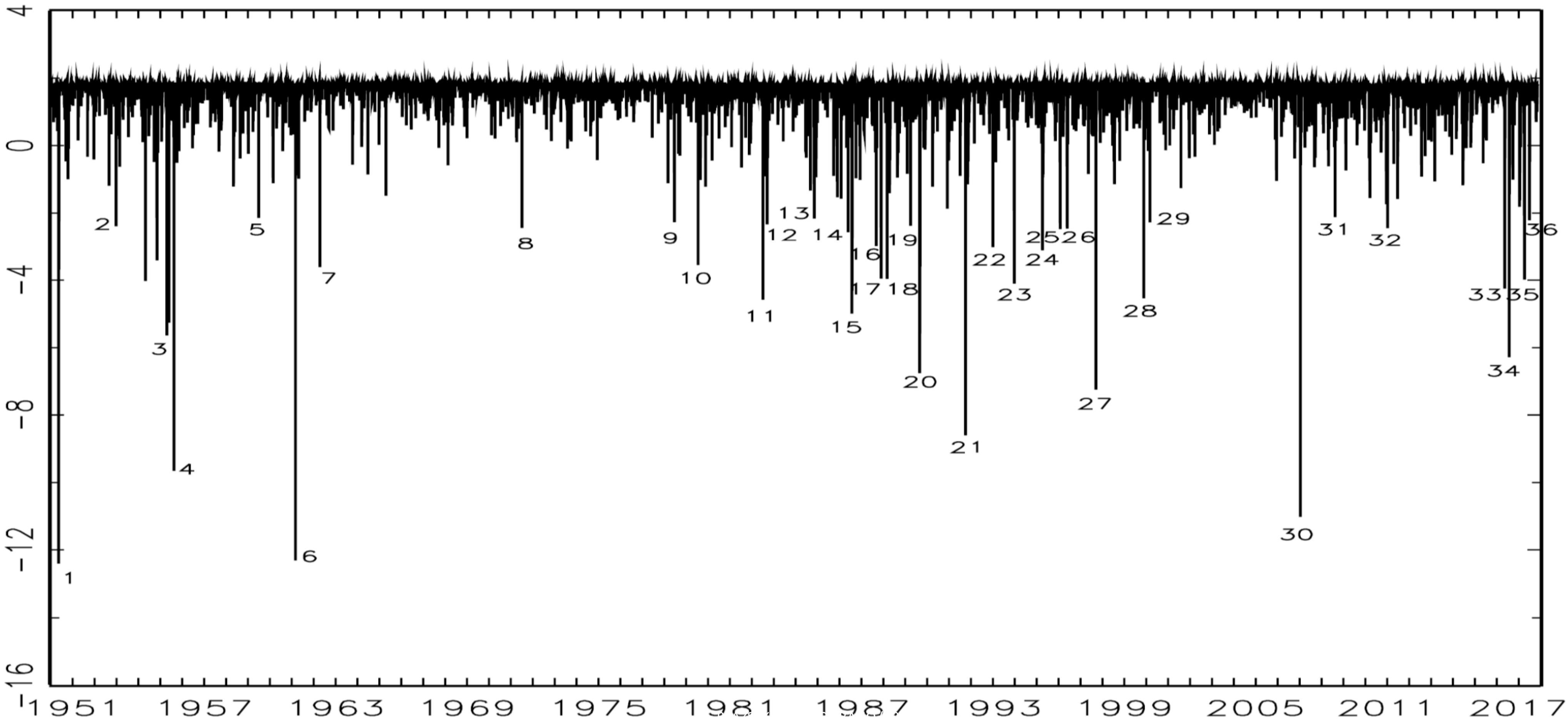
Skew-Gen-t (skewed generalized t):

3 shape parameters (skewness, peakedness, tail thickness)


EGB2 ($\xi_t = 0.4$, $\zeta_t = 0.4$)
EGB2 ($\xi_t = 1$, $\zeta_t = 0.4$)
EGB2 ($\xi_t = 0.4$, $\zeta_t = 1$)
NIG ($\nu_t = -0.8$, $\eta_t = 0$)
NIG ($\nu_t = -0.8$, $\eta_t = -1$)
NIG ($\nu_t = -0.8$, $\eta_t = 1$)
Skew-Gen-t ($\tau_t = -0.5$, $\nu_t = 0.2$, $\eta_t = 0.5$)
Skew-Gen-t ($\tau_t = 0$, $\nu_t = 0.2$, $\eta_t = 0.5$)
Skew-Gen-t ($\tau_t = 0.5$, $\nu_t = 0.2$, $\eta_t = 0.5$)

EGB2-DCS	AIC	AIC rank	BIC	BIC rank	HQC	HQC rank
Constant ξ_t and ζ_t	−6.9050	16	−6.9004	14	−6.9004	16
Dynamic ξ_t and ζ_t	−6.9072	9	−6.9009	11	−6.9051	9
Dynamic ξ_t and constant ζ_t	−6.9060	12	−6.9005	12	−6.9042	13
Constant ξ_t and dynamic ζ_t	−6.9055	15	−6.9000	16	−6.9037	15
NIG-DCS	**AIC**	**AIC rank**	**BIC**	**BIC rank**	**HQC**	**HQC rank**
Constant ν_t and η_t	**−6.9059**	13	**−6.9013**	10	**−6.9044**	12
Dynamic ν_t and η_t	**−6.9068**	10	**−6.9005**	13	**−6.9047**	11
Dynamic ν_t and constant η_t	**−6.9068**	11	**−6.9013**	9	**−6.9050**	10
Constant ν_t and dynamic η_t	**−6.9056**	14	**−6.9002**	15	**−6.9038**	14
Skew-Gen-*t*-DCS	**AIC**	**AIC rank**	**BIC**	**BIC rank**	**HQC**	**HQC rank**
Constant τ_t, ν_t and η_t	−6.9080	8	−6.9030	4	−6.9064	7
Dynamic τ_t, ν_t and η_t	−6.9099	1	−6.9022	7	−6.9074	6
Dynamic τ_t, ν_t and constant η_t	**−6.9097**	4	**−6.9029**	6	**−6.9075**	5
Dynamic τ_t, constant ν_t and dynamic η_t	−6.9099	2	−6.9031	3	−6.9076	3
Dynamic τ_t and constant ν_t, η_t	−6.9080	7	−6.9021	8	−6.9061	8
Constant τ_t and dynamic ν_t, η_t	**−6.9097**	3	**−6.9029**	5	**−6.9075**	4
Constant τ_t, dynamic ν_t and constant η_t	**−6.9096**	6	**−6.9037**	2	**−6.9077**	2
Constant τ_t, ν_t and dynamic η_t	**−6.9097**	5	**−6.9038**	1	**−6.9077**	1

Time-varying parameter that drives the degrees of freedom parameter for the Skew-Gen-t-EGARCH model. ☺



Thank you for your attention.